

Capacity of Electronic Health Records

Shao Chao

Xi'an University of post and Telecommunications
shaochao@xupt.edu.cn

Fan Li-Shuang

Xi'an University of post and Telecommunications

Xiao Qian

Xi'an University of post and Telecommunications

Mo Zeng-Tao

Xi'an University of post and Telecommunications

Abstract—This paper provides an overview of the capacity of multi-antenna channel in Shannon theoretic sense, and gives analysis results and the upper bound expressions of Shannon MIMO channel capacity when transmit and receive antennas have the same beam pattern and are aligned, or one has wider beam than (or the other), or their beams are unaligned. It also has been proved that when transmit and receive antennas have aligned beams, the capacity of multi-antenna channel will obtain its maximum. In the end, the simulation results show the theory analysis is valid.

Keywords—Shannon capacity of multiantenna channels, antenna beam patterns, beam align or disjoint.

I. INTRODUCTION

Wireless systems continue to strive for ever higher data rates. This goal is particularly challenging for systems that have power, bandwidth, and complexity limitations. However, another domain can be exploited to significantly enlarge channel capacity: the use of multiple transmit and receive antennas. The pioneering work by Telatar [1] and Foschini [2] ignited much interest in this area by predicting remarkable spectral efficiencies for wireless systems with multiple antennas when the channel exhibits rich scattering and its variations can be accurately tracked. Here, we focus on MIMO channel capacity in the Shannon theoretic sense that multi-antenna beams on both sides are aligned or shifted with a certain deviation.

Utilizing beamforming technique to increase the capacity of multi-antenna channels is an optimal transmission strategy. Jorswieck [3] on the assumption that transmitter knows the transmit and receive correlation matrices showed that transmitting in the directions of the eigenvectors of the transmit correlation matrix is the optimal transmission strategy. Sadek [4] extended the transmit beamforming in space-frequency coded MIMO-OFDM systems and proposed three design criteria for beamforming; among the algorithms the eigenvalue selection scheme, which locates the subspace associated with the largest eigenvalues in the eigenspace of the covariance matrices of the channels, provided the best performance. Lashkarian [5] derived an upper bound on the ergodic capacity of MIMO beamforming channels, and investigated the parameters such as cluster/ray arrival rates and power decay profiles on this upper bound.

In this paper, we go on utilizing the covariance matrices of receive and transmit antenna, assume that receiver knows the information of channel state and updates the beam to aim at transmit. We adopt Bartlett spatial spectrum [6] to make sense of adjusting receive beam to align it to the transmit antenna and vice versa.

We assume a wireless communication system employing N_T transmit and N_R receive antennas. We adopt the beamforming methods proposed in [7] to get the array antennas beam pattern on transmit or receive side or both. In the case when the beam is formed only on one side, the other side will be considered omnidirectionally. It is assumed that wireless MIMO channel is formed by array antenna beam on one or both sides. Based on this premise, we discuss MIMO channel capacity in the Shannon theoretic sense: this capacity is identical to the ability of the wireless channel transmitting information in bits per second per frequency bandwidth. We show the impacts of the antenna beams on the performance of the channel capacity with respect to that receive and transmit beams are aligned, overlapped, or non-overlapped. We formulate the capacity of multiantenna channels in all cases. With theoretical analysis and simulation confirmation, we can grasp the characteristics of multiantenna channels established by transmit and receive beamforming.

The remainder of this paper is organized as follows. In section II, we describe the MIMO channel model and establish the formulas of MIMO channel capacity. The effect of the spatial correlation on the channel capacity is analyzed in section III. The analytical results obtained in section III will be compared with simulation results in section IV. Finally, the conclusions are drawn in section V.

II. THE CAPACITY OF MULTIAN TENNA CHANNELS

First, we give the expression of differential entropy of random variables. Let $\xi \in \mathbb{C}^n$ be circularly symmetric complex Gaussian random vector with mean value μ and covariance matrix \mathbf{Q} , then ξ 's probability density function can be expressed as^[1]

$$\gamma_{\mu, \mathbf{Q}}(\mathbf{x}) = \det(\pi \mathbf{Q})^{-1} \exp\{-(\mathbf{x} - \boldsymbol{\mu})^H \mathbf{Q}^{-1} (\mathbf{x} - \boldsymbol{\mu})\} \quad (1)$$

