

# Recognition of Cyclostationary Signals Smoothed

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**Abstract**—In the identification process of modulated signal based cyclic spectrum, it is chief to be synchronized for the received signal. The article focuses on the reception signal which is not synchronized. When uniform distribution variable delay exists, it will not have cyclostationary, namely it is converted into a stable signal, which limits the usable range of cyclic spectrum identifying signals. Meanwhile, in this situation, through extracting characteristic parameters, simulation shows that the method is perfect for modulation identification under the condition that SNR is 0-50dB.

**Keywords**—cyclostationarity; uniform distribution; modulated signal recognition

## I. Introduction

With the demand of information increases, in order to send a large number of messages effectively in different channels, different modulation mode is devised for different communication system. Accordingly, the signal modulation recognition method attracts more and more attention. Modulation recognition methods mainly include two categories: (1) the maximum likelihood method of hypothesis testes based on the theory of decision-making characteristics; (2) the statistical pattern recognition method based on feature extraction. In the present, most recognition methods use statistical pattern recognition method, mainly about two aspects: on the one hand, analyse the signal characteristic, extract and choose parameters which reflect the characteristics of the signal difference better, such as wavelet theory, chaos theory, fuzzy theory, statistical theory and other mathematical analysis methods. On the other hand, the methods, which include neural networks, genetic algorithms, clustering algorithm, Kalman filtering, and other adaptive signal processing methods, can improve the automation and intelligence of recognition.

As the basic tool to study the signal cyclostationarity, the cyclic spectrum has advantages on high resolution、capability of anti-interference and noise, the low environmental sensitivity of the channel and so on. Then it is often regarded as the characteristic parameter of the signal recognition. However, [1] - [7] are only for the synchronization signal. When the received signal is not synchronized and a uniformly distributed random variable delay exists, the signal will not show cyclostationarity and become stationary signal.

This essay studies the cyclostationarity of the signal deeply. It is organized as follows. In Section II, a principle of

cyclostationary signals smoothed is given, and the simulation results based on cyclic spectrum are showed in Section III. In Section IV, the recognition methods of random delay modulated signals are proposed in the Gaussian and Section V draws a short conclusion of this paper.

## II. Cyclostationary Signals Smoothed

Suppose  $x(t)$  as a digital modulated signal, which is expressed as follows:

$$x(t) = a(t) \cos(2\pi f_c t + \varphi) \quad (1)$$

$$\text{where } a(t) = \sum_{n=-\infty}^{\infty} a_n q(t - nT_s), \quad q(t) = \begin{cases} 1, & |t| \leq T_s/2 \\ 0, & \text{other} \end{cases}$$

obviously, its mean value is 0, and its autocorrelation function is given as:

$$\begin{aligned} R_x(t, t+\tau) &= E[x(t)x(t+\tau)] = E[a(t) \cdot \cos(2\pi f_c t + \varphi) \cdot a(t+\tau) \cdot \cos(2\pi f_c (t+\tau) + \varphi)] \\ &= \frac{1}{2} E \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_n q(t - nT_s) \cdot a_m q(t+\tau - mT_s) \cdot (\cos(2\pi f_c (2t+\tau) + 2\varphi) + \cos(2\pi f_c \tau)) \right] \\ &= \frac{1}{2} E \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_n q(t - nT_s) \cdot a_m q(t+\tau - mT_s) \right] (\cos(2\pi f_c (2t+\tau) + 2\varphi) + \cos(2\pi f_c \tau)) \\ &= \frac{1}{2} (\cos(2\pi f_c (2t+\tau) + 2\varphi) + \cos(2\pi f_c \tau)) E \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_n q(t - nT_s) \cdot a_m q(t+\tau - mT_s) \right] \\ &= R_x(t+T, t+\tau+T) \end{aligned} \quad (2)$$

From formula (1) and (2), a general modulated signal is a second order cyclostationary random process on the time interval  $T$ , (make  $T = m/2f_c = T_s$ ). But if there is a uniformly distributed random variable  $D$  on  $[0, T]$ . Then  $x(t-D)$  is

$$x(t-D) = a(t-D) \cos(2\pi f_c (t-D) + \varphi) \quad (3)$$

The mean value and autocorrelation function of the delayed signal can be obtained as:

$$\begin{aligned} m_x(t-D) &= E(a(t-D) \cos(2\pi f_c (t-D) + \varphi)) \\ &= E \left( \sum_{n=-\infty}^{\infty} a_n q(t - nT_s) \cos(2\pi f_c (t-D) + \varphi) \right) \\ &= 0 \end{aligned} \quad (4)$$

$$\begin{aligned}
R_x(t, t+\tau) &= E[x(t-D)x(t-D+\tau)] = E[a(t-D) \cdot \cos(2\pi f_c(t-D) + \varphi) a(t+\tau-D) \cdot \cos(2\pi f_c(t+\tau-D) + \varphi)] \\
&= \frac{1}{2} E \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_n q(t-D-nT_s) \cdot a_m q(t+\tau-D-nT_s) \cdot (\cos(2\pi f_c(2t+\tau-2D) + 2\varphi) + \cos(2\pi f_c(\tau))) \right] \\
&= \frac{1}{2} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left[ E(a_n \cdot a_m \cdot q(t-D-nT_s) q(t+\tau-D-nT_s)) \cdot (E(\cos(2\pi f_c(2t+\tau-2D) + 2\varphi) + \cos(2\pi f_c(\tau)))) \right]
\end{aligned} \quad (5)$$

