Research of Workflow nets Application Based on the Petri Nets

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Abstrac—**This paper introduced basic knowledge of Petri nets and workflow, and invariant analysis technique of Petri nets models structure, and integrate the real application circumstances of the workflow of receive document in document management, constructs an integrated workflow model based on Petri net. Further more, the author utilize invariant analysis technique to provide the verification of the model.**

Key words-Petri Nets; Workflow nets; Performance analysis; Receive document

In recent years, Petri-net theories developed concurreny、synchrony、enlogy and net topology as primary content . Petri-net application has refer to every field of computer science , especially, workflow application field . Petri- net refered to the better maths base and visual modeling . and it has become implement of workflow study on analyzing and modeling.

Ⅰ.PETRI-NET DEFINITION AND FUNCTION RULE.

Petri-net definition：Petri-net is defined as four-dimension groups PN(P, T, F, M0), P is named place sets, P=(P1,P2,⋯)is not full sets.T is named transition sets,T=(T1,T2, …)is not full sets.F is flow relation, M0 is initial marking. In Petri-net , "O" denotes a place (middle state); rectangle denotes a transition ; the arrowhead from P to T(or from T to P) denotes a arc in flow relation.

Dynamic action is denoted by token, namely symbol " • " in Petri-net . T is activated by condition which token amounts from T to P are greater than arc amounts from P to T , namely "t" has occurring authority in "M"：for all "p∈P, p ∈ *t", M(p)≥W(p,t), "M" enables "t" to occur, "*t" is "t" input sets. W(p, t) is arc amounts from P to T. If T is activated ,token distribution will occur to alter in Petri-net, token amounts with transition will reduce in input place, but token amounts will increase in its output place ("t*"). "M" makes "t" become new marking M′ (successor marking)：for all "p∈P,

$$
M'(p) = \begin{cases}
M(p) - W(p,t) & p \in {}^{*}t \\
M(p) + W(t,p) & p \in t^{*} \\
M(p) - W(p,t) + W(t,p) & p \in {}^{*}t \cap t^{*} \\
M(P) & p \notin {}^{*}t \cap t^{*}
\end{cases}
$$

Ⅱ. WORKFLOW-NET BASED ON PETRI-NET

Workflow model transforms Petri-net, constructing Workflow-net relying on Petri-net instrument, accomplishing Workflow-net model to evaluate.

A..1 Workflow net definition

Definition 1:

One Petri-net is Workflow-net, W=(P , T , F) , iff：

（1）existing two places in W：place i is input place , ${}'i = \phi$; place o is output place，$o' = \phi$.

（2）Increasing a transition t^{*} between Place o and i , (namely ${}'t^{*} = \{o\}$ and $t^{*} = \{i\}$),then w' is a strong connectivity Petri-net.

Definition 2:

One Petri-net is reliable，iff：

（1）for any a reachable state from i to M , existing a executing sequence M which reaching state o , i is initial state and o is terminative state. Namely：$\forall M(i \xrightarrow{*} M) \Rightarrow M \xrightarrow{*} o$

（2）for any a reachable state from i to M，if existing a token in o at least , the state is terminative state , now ，the Workflow-net only has a token in o, other places should be empty. Namely：$\forall M(i \xrightarrow{*} M \wedge M \geq o) \Rightarrow (M = o)$

（3）no existing deadlock in (w,i), then Petri-net is flexible. Namely：$\forall t \in T, \exists M, M', then\ (i \xrightarrow{*} M \xrightarrow{*} M')$
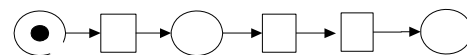
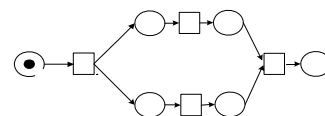*B. Basic structure on Workflow net*

