Existence, Uniqueness for Stochastic forest Evolution

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***Abstract*—This paper introduces random perturbations into the established system of stochastic forest evolution, and studies the system of stochastic forest evolution in Hilbert space, at the same time, using Kolmogorov's inequality and Burkholder-Davis-Gundy's inequality, analyzes the existence uniqueness of system of stochastic forest evolution.**

Key words-Existence; uniqueness; stochastic forest evolution system; $It\hat{o}$ equation

I. INTRODUCTION

There has been much recent interest in application of deterministic age-dependent mathematical models in population dynamics. Population system are often subject to environment noise[1,2].For example, Cushing[3], Henson and Cushing[4] investigate hierarchical age-dependent populations with intra-specific competition or predation. Allen and Thrasher[5] consider vaccination strategies in age-dependent populations. In addition, Pollard[8],Block and Allen[9]study the effects of adding stochastic terms to discrete-time age-dependent models that employ Leslie matrices.

Consider the following forest evolution system:

$$\begin{cases} \dfrac{\partial p}{\partial a}+\dfrac{\partial p}{\partial t}=-\mu(t,a)P+f(t,p), in Q=(0,A)\times(0,T), \\ P(0,a)=P_0(a), \qquad in[0,A], \\ P(t,0)=\gamma(t)\beta(t)\int_0^A \beta(t,a)P(t,a)da, \quad in[0,T], \end{cases} \tag{1}$$

where $P(t,a)$ is the age-area distribution density of forest. $\beta(t)$ is the ratio of reforested area to cut area $\gamma(t)$ is the reforestation percentage. By $\mu(t,a)$ is denoted the cut ratio. $f(t,p)$ denotes affects external environment for system, it is a reduction of area because forest fires and denudation .

Suppose that $-\mu(t,a)P+f(t,p)$ is stochastically perturbed with

$$-\mu(t,a)P+f(t,p)\to-\mu(t,a)P+f(t,p)+g(t,P)\dot{\omega}(t),$$

Here $\dot{\omega}(t)$ is white noise. Then this environmentally perturbed system may be described by the $It\hat{o}$ equation

$$\begin{cases} d_tP=-\dfrac{\partial p}{\partial a}dt-\mu(t,a)Pdt+f(t,p)dt+g(t,P)d\omega_t, \quad in Q=(0,A)\times(0,T) \\ P(0,a)=P_0(a), \qquad in[0,A] \\ P(t,0)=\gamma(t)\beta(t)\int_0^A \beta(t,a)P(t,a)da, \qquad in[0,T] \end{cases} \tag{2}$$

d_tP is the differential of P relative to t ,i.e, $d_tP=(\partial P/\partial t)dt$

A new stochastic differential equation model (2) a forest evolution dynamic system. is derived. It is an extension of Eq(1).

In this paper, we shall discussion the existence, uniqueness for a forest evolution dynamic system Eq.(2).

II. PRELIMINARIES

Let

$$V=H^1([0,A]) \equiv\left\{\varphi\,|\,\varphi\in L^2([0,A]),\frac{\partial\varphi}{\partial x_i}\in L^2([0,A]), \text{ where } \frac{\partial\varphi}{\partial x_i} \text{ is generalized partial derivatives}\right\}.$$

V is a Sobolev space. $H=L^2([0,A])$ such that $V\to H\equiv H'\to V'$.

V' is the dual space of V .We denote by $\|.\|,|.|$ and $\|.\|_*$ the norms in V , H and V' respectively ;by $\langle .,.\rangle$ the duality product between V ,V',and by $(.,.)$ the scalar product in H ,and m a constraint such that $m|x|\le m\|x\| \quad \forall x\in V$.

