

Comparison and Analysis of the Performance on Measurement Matrix to Spectrum Estimation

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Abstract—In the spectrum estimation algorithm based on Compressed Sensing, the selection of measurement matrix has significant influence on whether can estimate the signal power spectrum with high precision. In the article, it briefly describes the basic principles of spectrum estimation algorithm and the common measurement matrixes. In addition, it presents a detailed comparison and analysis of the construction method, pros and cons among random measurement matrix, structured measurement matrix and deterministic measurement matrix. On this basis, aiming at single-tone, multi-tone and QPSK signals, the feasibility of six kinds of measurement matrixes used in power spectrum estimation based on compressed sensing were validated and compared the estimation performance by simulations. The simulation results show that Toeplitz matrix can obtain the minimum NMSE at the same compression ratio.

Key words—Compressed Sensing(CS); Measurement Matrix; Power Spectrum Estimation(PSD)

I. INTRODUCTION

Spectrum sensing is one of the key technologies in Cognitive Radio, and paid more and more attentions. It has been put forward to a variety of effective spectrum sensing methods at present^{[1][2]}. However, whether they are based on the energy detection, feature detection, or based on the matched filter detection algorithm, it needs sample the received signal at Nyquist rate first of all. So it requires high speed ADC for wideband spectrum sensing. For one hand, the implementation and technical difficulty of the system are increased obviously; for another hand, it also increases the data's analysis and processing. The recent proposed Compressed Sensing^[3] (CS) theory provides a new solution solving the problems. The first study of spectrum sensing algorithm based on CS was proposed in [4]. On the basis of this, the researchers put forward a variety of spectrum sensing algorithm based on CS^{[5][6][7]}. Generally speaking, reconstructing signal has very high complexity, and spectrum sensing in communication system would like the signal characters (such as power spectrum density, center frequency, bandwidth and so on) rather than the figures. Therefore, a new spectrum sensing algorithm was proposed in [8] which called

multi-taper method combined with singular-value decomposition based on CS(MTM-SVD-CS). The algorithm can reduce the sampling rate, and give a power spectrum estimation through the low rate measurements directly which obtained from the measurement matrix. It can avoid the complicate operation by reconstructing the original signal to a large extent. The paper gives a simple research on the performance of measurement matrixes based on this algorithm.

Measurement matrix plays a crucial role in sampling the data and reconstructing the signal spectrum. It has been presented a variety of measurement matrixes for different applications, and mainly divided into random measurement matrix, structured measurement matrix and deterministic measurement matrix^[9]. The orthogonal requirements of measurement matrixes under the condition of no noise, limited noise and Gauss noise were studied in [10], and proposed a chaotic sequence which can be regarded as measurement matrix in [11]. However, it is difficult to compare and study measurement matrixes for their different backgrounds, and it brings troubles for choosing one in practice. So, on the basis of comparing and analyzing the construction method and pros and cons among all kinds of measurement matrixes in this paper, a detailed simulation was offered to compare the estimation performance of six kinds of measurement matrixes at last. The simulation results show that Toeplitz matrix is superior to the other matrixes.

II. SPECTRUM ESTIMATION BASED ON NON-RECONSTRUCTION OF COMPRESSED SENSING

The principle diagram of the algorithm is as figure 1.

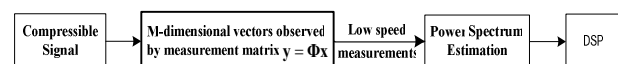


Figure.1 The diagram of Spectrum Estimation Based on Non-Reconstruction of Compressed Sensing

Firstly, using a $M \times N$ measurement matrix Φ transform the wideband compressible signal x into a lower speed digital signal y_M by sampling at the sub-Nyquist rate. Φ must satisfy some determinate conditions and can make x from \mathbf{R}^N to

\mathbf{R}^M ($M < N$) and not lose the main information which is used for reconstructing signals nearly intact, \mathbf{x} here is a sparse signal. So it can be said, measurement matrixes directly determine whether CS can realize successfully.