where $\det(\mathbf{A})$ denotes the determinant of matrix \mathbf{A} . Here, only the stationary processes are considered; therefore, the random variable vector $\boldsymbol{\xi}$ could be assumed with zero mean and covariance matrix \mathbf{Q} . Then the differential entropy of the complex Gaussian random vector $\boldsymbol{\xi}$ with covariance matrix \mathbf{Q} is given by^[1] $\mathcal{H}(\boldsymbol{\xi}) = \mathcal{E}_{\gamma_{\mu, \mathbf{Q}}}[-\log \gamma_{\mu, \mathbf{Q}}(\mathbf{x})]$. By some simple derivation, we have the following results on $\mathcal{H}(\boldsymbol{\xi})$,

$$\begin{aligned} \mathcal{H}(\boldsymbol{\xi}) &= \log \det(\pi \mathbf{Q}) + (\log e) \mathcal{E}[\mathbf{x}^H \mathbf{Q}^{-1} \mathbf{x}] \\ &= \log \det(\pi \mathbf{Q}) + (\log e) \text{tr} \left\{ \mathcal{E}[\mathbf{x} \mathbf{x}^H] \mathbf{Q}^{-1} \right\} \\ &= \log \det(\pi \mathbf{Q}) + (\log e) \text{tr} \{ \mathbf{I} \} \\ &= \log \det(\pi e \mathbf{Q}) \end{aligned} \quad (2)$$

According to the above formulation, we have the conclusion of that circularly symmetric complex Gaussian random variables are differential entropy maximizers, the maximum value is expressed in (2)

In this paper, the MIMO channel signal model is adopted as follows,

$$\mathbf{r} = \mathbf{H} \mathbf{s} + \mathbf{n} \quad (3)$$

where $\mathbf{s} \in \mathbb{C}^{N_T \times 1}$ is the transmitted signal vector and $\mathbf{r} \in \mathbb{C}^{N_R \times 1}$ is the receive signal vector, $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix, $\mathbf{n} \in \mathbb{C}^{N_R \times 1}$ is the circularly symmetric complex Gaussian random additive noise vector with zero mean and covariance matrix $\sigma_n^2 \mathbf{I}$, that is, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I})$.

Given the definite channel coefficients \mathbf{H} and constraint total transmit power \mathcal{P} , the ability of channel transmit the information data \mathbf{s} is denoted as the mutual information of the received signal \mathbf{r} and transmitted signal \mathbf{s} , which can be expressed as

$$\mathcal{I}(\mathbf{r}, \mathbf{s}) = \mathcal{H}(\mathbf{r}) - \mathcal{H}(\mathbf{r} | \mathbf{s}) = \mathcal{H}(\mathbf{r}) - \mathcal{H}(\mathbf{n}) \quad (4)$$

where $\mathcal{H}(\mathbf{r} | \mathbf{s})$ denotes the conditional differential entropy of \mathbf{r} when \mathbf{s} is known, which equals to the differential entropy of additive noise vector \mathbf{n} when the channel coefficients \mathbf{H} are determined.

Based on the above formulations and conclusions, the mutual information of \mathbf{r} and \mathbf{s} , $\mathcal{I}(\mathbf{r}, \mathbf{s})$, would reach its maximum value if and only if \mathbf{r} and \mathbf{n} in (3) subject to circularly symmetric complex Gaussian distribution, under the assumptions of the channel coefficients \mathbf{H} are definite, and the transmit signal \mathbf{s} and additive noise \mathbf{n} are subject to circularly symmetric complex Gaussian distribution, all of which can be assured. On the other hand, the quantity of $\mathcal{I}(\mathbf{r}, \mathbf{s})$ expresses the uncertainty of the transmitted signal \mathbf{s} : if the probability distribution density function of \mathbf{s} is denoted as $p(\mathbf{s})$, then the channel capacity can be expressed as designing the transmitted signal \mathbf{s} under the constraint of total power \mathcal{P} to maximize the mutual information $\mathcal{I}(\mathbf{r}, \mathbf{s})$, which could be represented as,

$$\max_{p(\mathbf{s}), \text{tr}[\mathcal{E}\{\mathbf{s}\mathbf{s}^H\}] \leq \mathcal{P}} \mathcal{I}(\mathbf{r}, \mathbf{s}) \quad (5)$$