Let ω_t be a Wiener process defined on complete probability space (Ω,F,P) ,and taking its values in the separable Hilbert space K, with increment covariance operator W .Let $(F_t)_{t\ge0}$ be the σ - algebra generated by $\{\omega_s,0\le s\le t\}$,then ω_t is a martingale relative to $(F_t)_{t\ge0}$ and we have the following representation of ω_t :

$\omega_t=\sum_{i=1}^{\infty}\beta_i(t)e_i$,$\{e_i\}_{i\ge1}$ is an orthonormal set of eigenvectors of W , $\beta_i(t)$ are mutually independent real Wiener processes with incremental covariance $\lambda_i>0$,$We_i=\lambda_ie_i$ and $\mathrm{tr}W=\sum_{i=1}^{\infty}\lambda_i<\infty$

('tr' denotes the trace of an operator [13]). For an operator $B\in\Gamma(K,H)$ be the space of all bounded linear operators from K into H , we denote by $\|B\|_2$ its Hilbert-Schmidt norm, i.e.

$$\|B\|_2^2=tr\,(BWB^T)\,.$$

In this paper, ω_t is a real standard Wiener process. Let $C = C([0,T],H)$ be the space of all continuous function from $[0,T]$ into H with sup-norm $\|\psi\|_C = \sup_{0\le s\le T}|\psi(s)|$, $L_V^P = L^P([0,T];V)$ and $L_H^P = L^P([0,T];H)$.

Consider the following nonlinear stochastic equation:

$$\begin{cases} P_t = P_0 - \int_0^t \frac{\partial P_s}{\partial \alpha} ds - \int_0^t \mu(s,a)P_s ds \\ \quad + \int_0^t f(s,P_s)ds + \int_0^t g(s,P_s)d\omega_s, & \forall t\in[0,T] \quad (*) \\ P(t,0) = \gamma(t)\beta(t)\int_0^A \beta(t,a)P_t da & \forall t\in[0,T] \end{cases}$$

Where $P_t = P(t,a), P_0 = P(0,a)$.

The objective in this paper is that, we hopefully find a unique process $P_t \in I^P(0,T;V)\cap L^2(\Omega;C(0,T;H))$, such that (*) hold.

For this objective, we assume the following conditions are satisfied: $\mu(t,a),\beta(t,a)$ are nonnegative measurable, and

$$\begin{cases} 0\le \mu_0 \le \mu(t,a) < \infty \quad in\, Q, \\ 0\le \beta(t) \le \bar{\beta} < \infty \quad in\, Q, \\ 0\le \gamma(t) \le \bar{\gamma} < \infty \quad in\, Q. \end{cases}$$

Let $f(t,\cdot): L_H^2 \to H$ be a family operator's defined a.e.t. and satisfy:

(a.1) $f(t,0)=0$;

(a.2) $\exists k_1 > 0$ such that

$|f(t,y) - f(t,x)| \le k_1\|y-x\|_c, \forall x,y\in C, \quad a.e.t.$

Let $g(t,\cdot): L_H^2 \to \Gamma(K,H)$, the family of nonlinear operator defined a.e.t., $g(t,x)\in\Gamma(K,H)$ and satisfy

(b.1) $g(t,0)=0$;

(b.2) there exists $k_2>0$ such that

$$\|g(t,y)-g(t,x)\|_2 \le k_2\|y-x\|_c \quad \forall x,y\in C, \quad a.e.t.$$

$f(t,v)$ and $g(t,v)$ are Lebergue miserable $\forall v\in L_H^2$, satisfying following condition(H):

There exist constants $\alpha>0, \xi>0, \lambda\in R$, and a non-negative continuous function $\gamma(t), t\in R_+$, such that

$$2\langle f(t,v),v\rangle + \|g(t,v)\|_2^2 \le -\alpha\|v\|^2 + \lambda|v|^2 + \gamma(t)e^{-\xi t}, \quad v\in V, \quad a.e.t.,$$

Where, for arbitrary $\delta>0, \gamma(t)$ satisfies $\gamma(t) = o(e^{\xi t})$, as $t\to\infty$, *i.e.*, $\lim_{t\to\infty}\gamma(t)/e^{\xi t} = 0$

Remark:

Observe that, owing to continuity and sub exponential growth of the term, there exists a positive constant $\bar{\gamma}$ such that $\gamma(t)e^{\xi t} \le \bar{\gamma}$ for all $t\in R^+$. As a consequence, (H) implies

$$2\langle f(t,v),v\rangle + \|g(t,v)\|_2^2 \le -\alpha\|v\|^2 + \lambda|v|^2 + \bar{\gamma}, \quad v\in V, \quad a.e.t.$$

III. Existence and Uniqueness of Solutions

A *Uniqueness of solutions*

Now we shall prove that there exists at most one solution of (*).This result will be deduced mainly from $It\hat{o}$ formula.