This paper introduced an algorithm based on compressive sensing and multi-taper method combined with singular-value decomposition (MTM-SVD-CS)^[8] to simulate and evaluate the performance of measurement matrixes. Now give an introduction to the algorithm.

Given a time series $\{x(n), 0 \leq n \leq N-1\}$, an orthonormal sequence of K Slepian tapers denoted by v_n^k . The associated eigenspectra defined by the Fourier transforms

$$Y_k(f) = \sum_{n=0}^{N-1} x(n) v_n^k e^{-j2\pi f n}, k = 0, 1, \dots, K-1 \quad (1)$$

$Y_k(f)$ is k -th eigenspectral, K is the number of windows, the length of DPSS is N . Equation (1) can be rewritten in the form of matrix as

$$\mathbf{Y}_k = \mathbf{F}_N (\mathbf{v}^k \mathbf{x}_n) \quad k = 0, 1, \dots, K-1 \quad (2)$$

\mathbf{v}^k is a $N \times N$ matrix and can be represented as

$$\mathbf{v}^k = \begin{bmatrix} v_1^{(k)} & 0 & \dots & 0 \\ 0 & v_2^{(k)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_N^{(k)} \end{bmatrix} \quad (3)$$

From CS theory

$$\mathbf{y}_m = \Phi \mathbf{x}_n, m = 1, 2, \dots, M \quad (4)$$

Inserting equation (2) into (4), it is

$$\mathbf{y}_m = \Phi \mathbf{x}_n = \Phi (\mathbf{v}^k)^{-1} \mathbf{F}_N^{-1} \mathbf{Y}_k \quad (5)$$

The k -th eigenspectrum \mathbf{Y}_k can be reconstructed by OMP or ROMP algorithm. And it can be expressed as

$$A(f) = [Y_1(f), Y_2(f), \dots, Y_K(f)] \quad (6)$$

Singular value decomposition of $A(f)$ is

$$A(f) = \sum_{k=0}^{K-1} \sigma_k(f) U_k(f) V_k^H(f) \quad (7)$$

$\sigma_k(f)$ is a singular value of $A(f)$, and $|\sigma_0(f)| \geq |\sigma_1(f)| \geq \dots \geq |\sigma_K(f)|$, therefore $|\sigma_0(f)|^2$ is an estimation of PSD.

III. CLASSIFICATION AND COMPARISON OF MEASUREMENT MATRIX

Now, the measurement matrix which has been put forward to dividing into three types on the whole^[9]: random measurement matrix, structured measurement matrix and deterministic measurement matrix.

A. Random Measurement Matrix

Random measurement matrix includes Gaussian, Bernoulli, sub-gaussian^[12], very sparse random matrix^[13] and so on. The common ground of these matrixes is: the elements in matrixes submit to one distribution independently, they are irrelevance with most sparse signals and demand less measurements to

recovery exactly. But they all need biggish storage space and upper complexity.

Each element in Gaussian random matrix satisfies that $\Phi \in \mathbf{R}^{M \times N}$: $\Phi(i, j) \sim N(0, 1/\sqrt{M})$. N is the signal length, K denotes sparse degree, So it meets RIP to a great extent only requires $M \geq cK \log(N/K)$ measurements, c is a small constant.

Each element in Bernoulli random matrix meets that $\Phi \in \mathbf{R}^{M \times N}$: $\Phi_{i,j} = \begin{cases} +1/\sqrt{M} & \text{probability } 0.5 \\ -1/\sqrt{M} & \text{probability } 0.5 \end{cases}$, the characters are similarity to Gaussian matrix.

B. Structured Randomly Measurement Matrix

Structured randomly measurement matrix includes Fourier, part of orthogonal, Hadamard, Toeplitz random measurement matrixes and so on. These kinds of matrixes extract M rows from an $N \times N$ orthogonal matrix.