Denoting the maximum value as $C(\mathbf{H}, \mathcal{P})$, it can be taken in that the channel capacity relies on the channel coefficients \mathbf{H} and the total transmitted signal power \mathcal{P} . With some manipulations, the capacity of the channel $C(\mathbf{H}, \mathcal{P})$ can be formulated as follows,

$$C(\mathbf{H}, \mathcal{P}) = \max_{\text{tr}[\mathbf{Q}] \leq \mathcal{P}} \log_2 \det[\mathbf{H} \mathbf{Q} \mathbf{H}^H + \mathbf{I}_{N_R}]$$

Here, we have utilized the assumption of $\text{tr}[\mathcal{E}\{\mathbf{s}\mathbf{s}^H\}] = \text{tr}[\mathbf{Q}]$. Let $\gamma = (\mathcal{P}/\sigma_n^2)/N_T$ denote the transmitted signal power uniformly distributed on each sensor, and constrain $\text{tr}[\mathbf{Q}] \leq N_T$, then the channel capacity $C(\mathbf{H}, \mathcal{P})$ can be expressed as

$$C(\mathbf{H}, \mathcal{P}) = \max_{\text{tr}[\mathbf{Q}] \leq 1} \log_2 \det[\gamma \mathbf{H} \mathbf{Q} \mathbf{H}^H + \mathbf{I}_{N_R}].$$

The channel capacity $C(\mathbf{H}, \mathcal{P})$ is also called error-free spectral efficiency or the transmit rate in bits per second per hertz (b/s/Hz). By using the determinant identity $\det[\mathbf{A}\mathbf{B} + \mathbf{I}_N] = \det[\mathbf{B}\mathbf{A} + \mathbf{I}_M]$ for matrices $\mathbf{A} \in \mathbb{C}^{N \times M}$ and $\mathbf{B} \in \mathbb{C}^{M \times N}$, we also have

$$C(\mathbf{H}, \mathcal{P}) = \max_{\text{tr}[\mathbf{Q}] \leq 1} \log_2 \det[\gamma \mathbf{Q} \mathbf{H}^H \mathbf{H} + \mathbf{I}_{N_T}]$$

Furthermore, we have the ergodic MIMO capacity as followings^[1],

$$\begin{aligned} C(\mathcal{P}) &= \max_{\text{tr}[\mathbf{Q}] \leq 1} \mathcal{E}_H \{ C(\mathbf{H}, \mathcal{P}) \} \\ &= \max_{\text{tr}[\mathbf{Q}] \leq 1} \log_2 \det[(\mathcal{P}/\sigma_n^2) \mathbf{Q} \mathbf{R}_H + \mathbf{I}_{N_T}] \end{aligned} \quad (6)$$

where $\mathbf{R}_H = \mathbf{H}^H \mathbf{H} / N_T$ is the correlation matrix of the MIMO channel \mathbf{H} . In (6), we have used the assumption of constraint $\text{tr}\{\mathbf{R}_H\} \leq N_T$. And in the following, matrices \mathbf{Q} and \mathbf{R}_H are considered as spatial correlation matrices of transmit and receive array, they have some specified spatial response characteristic respectively.

III. THE EFFECT OF SPATIAL CORRELATION ON CHANNEL CAPACITY

In the following, we analysis the channel capacity of (6) with respect to the spatial responses of the transmit and receive array. We show that all of the impact factors on the channel capacity $C(\mathcal{P})$ that could be attributed to the relationship of the spatial response of transmit and receive antenna, or the relationship of principle [eigenvector](#) of matrix \mathbf{Q} and \mathbf{R}_H .

Assume that there are eigen-decompositions of $\mathbf{Q} = \mathbf{U}_s \boldsymbol{\Sigma}_s \mathbf{U}_s^H$ and $\mathbf{R}_H = \mathbf{U}_H \boldsymbol{\Sigma}_H \mathbf{U}_H^H$, where the eigenvalues of matrix \mathbf{Q} and \mathbf{R}_H are arranged in descendant order. Then the impacts of transmit and receive array beams on the channel capacity $C(\mathcal{P})$ are determined by the factors of $\boldsymbol{\Sigma}_s \mathbf{U}_s^H \mathbf{U}_H \boldsymbol{\Sigma}_H$ or $\boldsymbol{\Sigma}_H \mathbf{U}_H^H \mathbf{U}_s \boldsymbol{\Sigma}_s$. In the simulations, we see that matrix $\mathbf{U}_s^H \mathbf{U}_H$ or $\mathbf{U}_H^H \mathbf{U}_s$ are diagonal, therefore, the number and location of nonzero diagonal entries of matrix $\mathbf{U}_s^H \mathbf{U}_H$ or $\mathbf{U}_H^H \mathbf{U}_s$ is

the key factors for determining the channel capacity $C(\mathcal{P})$.