Theorem 3.1 *Assume the preceding hypotheses hold. Then exists at most one solution of (*) in* $I^2(0,T;V)\cap L^2(\Omega;C(0,T;H))$

.Proof: Suppose $P_{1t}, P_{2t}\in I^2(0,T;V)\cap L^2(\Omega;C(0,T;H))$ are two solutions of (*). Then, applying $It\hat{o}$ formula to $|P_{1t}-P_{2t}|^2$,we obtain

$$\begin{aligned} |P_{1t}-P_{2t}|^2 &= 2\int_0^t \left\langle -\frac{\partial P_{1s}}{\partial a} + \frac{\partial P_{2s}}{\partial a} - \mu(s,a)(P_{1s}-P_{2s}), P_{1s}-P_{2s}\right\rangle ds \\ &+ 2\int_0^t (f(s,P_{1s}) - f(s,P_{2s}), P_{1s}-P_{2s})ds \\ &+ 2\int_0^t (P_{1s}-P_{2s}, (g(s,P_{1s}) - g(s,P_{2s}))d\omega_s) \\ &+ \int_0^t \|g(s,P_{1s}) - g(s,P_{2s})\|_2^2 ds \end{aligned}$$

Therefore, we get that

$$\begin{aligned} |P_{1t}-P_{2t}|^2 &\le A(\bar{\gamma}\bar{\beta}\bar{\mu})^2\int_0^t |P_{1s}-P_{2s}|^2 ds \\ &+ 2\int_0^t |P_{1s}-P_{2s}||f(s,P_{1s}) - f(s,P_{2s})|ds - 2\mu_0\int_0^t |P_{1s}-P_{2s}|^2 ds \\ &+ \int_0^t \|g(s,P_{1s}) - g(s,P_{2s})\|_c^2 ds + 2\int_0^t (P_{1s}-P_{2s}, (g(s,P_{1s}) - g(s,P_{2s})))d\omega_s. \end{aligned}$$

Now, it follows from (a.2) and (b.2) that for any $t\in[0,t]$

$$\begin{aligned} &E\sup_{0\le s\le t}|P_{1s}-P_{2s}|^2 \\ &\le (|A(\bar{\gamma}\bar{\beta}\bar{\mu})^2 - 2\mu_0| + 1)\int_0^t E|P_{1s}-P_{2s}|^2 ds + (k_1^2+k_2^2)\int_0^t E\|P_{1s}-P_{2s}\|_C^2 ds \quad (3) \\ &+ 2E\sup_{0\le s\le t}\int_0^s (P_{1r}-P_{2r}, (g(r,P_{1r}) - g(r,P_{2r}))d\omega_r). \end{aligned}$$

However, by Burkholder-Davis-Gundy's inequality, $K>0$ we have

$$\begin{aligned} &E\sup_{0\le s\le t}|P_{1s}-P_{2s}|^2 \\ &\le 2(|A(\bar{\gamma}\bar{\beta}\bar{\mu})^2 - 2\mu_0| + 1 + k_1^2 + k_2^2 + 2Kk_2^2) \\ &\times \int_0^t E\sup_{0\le s\le t}|P_{1r}-P_{2r}|^2 ds, \forall t\in[0,T]. \end{aligned}$$

Now, Gronwall's lemma obviously implies uniqueness.

B *Existence of strong solutions*

In order to prove the existence of solution for Eq.(*), we shall first prove the following lemmas.