Then it can be got as:

$$\begin{aligned}
&E(\cos(2\pi f_c(2t+\tau-2D) + 2\varphi)) \\
&= \frac{1}{T} \int_0^T \cos(2\pi f_c(2t+\tau-2D) + 2\varphi) dD = 0
\end{aligned} \quad (6)$$

Put (6) into (5), the autocorrelation function will be similar to:

$$\begin{aligned}
R_x(t, t+\tau) &= \frac{1}{2} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left[ E(a_n \cdot a_m \cdot q(t-D-nT_s) q(t+\tau-D-nT_s) \cos(2\pi f_c(\tau))) \right] \\
&= \frac{1}{2} E \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_n a_m q(t-D-nT_s) q(t+\tau-D-nT_s) \cos(2\pi f_c(\tau)) \right] \\
&= \frac{1}{2} \cos(2\pi f_c(\tau)) \cdot E \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_n a_m q(t-D-nT_s) q(t+\tau-D-nT_s) \right] \\
&= R_x(\tau)
\end{aligned} \quad (7)$$

when  $\tau = 0$ , the variance of delayed signal is:

$$R_x(0) = \frac{1}{2} E \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_n a_m q(t-D-nT_s) q(t-D-nT_s) \right] \quad (8)$$

From formula (4) and (7), it can be deduced that the delayed signal  $x(t-D)$  obeys wide-stationarity, and mean value is 0. Similarly, by formula (8), the variance is constant, only depends on the modulation method.

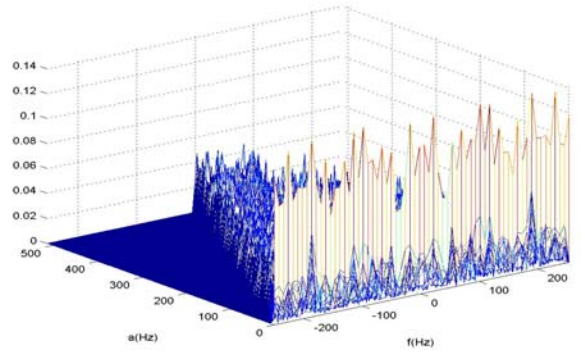
According to the analysis above, the variances of 2ASK, BPSK and 2FSK have features. For 2ASK, its variance is  $\frac{N}{P}$  (where  $N$  is the number of samples,  $P$  is the probability of code 1). However, for BPSK, whose binary codes are -1 and 1, so its variance is larger than 2ASK's. And it is also inferred that 2FSK can be viewed as the composition of two 2ASK signals, this is to say, 2FSK's variance is larger than 2ASK's, too. So we can choose the difference of variances as the characteristic parameter to distinguish 2ASK, BPSK and 2FSK.

### III. Simulation Experiments

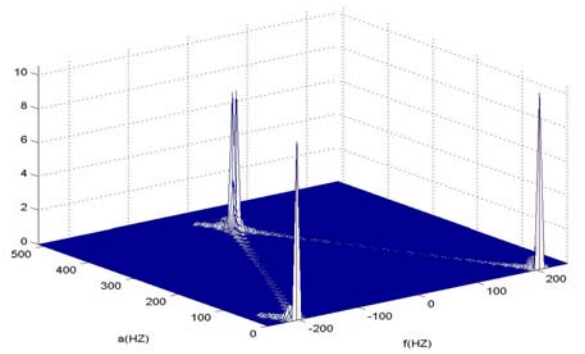
The original signal is the second order cyclostationary, which exists spectral lines at  $\alpha$  cross-section, however, the

modulated signal with a uniform variable delay is stationary. The random delay signal's and the original signal's cyclic spectrum structures are shown in Fig.1, using frequency-smoothed method in [8] to estimate signal's spectral density.