The structure of part of orthogonal matrix is: building U , an $N \times N$ orthogonal matrix, we can get a $M \times N$ matrix which extract M rows from U randomly. In the premise of matrix size, to make signal reconstruct accuracy, the sparsity K should meet the expression that $K \leq cM / (\mu^2 (\log(N))^6)$, and $\mu = \sqrt{M} \max_{i,j} |U_{i,j}|$. When $\mu = 1$, part of orthogonal matrix is part of Fourier matrix and the Fourier matrix satisfies that $K \leq cM / (\log(N))^6$. The matrix utilizes the high speed of fast Fourier transform, but it is incoherent with signals only in the time domain or frequency domain.

Partial Hadamard matrix is structured in a way that sampling M rows from a generated $N \times N$ Hadamard matrix. Because of the inherent characteristics of Hadamard, N must match the condition of $N = 2^k$, $k = 1, 2, \dots$, this greatly limits the application of the matrix.

Toeplitz matrix is constructed of the generated vector, the process is through the cyclic shift to realize. Above all, generating a vector $u = (u_1, u_2, \dots, u_N) \in \mathbf{R}^N$ and U which is the corresponding rotation matrix, then selecting one of the M rows to structure $M \times N$ Toeplitz matrix. Typically, the value of u is ± 1 and each element submits independently to Bernoulli distribution. The cyclic shift can be easily implemented in hardware, which is the main reason for Toeplitz matrix widely studied and applied. For K -sparsity signal, when K and M satisfy $M \geq cK / \log(N/\varepsilon)$, the original sparse signal can accurately reconstruct at the probability of $1 - \varepsilon$.

C. Deterministic Measurement Matrix

Deterministic measurement matrix is put forward to overcoming the shortcomings of random matrixes for long time simulation and difficult to achieve in equipment. It has an irreplaceable excellent quality. Ronald A.Devore^[14] proposed polynomial deterministic matrix which is a new research direction. But the research in this field has just started, there still are many problems need to be further studied.

Gaussian matrix, Toeplitz matrix, Fourier matrix, Hadamard matrix, part of orthogonal matrix and Bernoulli matrix are the mainly matrixes compared in the paper.

IV. SIMULATION ANALYSIS

Simulation analysis 1: Under the influence of white Gaussian noise, estimating the PSD of single-tone, multi-tone and QPSK signals based on MTM-SVD-CS, the article is simulated the effect which measurement matrixes have on the signal spectrum estimation. Six kinds of matrixes are talked above, compression ratio $M/N=0.7$. Signal parameter sets as shown in Table 1, and spectrum estimation shows in figure2 to figure4.

TABLE I SPECIFIC PARAMETER SETTING OF SIGNALS

Signal Style	Length/ Sampling Point	Sampling Rate	Specific Parameters	SNR	Simulation Number
Single-tone	$N/nfft=1024$	$f_s=1024$	Center frequency =400	25	1000
Multi-tone	$N/nfft=1024$	$f_s=1024$	Multi-tone number=7	25	1000
QPSK	$N/nfft=2048$	$f_s=1600$	symbols=128 Center frequency =2000	25	1000