We consider the equations

$$P_t^1 = P_0 + \int_0^t [-\frac{\partial P^1}{\partial a} - \frac{A(\bar{\gamma}\bar{\beta}\bar{\mu})^2}{2}P_s^1]ds, t\in[0,T],$$

$$\begin{aligned} &2E\int_0^t |f(P_s^n) - f(P_s^{n-1})||P_s^{n+1} - P_s^n|ds \\ &\le \frac{1}{4T}E\int_0^t |P_s^{n+1} - P_s^n|ds| + 4k_1^2 TE\int_0^t \|P_s^n - P_s^{n-1}\|_C^2 ds \\ &\le \frac{1}{4}E\left[\sup_{0\le r\le t}|P_r^{n+1} - P_r^n|\right] + 4k_1^2 T\int_0^t E\left[\sup_{0\le r\le s}|P_r^n - P_r^{n-1}|^2\right]ds. \end{aligned}$$

$$P^1(t,0)=\gamma(t)\beta(t)\int_0^A \beta(t,a)P_t^1 da, t\in[0,T], \quad (4)$$

$$P_t^{n+1}=P_0+\int_0^t[-\frac{\partial P_s^{n+1}}{\partial a}-\frac{A(\bar{\gamma}\bar{\beta}\bar{\mu})^2}{2}P_s^{n+1}]ds+\int_0^t\frac{A(\bar{\gamma}\bar{\beta}\bar{\mu})^2}{2}P_s^n ds-\int_0^t\mu(s,a)P_s^n ds \quad (5)$$

$$+\int_0^t f(s,P_s^n)ds+\int_0^t g(s,P_s^n)d\omega_s, t\in[0,T], \quad \forall n\geq 1,$$

$$P^{n+1}(t,0)=\gamma(t)\beta(t)\int_0^A \mu(t,a)P_t^{n+1}da, t\in[0,T], \forall n\geq 1. \quad (6)$$

Lemma 3.1 $\{P_t^n\}$ *is a Cauchy sequence in* $L^2(\Omega;C(0,T;H))$.

Proof: For $n>1$ and the process $P_t^{n+1}-P_t^n$, it following from $It\hat{o}'s$ formula:

$$|P_t^{n+1}-P_t^n|^2=2\int_0^t\left\langle -\frac{\partial P_s^{n+1}}{\partial a}+\frac{\partial P_s^n}{\partial a}, P_s^{n+1}-P_s^n\right\rangle ds-2\int_0^t(\mu(s,a)(P_s^n-P_s^{n+1}),P_s^{n+1}-P_s^n)ds$$

$$-A(\bar{\gamma}\bar{\beta}\bar{\mu})^2\int_0^t|P_s^{n+1}-P_s^n|^2 ds+A(\bar{\gamma}\bar{\beta}\bar{\mu})^2\int_0^t(P_s^{n+1}-P_s^n,P_s^n-P_s^{n-1})ds$$

$$+2\int_0^t(f(P_s^n)-f(P_s^{n-1}),P_s^{n+1}-P_s^n)ds+2\int_0^t(P_s^{n+1}-P_s^n,(g(P_s^n)-g(P_s^{n-1}))d\omega_s)+\int_0^t\|g(P_s^n)-g(P_s^{n-1})\|_2^2 ds,$$

Where, by definition, $P_t^n:=P^n(t,a), f(P_t^n):=f(t,P_t^n)$ and $g(P_t^n):=g(t,P_t^n)$

$$|P_t^{n+1}-P_t^n|^2$$

$$\leq\left|A(\bar{\gamma}\bar{\beta}\bar{\mu})^2-2\mu_0\right|\int_0^t|P_s^{n+1}-P_s^n||n_s-P_s^{n-1}|ds+2\left|\int_0^t(P_s^{n+1}-P^n,(g(P_s^n)-g(P_s^{n-1}))d\omega_s\right| \quad (9)$$

$$+2\int_0^t|f(P_s^n)-f(P_s^{n-1})||P_s^{n+1}-P_s^n|ds+\int_0^t\|g(P_s^n)-g(P_s^{n-1})\|_2^2 ds$$