In the following, we only consider the case of $N_T \leq N_R$ (the case of $N_T \geq N_R$ can be discussed in same way). General speaking, if transmit or receive beams have specified pattern, then the spatial response of transmit and receive matrix \mathbf{Q} and \mathbf{R}_H are rank deficiency. Therefore, there are $\text{rank}(\mathbf{Q}) < N_T$, and $\text{rank}(\mathbf{R}_H) < N_T$, then for (6) we have,

$$\begin{aligned} & \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \mathbf{Q} \cdot \mathbf{R}_H) \\ &= \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \mathbf{U}_s \boldsymbol{\Sigma}_s \mathbf{U}_s^H \cdot \mathbf{U}_H \boldsymbol{\Sigma}_H \mathbf{U}_H^H) \\ &= \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \boldsymbol{\Sigma}_s (\mathbf{U}_s^H \mathbf{U}_H) \boldsymbol{\Sigma}_H (\mathbf{U}_H^H \mathbf{U}_s)) \end{aligned}$$

It is seen obviously from above expressions that the relationships of principle eigenvector of matrix \mathbf{Q} and \mathbf{R}_H absolutely determine the channel capacity $C(\mathcal{P})$. These relationships are that transmit and receive antenna beams being aligned, or one overlapped other, or one diverged from other, ultimately. These attribute to the relationships of matrices \mathbf{U}_H and \mathbf{U}_s , there are three cases, which are,

- 1) $\mathbf{U}_s^H \cdot \mathbf{U}_H = \mathbf{U}_H^H \cdot \mathbf{U}_s = \text{diag}\{\lambda_1, \dots, \lambda_N, 0, \dots, 0\} = \mathbf{A} \preceq \mathbf{I}$;
 - 2) $\boldsymbol{\theta} \preceq \mathbf{U}_s^H \cdot \mathbf{U}_H = \mathbf{U}_H^H \cdot \mathbf{U}_s = \text{diag}\{\tilde{\lambda}_1, \dots, \tilde{\lambda}_{N_0}, 0, \dots, 0\} = \tilde{\mathbf{A}} \preceq \mathbf{A}$,
- that is $N_0 < N$, and $\tilde{\lambda}_i \leq \lambda_i$ for $i = 1, 2, \dots, N_0$;
- 3) $\mathbf{U}_s^H \cdot \mathbf{U}_H = \mathbf{U}_H^H \cdot \mathbf{U}_s \approx \boldsymbol{\theta}$.

where $N \leq \min\{\text{rank}(\mathbf{Q}), \text{rank}(\mathbf{R}_H)\}$ and $\mathbf{A} \preceq \mathbf{B}$ means matrix $\mathbf{B} - \mathbf{A}$ is non-negative definite.

In case 1), we have the channel capacity $C(\mathcal{H}, \mathcal{P})$ equals to

$$\begin{aligned} & \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \mathbf{Q} \cdot \mathbf{R}_H) \\ &= \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \mathbf{U}_s \boldsymbol{\Sigma}_s \mathbf{U}_s^H \cdot \mathbf{U}_H \boldsymbol{\Sigma}_H \mathbf{U}_H^H) \\ &= \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \mathbf{A} \boldsymbol{\Sigma}_s \mathbf{A} \boldsymbol{\Sigma}_H) \\ &= \prod_{i=1}^N [1 + (\mathcal{P}/\sigma_n^2) \lambda_i^2 \cdot \lambda_i(\boldsymbol{\Sigma}_s) \lambda_i(\boldsymbol{\Sigma}_H)] \end{aligned} \quad (7)$$