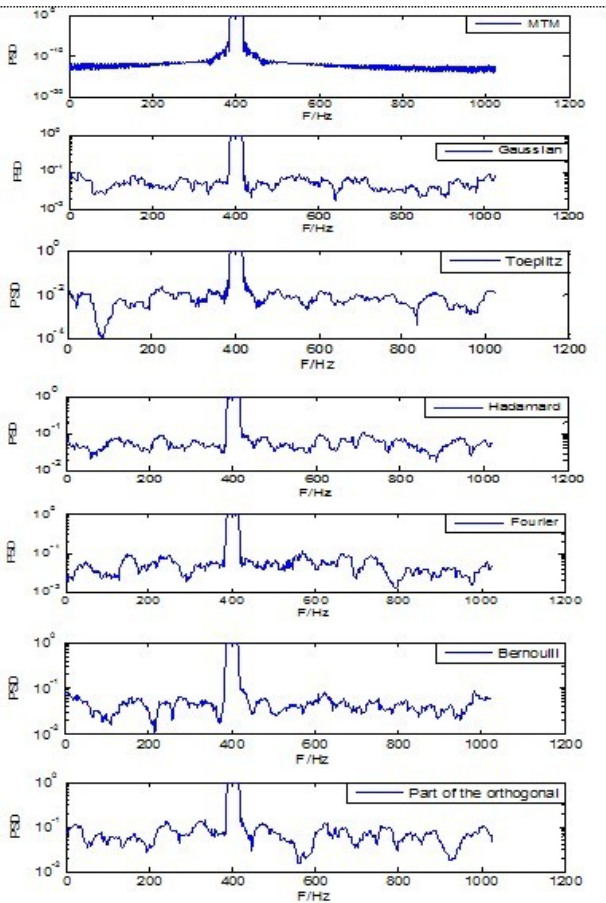


Figure.2 PSD of Single-tone

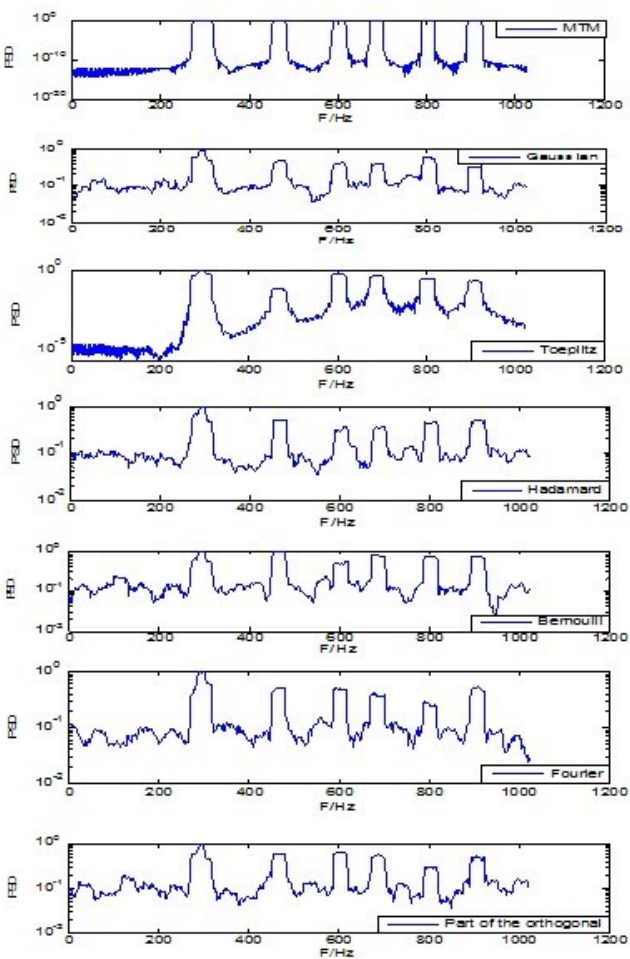


Figure.3 PSD of Multi-tone

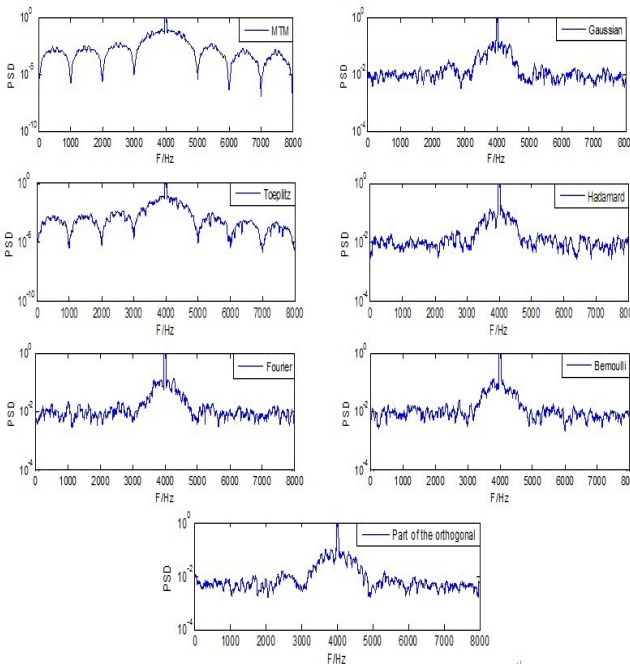


Figure.4 PSD of QPSK

From the three diagrams it can be found: These measurement matrixes can be approximated to recover the signal power spectrum when the compression rate is 0.5. It means that these stochastic matrixes can be successfully used in CS. It is noted that, after CS, the power spectrum comes out the apparent loss in amplitude.

Simulation analysis 2: The six kinds of matrixes have an effect on signals spectrum estimation at different compression rate. Signal parameters setting as Table 1. The expression of normalized error minimization is

$$NMSE = \frac{1}{N} \sum_{n=1}^N \left[\frac{(\hat{s}(n) - s(n))^2}{s^2(n)} \right] \quad (8)$$

s is the PSD vectors in Nyquist rate, \hat{s} is the estimation PSD vectors in CS rate. Compression rate M/N changes from 0 to 1. The spectrum estimation $NMSE$ with compression rate curves is shown in figure 5 to figure 7.

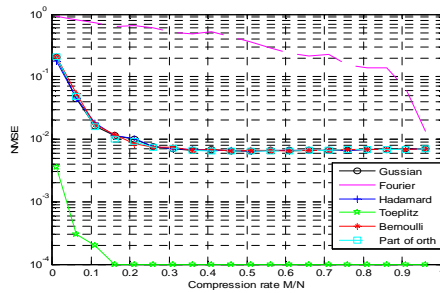


Figure.5 M/N and $NMSE$ curve-Single-tone

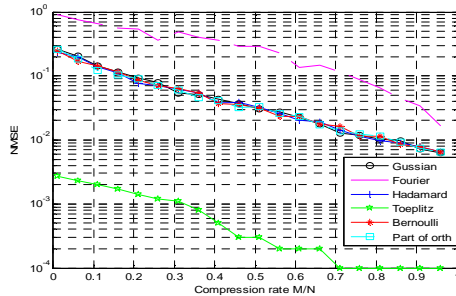


Figure.6 M/N and $NMSE$ curve-Multi-tone

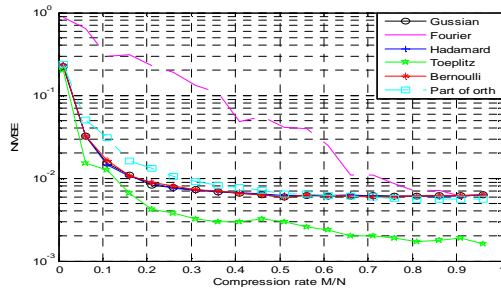


Figure.7 M/N and $NMSE$ curve-QPSK

Thus, there are some conclusions as follows:

(1) As the compression rate increases, $NMSE$ of power spectrum estimation decreases gradually; Toeplitz matrix gets the minimum $NMSE$, and Fourier matrix gets the maximum $NMSE$; and other four measurement matrixes have the similar properties.

(2) For different signals, it can be found a certain compression rate which makes these stochastic matrixes recovery the signal power spectrum in high probability;

(3) The more sparse the signal is, the better performance for recovering power spectrum at the same compression rate (by single-tone and multi-tone);

V. THE CONCLUSION

By summarizing the classification of the Measurement Matrix, a detailed simulation was given to compare the estimation performance of six kinds of measurement matrixes, the conclusion is made that Toeplitz matrix has the optimal performance. The conclusion is very important in selecting measurement matrix in compressive spectrum sensing. As a result of these random matrixes is random and difficult to realize in hardware circuit, deterministic matrix is studied and whether existing an optimal deterministic matrix for a stable PSD algorithm is the next step to solve.

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