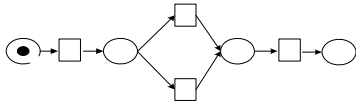Figure 1 Serial structure

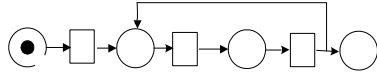Figure 2 Parallel structure

Figure 3 Choosing structure

Figure 4 Circulating structure

III. ANALYSIS METHOD THEORY BASED ON INVARIANT

The method uses algebra calculation to transact system, and supposes basic net of Petri-net is simplex, namely $\forall p \in P, {}^{*}p \cap p^{*} = \phi$ .T_invariant explains that marking will resume initial state afte transition sets occur, T_invariant reflects transition sets making state return , utilizing it to study periodicity performance, etc; P_ invariant reflects one weighted conservation on all token amounts in part places, utilizing it to study mutex、 dead lock and error detection, etc.

In Petri-net analysis technology, it has great merit for Petri-net to apply invariant analysis method to confirm Petri-net performance based on simple linearity algebra equation.

A..Petri-net state equation.

In Petri-net , mk denotes Petri-net marking after running k(k≥0) time , then the marking after running "k+1" times is：

$$m_{k+1} = m_k + Cu_k, k \geq 0 \qquad (1)$$

C is incidence matrix，uk is activated count vector，it is one "m×1" vector，the i element denotes activated times by transition ti in running "k+1"，formula (1) is named Petri-net state equation. All tokens in place is not negative, so running legally will ensure ：$m_k + Cu_k \geq 0$ ， for all $k \geq 0$

B. P_ invariant and T_invariant

I ）P_ invariant

One P_ invariant is one "n×1" vector x(x≥0), and meets condition：

$$x^T C = 0 \qquad (2)$$

left multiply " x^T " both sides in formula (1) ：
$x^T m_{k+1} = x^T m_k + x^T Cu_k$

$$x^T m_{k+1} = x^T m_k, k \geq 0 \qquad (3)$$

Beginning on deducing 'gradually from "k=0"：
$x^T m_0 = x^T m_1 = x^T m_2 = x^T m$＝constant

$$x^T m = x^T m_0 = \text{constant} \qquad (4)$$

Formula (4) denotes the sum of initial tokens which all weighted places by p_invariant are constant , namely the sum of weighted tokens in all places from "m0" to reachable

"m" are constant, the process is called that the places are covered with p_invariant.

Based on incidence matrix，utilizing p_invariant to analyze limit and conservation performance in Petri-net structure：

（1）supposes it has only one "n×1" vector x (x>0)，makes $x^T C \leq 0$, then the Petri-net structure is limit.

（2）supposes it has only one "n×1" vector x (x>0) ，makes $x^T C = 0$, then the Petri-net structure is conservation.

II ）.T_invariant
One T_invariant is one "m×1" vector y(y≥0) , and meets condition：

$$Cy = 0 \qquad (5)$$

Suppose one transition sequence is activated (the sequence count vector is "u")，token return from initial marking to initial marking， then based on formula (2)：

$$m_0 = m_0 + Cu \qquad (6)$$

Concluded ： $Cu = 0$ Then "u" is one T_invariant，namely "y=u" , it denotes that element （element k≥0)in T_invariant is times activated by transition in transition sets returning from m0 to m0. It is important applying validity of transition sets in T_invariant to analyze activity of Petri-net structure .

Estimating Petri-net structure reliability to utilize limit and activity.

Petri-net invariant is not exclusive , calling invariant which is not other invariant linearity combination basic invariant.from linearity algebra，supposing that the order of matrix is r=rank(C)，then the matrix has "n-r"P_invariant and "m-r"T_invariant. The two invariants are attained by solving linearity equation x^{T}C=0 and Cy=0.

IV. ANALYSIS OF APPLYING PROBLEM

Document management adapted to come true ,in OA(Office Automation) system, document is transferred from a dept to antother dept , which based on workflow technology. Sending document and receiving document are center of document management , utilizing workflow-net technology to realize document transferred is efficient.