It is easy to deduce.Consequently, (9) yields

$$E[\sup_{0\leq\theta\leq t}|P_\theta^{n+1}-P_\theta^n|^2]$$

$$\leq|A(\bar{\gamma}\bar{\beta}\bar{\mu})^2-2\mu_0|E\int_0^t|P_s^{n+1}-P_s^n||P_s^n-P_s^{n-1}|ds$$

$$+2E[\sup_{0\leq\theta\leq t}|\int_0^\theta(P_s^{n+1}-P_s^n,(g(P_s^n)-g(P_s^{n-1}))d\omega_s)| \quad (10)$$

$$+2E\int_0^t|f(P_s^n)-f(P_s^{n-1})||P_s^{n+1}-P_s^n|ds+E\int_0^t\|g(P_s^n)-g(P_s^{n-1})\|_2^2 ds].$$

On the other hand, we can get from (b.2)

$$E\int_0^t\|g(P_s^n)-g(P_s^{n-1})\|_2^2 ds\leq k_2^2E\int_0^t\sup_{0\leq r\leq s}|P_r^n-P_r^{n-1}|^2 ds. \quad (12)$$

In a similar manner, from (a.2) we can obtain

$$2E\int_0^t|f(P_s^n)-f(P_s^{n-1})||P_s^{n+1}-P_s^n|ds$$

$$\leq\frac{1}{4T}E\int_0^t\|P_s^{n+1}-P_s^n|ds|+4k_1^2TE\int_0^t\|P_s^n-P_s^{n-1}\|_C^2 ds$$

$$\leq\frac{1}{4}E\left[\sup_{0\leq r\leq t}|P_r^{n+1}-P_r^n|\right]+4k_1^2T\int_0^tE\left[\sup_{0\leq r\leq s}|P_r^n-P_r^{n-1}|^2\right]ds.$$

Now, Burkholder-Davis-Gundy's inequality implies

$$2E\left[\sup_{0\leq r\leq t}\left|\int_0^r(P_s^{n+1}-P_s^n,(g(P_s^n)-g(P_s^{n-1})))d\omega_s\right|\right]$$

$$\leq 6E\left[(\sup_{0\leq r\leq t}|P_r^{n+1}-P_r^n|^2)\int_0^t\|g(P_s^n)-g(P_s^{n-1})\|_2^2\right]^{1/2}$$

$$\leq\frac{1}{4}E\left[\sup_{0\leq r\leq t}|P_r^{n+1}-P_r^n|^2\right]+72k_2^2\int_0^tE\left[\sup_{0\leq r\leq s}|P_r^n-P_r^{n-1}|^2\right]ds. \quad (13)$$

If we set

$$\varphi^n(t)=E\left[\sup_{0\leq\theta\leq t}|P_\theta^{n+1}-P_\theta^n|^2\right], \quad (14)$$

Then from (10)-(13), it could be deduced that there exists a positive constant $c>0$ such that

$$\varphi^n(t)\leq\frac{3}{4}\varphi^n(t)+c\int_0^t\varphi^{n-1}(s)ds \quad (15)$$

consequently there exists $k>0$ such that

$$\varphi^n(t)\leq k\int_0^t\varphi^{n-1}(s)ds. \quad (16)$$

By iteration from (16), we get

$$\varphi^n(t)\leq\frac{K^{n-1}T^{n-1}}{(n-1)!}\varphi^1(T) \quad \forall n>1 \quad \forall t\in[0,T]. \quad (17)$$

Therefore

$$E\left[\sup_{0\leq\theta\leq T}|P_\theta^{n+1}-P_\theta^n|^2\right]\leq\frac{K^{n-1}T^{n-1}}{(n-1)!}\varphi^1(T) \quad \forall n>1. \quad (18)$$

Obviously, (18) implies that $\{P_t^n\}$ is a Cauchy sequence $L^2(\Omega;C(0,T;H))$.

Lemma 3.2 *The sequence* $\{P_t^n\}$ is bounded in $I^2(0,T;V)$.