In case 2), we have the channel capacity $C(\mathcal{P})$ equals to

$$\begin{aligned} & \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \mathbf{Q} \cdot \mathbf{R}_H) \\ &= \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \mathbf{U}_s \boldsymbol{\Sigma}_s \mathbf{U}_s^H \cdot \mathbf{U}_H \boldsymbol{\Sigma}_H \mathbf{U}_H^H) \\ &= \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \tilde{\mathbf{A}} \boldsymbol{\Sigma}_s \tilde{\mathbf{A}} \boldsymbol{\Sigma}_H) \\ &= \prod_{i=1}^{N_0} [1 + (\mathcal{P}/\sigma_n^2) \tilde{\lambda}_i^2 \cdot \lambda_i(\boldsymbol{\Sigma}_s) \cdot \lambda_i(\boldsymbol{\Sigma}_H)] \\ &\leq \prod_{i=1}^{N_0} [1 + (\mathcal{P}/\sigma_n^2) \lambda_i^2 \cdot \lambda_i(\boldsymbol{\Sigma}_s) \lambda_i(\boldsymbol{\Sigma}_H)] \\ &< \prod_{i=1}^N [1 + (\mathcal{P}/\sigma_n^2) \lambda_i^2 \cdot \lambda_i(\boldsymbol{\Sigma}_s) \lambda_i(\boldsymbol{\Sigma}_H)] \end{aligned} \quad (8)$$

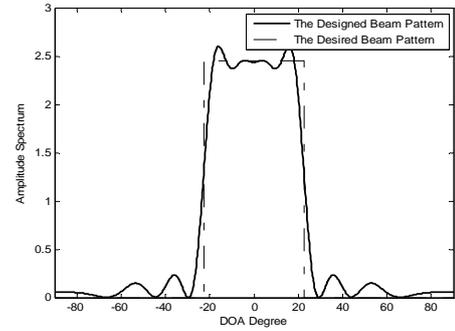
In case 3), we have the channel capacity $C(\mathcal{H}, \mathcal{P})$ equals to

$$\begin{aligned} & \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \mathbf{Q} \cdot \mathbf{R}_H) \\ &= \det(\mathbf{I}_{N_T} + (\mathcal{P}/\sigma_n^2) \cdot \mathbf{U}_s \boldsymbol{\Sigma}_s \mathbf{U}_s^H \cdot \mathbf{U}_H \boldsymbol{\Sigma}_H \mathbf{U}_H^H) \\ &\approx \det(\mathbf{I}_{N_T}) = 1 \end{aligned} \quad (9)$$

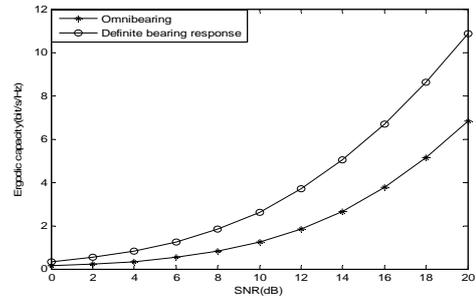
From the expressions (7)-(9), it can be seen that as the correlation of spatial response of transmit and receive decrease, declines the channel capacity $C(\mathcal{P})$.

IV. SIMULATION RESULTS

In all the simulations, it is assumed that there are 10 transmit sensors and 10 receive sensors (the simulations could be implemented in the same way for the cases there are different number sensor in transmit and receive). In the figures, it is shown that spatial correlation matrix of transmit or receive antenna having specified beampattern which is generated based on the principle proposed in [7], and the spatial correlation matrices are produced by the software `vcx`[8]. Then the spatial covariance matrices are employed in the simulations of computation of multiantenna channels capacity.



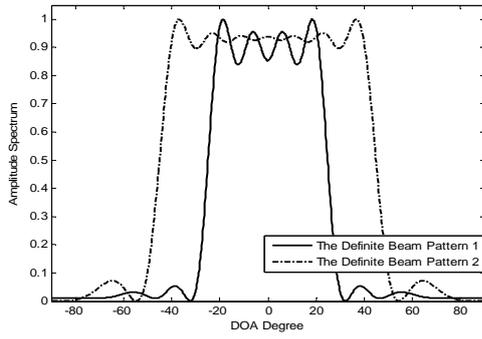
(a) The designed beampattern in transmitter and receiver



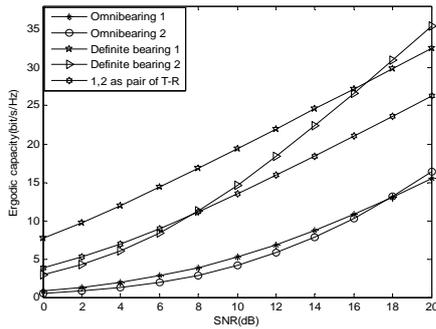
(b) The capacity of multiantenna channels

Figure 1. The capacity in aligned beam and omnibearing cases

In the figure 1(a), it is shown that the designed and desired array beampattern of the transmit antenna, and in the figure 1(b), it is shown that the MIMO channel capacity in the case of receive array antenna have same beampattern as the transmit or it is omnidirectional. It could be seen that the channel capacity of the former case is super than the latter one. These reveal that when transmit and receive spatial response beam are aligned, the transmission is more efficiency. In the viewpoint of Shannon, the former case corresponds to $\mathbf{R}_H = \mathbf{Q}$ and latter to $\mathbf{R}_H = \mathbf{I}$ in (7).