A. Workflow-net reliability analysis.

Figure 5 is dispatch document manage workflow model, utilizing it to analyze workflow-net model construction、 analysis and the same technology.
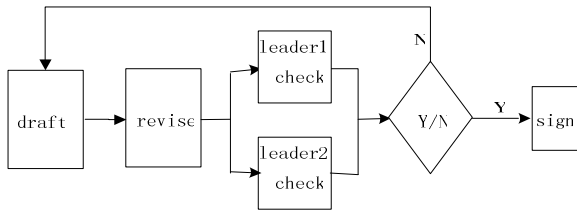
Figure 5 dispatch document workflow model

The flow process as follows：user drafts document, submitting, secretary transacts document with flow of serial structure and parallel structure based on sending document request. After Checking , transacting document has two choice：sign or draft .In all the flow process , administer can check and print flow process tracking table at any moment.

Figure 6 is workflow-net model of Figure 5. It includes six transition:draft、revise、leader1 and leader 2 check、auditing、sign. Workflow process commences on token p1, and p8 in the end, the role denotes transition in each process
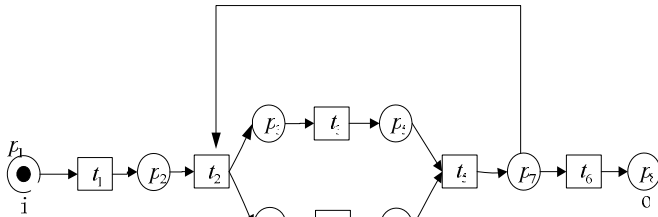

Figure 6 dispatch document workflow-net model

t_1.draft t_2.revise t_3.leader1 check t_4. Leader2 check t_5. auditing t_6.sign

utilizing workflow-net definition to analyze model reliability：

（1）Increasing a transition t^* between Place o and i , (namely $'t^* = \{o\}$ and $t^* = \{i\}$), then w is a strong connectivity Petri-net.

（2）each node($x \in P \cup T$)belongs to path from i to o

（3）no existing deadlock in model , each transition is executed.

The model is reliable from theory , in accord with workflow-net definition, however, in practice, the model has logic error：executing t_2 , appearing and-join of Parallel structure . t_2 has two input places , $'t_2 = （p_2, p_7）$, only existing tokens in p_2 and p_7, t_2 can occur, the conclusion is not right. In practice, any one token exists in p_2 or p_7 , namely drafting document is finished or auditing didn't pass , workflow should enter directly into revising state. The model is revised as Figure 7
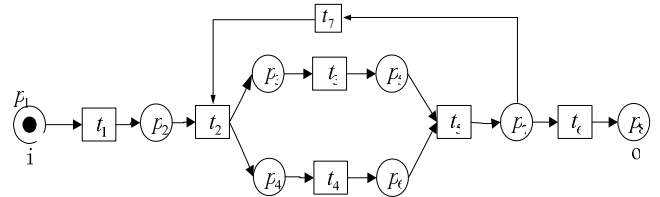

Figure 7 dispatch document workflow-net improvement model

Increasing a transition t_7 (return document) , if transition t_5 is not past , the document will be returned t_2 via t_7, now , any one token exists in p_2 or p_7, t_2 will occur , the improvement model (Figure 3)accords with not only Petri-net theory but sending document workflow in practice.

Further more , the improvement model is free choice Petri-net model. For any two transition of the improvement model t_1 and t_2 , only existing $^*t_1 \cap ^*t_2 \neq \phi$, then $^*t_1 = ^*t_2$, for example , $^*t_3 = ^*t_4 = ^*t_7$, the same as , the improvement model is proved as right structure via the analysis.