Proof: Indeed, applying $It\hat{o}'s$ formula to $|P_t^n|^2$ with $n\geq 2$ immediately yields

$$E|P^n(T)|^2=2E\int_0^T<-\frac{\partial P_s^n}{\partial a},P_s^n>ds-2\int_0^T(\mu(s,a)P_s^{n-1},P_s^n)ds$$

$$-A(\bar{\gamma}\bar{\beta}\bar{\mu})^2E\int_0^T|P_s^n|^2 ds+E|P_0|^2+2E\int_0^T(f(P_s^{n-1}),P_s^n)ds \quad (19)$$

$$+A(\bar{\gamma}\bar{\beta}\bar{\mu})^2E\int_0^T(P_s^n,P_s^{n-1})ds-2E\int_0^T(f(P_s^{n-1}),P_s^{n-1})ds$$

$$+2E\int_0^T(f(P_s^{n-1}),P_s^{n-1})ds+E\int_0^T\|g(P_s^{n-1})\|_2^2 ds.$$

Since $\{P^n\}$ is convergent in $L^2(\Omega;C(0,T;H))$, it will be bounded in this space. Now, it is not difficult to check that there exists positive constant $k'>0$ .We will estimate one of those terms. First, we observe that

$$2E\int_0^T|f(P_s^{n-1})|(|P_s^n|+|P_s^{n-1}|)ds$$

$$\leq 2k_1E\int_0^T\|P_s^{n-1}\|_C(|P_s^n|+|P_s^{n-1}|)ds$$

$$\leq k_1E\int_0^T\left[\|P_s^{n-1}\|_C^2+(|P_s^n|+|P_s^{n-1}|)^2\right]ds$$

$$\leq Tk_1E(\sup_{0\leq\theta\leq T}|P_\theta^{n-1}|^2)+2k_1T\left[E(\sup_{0\leq\theta\leq T}|P_\theta^n|^2)+E(\sup_{0\leq\theta\leq T}|P_\theta^{n-1}|^2)\right]$$

$$=Tk_1\|P_t^{n-1}\|_{L^2(\Omega;C(0,T;H))}+2k_1T\left[\|P_t^n\|_{L^2(\Omega;C(0,T;H))}+\|P_t^{n-1}\|_{L^2(\Omega;C(0,T;H))}\right],$$

Which, in addition to (H), lead to the following inequality

$$\alpha\int_0^TE\|P_s^{n-1}\|^2 ds\leq-2E\int_0^T(f(P_s^{n-1}),P_s^{n-1})ds-E\int_0^T\|g(P_s^{n-1})\|_2^2 ds$$

$$+|\lambda|T\|P_t^{n-1}\|^2_{L^2(\Omega;C(0,T;H))}+\int_0^T\gamma(s)e^{-\xi s}ds.$$

Since $\{P^n\}$ is convergent in $L^2(\Omega;C(0,T;H))$.Therefore, there exist a constant k' such that $\int_0^TE\|P_s^{n-1}\|^2 ds\leq k'$

Lemma 3.2 is proved.

Theorem 3.2 *Assume the preceding hypotheses and* $A(\bar{\gamma}\bar{\beta}\bar{\mu})^2=0$ *hold. Then, there exist a unique process* $P_t\in I^2(0,T;V)\cap L^2(\Omega;C(0,T;H))$ *such that*

$$P_t=P_0+\int_0^t\left[\frac{\partial P_s}{\partial a}+f_1(s)\right]ds+M(t), P \quad a.s. \quad \forall t\in[0,T],$$

Where $f_1\in I^2(0,T;V), P_0\in L^2(\Omega,F_0,P;H)$ and M_t is an H-valued continuous, square integrable $F_t-martingale$. In addition, the following energy equality also holds:

$$|P_t|^2=|P_0|^2+2\int_0^t\left\langle\frac{\partial P_s}{\partial a},P_s\right\rangle ds+2\int_0^t(f_1(s),P_s)ds+2\int_0^t(P_s,dM_s)+tr\langle\langle M\rangle\rangle_t, \quad P \quad as. \quad \forall t\in[0,T],$$

$\langle\langle M\rangle\rangle_t$ denotes the quadratic variation of M_t.

Proof: See Metiver and Pellaumail [14].

Now we are in a position to prove the existence of solution to the problem （*）.

Theorem 3.3 *Assume (a.1)-(a.2), (b.1)-(b.2), and (H) hold,* $P_0 \in I^2(0,T;V) \cap L^2(\Omega;C(0,T;H))$, *there exists a unique solution of the problem* （*） *in* $g(P_t^n) \to g(P_t)(in\, L^2(\Omega;L^\infty(0,T;\Gamma(K,H))))$.