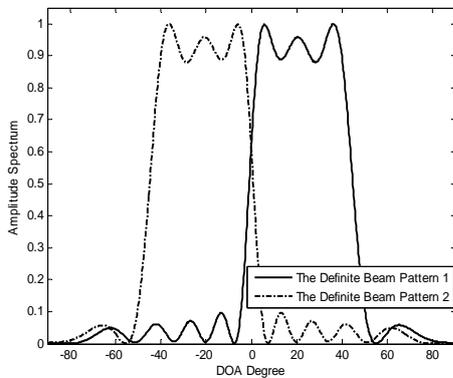
(a) The designed beampattern in transmitter and receiver



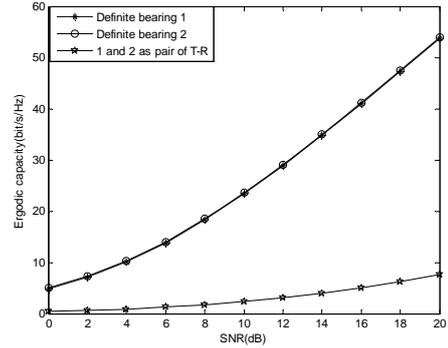
(b) The capacity of multiantenna channels

Figure 2. The capacity of aligned, overlapped and omnibearing beam

In figure 2(a), it is shown two cases of beampattern, one is wider than another. It is assumed that one is transmit array beam and another is receive, the capacity of the MIMO channel in the case of: 1) receive array antenna beampattern same as that of the transmit; 2) receive array antenna beampattern is omnidirectional in both cases, 3) making the wider or narrower beam one as transmit beam and other as receive beam. The multiantenna channel capacity about all cases are plotted in figure 2(b). It is seen that the performance of the narrower beam is super than the wider one in the lower signal-to-noise ratio region, and inferior to the wider beam in the relatively higher signal-to-noise ratio region, but the beampattern aligned cases proved best after all. In the viewpoint of Shannon, this corresponds to the case of $\text{rank}(\mathbf{Q}) \neq \text{rank}(\mathbf{R}_H)$ as in (8).



(a) The designed beampattern in transmitter and receiver



(b) The capacity of multiantenna channels

Figure 3. The capacity of diverge beam

In figure 3(a), the case of spatial response of transmitter and receiver array beampattern that diverge as if become disjoint are plotted. And in figure 3(b), the capacity of the MIMO channel in the case of: 1) receive array antenna beampattern same as the transmit, 2) the divergence beams one as transmit other as receive, are plotted. It is seen that in the divergence beampattern case, the transmission efficiency degenerated very seriously compares to the aligned cases. In the viewpoint of Shannon, this corresponds to (9) in our analysis.

It could be seen that all implemented cases are satisfied with requirement of theory analysis, and that all the experiment results are consistent with the theory analysis.

V. CONCLUSION

In general principle, when the array antenna beampattern has special directional, its covariance matrix is rank deficiency. This reveals that some of the antenna sensors are strongly correlated. The correlation between the antenna sensors has negative impact on the capacity of multiantenna channels [9]. In the viewpoint of diversity, as array antenna beam become narrow, increase the correlation between the antenna sensors, reduce the rank of the covariance matrix, and decrease the order of diversity, finally, degenerate the performance of the systems [10]. All of these seem contradict to our theory analysis and simulation results. In fact, our conclusion could be explained with the algorithm of water-filling [11]. When transmit beam is aligned to that of receive array, then the principle components of the covariance matrices of both side are matched each other, this is equivalent to the water-filling algorithm in transmit. This also could be explained with matched filter, when receive adjusts its beam aiming at transmit, a matched filter is established between transmit and receive, it is an optimal transmission strategy.

With the above discussion, we conclude that transmit and receive array antenna beam aligning is positive for system performance. This could be realized by feedback the information of covariance matrix of receive [3] or the coverage of beam of receive, or receive adjusts its beam aiming at transmit and matching the beampattern of transmit actively. The research is support under Grant 2012K06-27.

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