B. Workflow-net invariant analysis method

As follows , Resorting of invariant analysis method ,for workflow model based on Petri-net , utilizing invariant analysis method, analyzing to accept document in document management, construct accept document management workflow model as Figure 8.
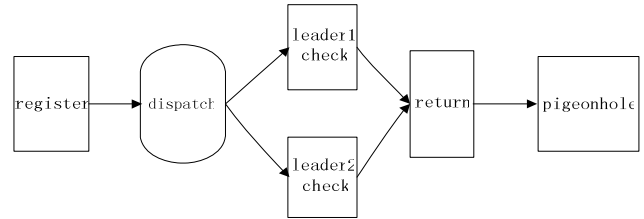

Figure 8 accept document management workflow model

Figure 9 is workflow-net model of Petri-net on figure 8. Workflow process commences on token p_1, and p_8 in the end, the role denotes transition in each process . p_1 concludes a transacting document , equal to a token, at the same time t_1 consumes resource of p_1; t_1 produces new token and puts it into p_2 , then t_2 begins to run. If producing a token between p_3 and p_4, t_3 and t_4 will execute at once, then t_5 runs. T_5 ensures parallel transition executing accurately. Producing to transacting document in P_7, after t_6 is pigeonholing , reach the lastest state p_8 .
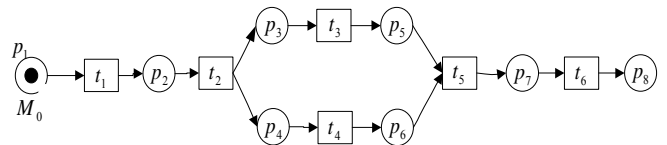

Figure 9   accept document Workflow-net model

The model(Figure 9) commences on M_0 , all likely states can attain in the reachable tree of model.Utilizing invariant

analysis method and Workflow-net theory to analyze the performance of model .

1)   Workflow-net activity

Workflow-net activity can be proved by constructing states equation , based on invariant analysis method. By formula $M=M_0+CU$, supposing initial marking $M_0=(1,0,0,0,0,0,0,0)$, terminating marking $M=(0,0,0,0,0,0,0,1)$, constructs the incidence matrix :

$$C = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\because$ $(1,0,0,0,0,0,0,0)^T+CU=(0,0,0,0,0,0,0,1)^T$, $\therefore$

$U=(1,1,1,1,1,1)$,pass transition sequence $T_1 \cdots T_6$, all the workflow process is from initial marking to terminating marking , so the Workflow-net is active.

2)   Workflow-net boundary

Boundary problem may transform states equation to solve problem. For model (Figure 9), $\because \exists Y>0, CY \leq 0$, $\therefore Y=0$,so the Workflow-net is not boundary , the model should be improved. Improving model is as follows Figure 10.
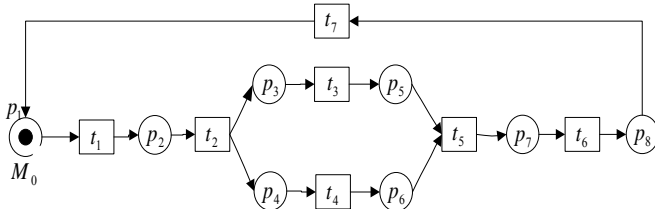

Figure 10   accept document Workflow-net improvement model

In improvement model(Figure 10), Increasing a transition t_7 , accept document process didn't execute one time ,but execute circularly.

constructs the incidence matrix over again:

$$C = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}; CY \leq 0 \Rightarrow \begin{cases} -y_1+y_7 \leq 0 \\ y_1-y_2 \leq 0 \\ y_2-y_3 \leq 0 \\ y_2-y_4 \leq 0 \\ y_3-y_5 \leq 0 \\ y_4-y_5 \leq 0 \\ y_5-y_6 \leq 0 \\ y_6-y_7 \leq 0 \end{cases}$$

$Y \geq 0$, so the improvement model is boundary.

V . CONCLUSION

This article introduced basic knowledge of Petri-net and Workflow-net ， utilizing Workflow-net definitions and invariant analysis method to analyze Petri-net performance. In the complicated OA system , applying Workflow-net to construct model , applying reachable tree and incidence matrix method to analyze system feasibility、reliability、security ,etc. in practice, the methods have an better effect on Petri-net performance analysis. The methods make system project and performance optimal, more application of the technology demands to study and explore.

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