Proof: Uniqueness hold from Theorem 3.1.

By virtue of (a.1), the family $A_1(t,g): V \to V'$ defined as $A_1(t,P_t) = -\frac{\partial P_t}{\partial a} - \frac{A(\overline{\gamma}\overline{\beta}\overline{\mu})^2}{2} P_t$, satisfies the assumptions in Theorem 3.2. Consequently, (5)-(7) has a unique solution

$$P_t^1 \in I^2(0,T;V) \cap L^2(\Omega;C(0,T;H)).$$

We note that from (a.2) and (b.2), it follows:

(i) The mapping $(t,\omega) \in (0,T) \times \Omega$ a $f(t,P_t^1) \in H$ belongs to $I^2(0,T;H)$;

(ii) The mapping $(t,\omega) \in (0,T) \times \Omega$ a $g(t,P_t^1) \in \Gamma(K,H)$ belongs to the space $I^2(0,T;\Gamma(K,H))$ and therefore $\int_0^T g(t,P_s^1)d\omega_s$ is a continuous and square integrals

$F_1 -$martingale.

Consequently, we can use Theorem 3.2 and get that there exist a unique process

$P_t^1 \in I^2(0,T;V) \cap L^2(\Omega;C(0,T;H))$, which is the solution of (5)-(7) for n=1. By recurrence, we obtain a sequence of solutions for (5)-(7), $\{P_t^n\}_{n\geq 1} \subset I^2(0,T;V) \cap L^2(\Omega;C(0,T;H))$.

Now we want to prove that the sequence $\{P_t^n\}$ is convergent to a process P_t in $I^2(0,T;V) \cap L^2(\Omega;C(0,T;H))$.

which will be the solution of （*）.

First, we observe that Lemma 3.1implies that there exists in $P_t \in L^2(\Omega;C(0,T;H))$ such that $P_t^n \to P_t$ in $L^2(\Omega;C(0,T;H))$.Since (a.2) and (b.2), Have $f(P_t^n) \to f(P_t)(in L^2(\Omega;L^\infty(0,T;H)))$, and $g(P_t^n) \to g(P_t)(in\, L^2(\Omega;L^\infty(0,T;\Gamma(K,H))))$.

Let $DP_t = \frac{\partial P_t}{\partial t} + \frac{\partial P_t}{\partial a}$.

So $DP_t^n = -\frac{A(\overline{\gamma}\overline{\beta}\overline{\mu})^2}{2} P_t^n dt - \mu(t,a)P_t^{n-1}dt + \frac{A(\overline{\gamma}\overline{\beta}\overline{\mu})^2}{2} P_t^{n-1}dt$
$+ f(t,P_t^{n-1})dt + g(t,P_t^{n-1})d\omega_t$.

By preceding analysis, we easily obtain that

$$\left\| DP_t^n \right\|_{V'} \leq M \leq \infty.$$

On the other hand, by virtue of Lemma 3.2 $\{P_t^n\}$ has a subsequence which is weakly convergent in $I^2(0,T;V)$. But, since $P_t^n \to P_t$ in $L^2(\Omega;C(0,T;H))$, we can assure that $P_t^n \to P_t$ weakly in $I^2(0,T;V)$(in the sequel, we will denote $P_t^n \to P_t$ in $I^2(0,T;V)$).In conclusion, we have proved

$$P^n \to P \quad in\, L^2(\Omega;C(0,T;H)), \tag{21}$$

$$f(P_t^n) \to f(P_t) \quad in\, L^2(\Omega;L^\infty(0,T;H)), \tag{22}$$

$$g(P_t^n) \to g(P_t) \quad in\, L^2(\Omega;L^\infty(0,T;\Gamma(K,H))), \tag{23}$$

$$P_t^n \to P_t \quad in\, I^2(0,T;V), \tag{24}$$

$$DP_t^n \to h \quad in\, L^2(\Omega\times(0,T);V')$$

Since the differential operator is continuous, so $DP = h$.

Theorem 3.3 is completed.

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