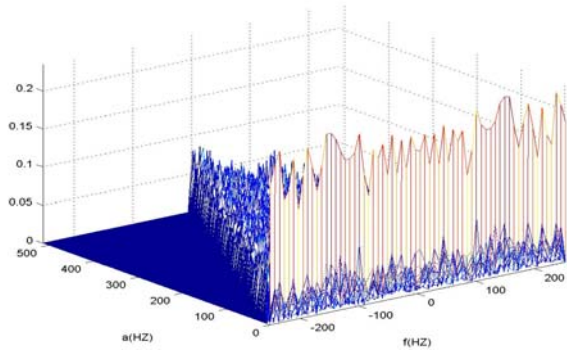
The simulation parameters: 2ASK signal: the symbol rate is 2, the carrier frequency is 204Hz; the sampling frequency is 512Hz; the number of frequency-domain smoothing points is 80; BPSK signal simulation parameters are the same; the 2FSK signal: other parameters are constant, except  $f_1 = 204Hz$ ,  $f_2 = 180Hz$ . But when  $f_1 = 204Hz$ ,  $f_2 = 202Hz$  and other parameters are the same (labeled 2FSK-t, its frequency spectrum shows a single peak), the cyclic spectrum after a random delay and 2FSK-t signal's cyclic spectrum are graphed in Fig.1 (g), (h).



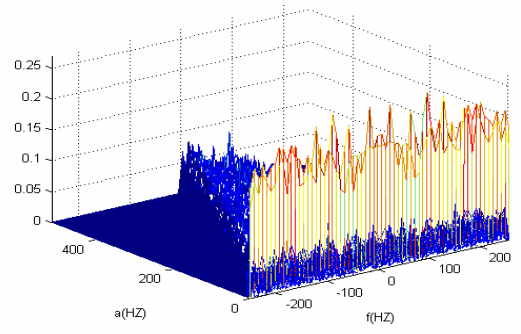
(a) Random delay 2ASK signal's cyclic spectrum.



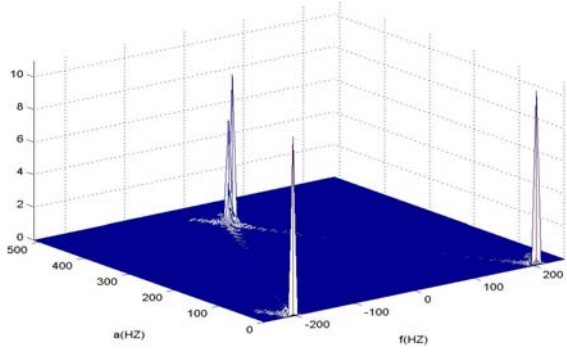
(b) Original 2ASK signal's cyclic spectrum



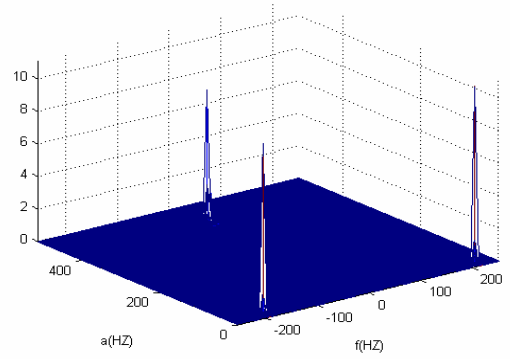
(c) Random delay BPSK signal's cyclic spectrum



(g) Random delay 2FSK-t signal's cyclic spectrum.



(d) Original BPSK signal's cyclic spectrum

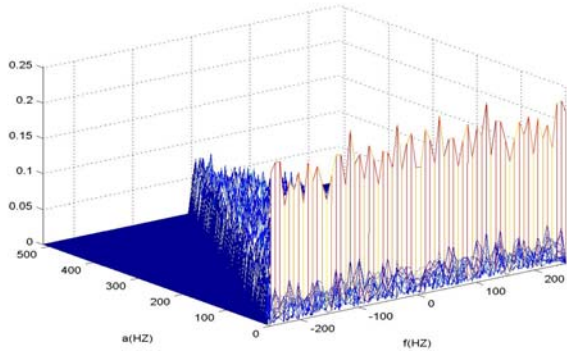


(h) Original 2FSK-t signal's cyclic spectrum

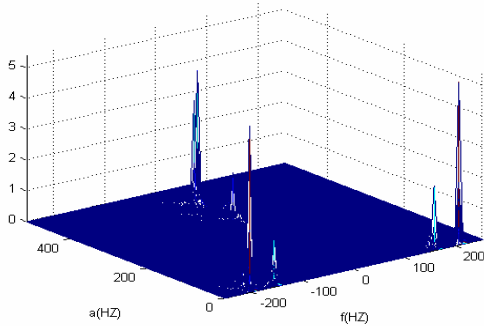
Figure 1. Random delay modulated signal's and the original modulated signal's cyclic spectrum.

In Fig.1, it needs to be emphasized that: in Fig.1-(g)、(h), the original 2FSK-t cyclic spectrum appears spectral lines at the center frequency  $f_1' = \frac{f_1 + f_2}{2}$   $f_2' = -\frac{f_1 + f_2}{2}$  and

$\alpha = f_1 + f_2$ . While compared to the delay 2FSK signal cyclic spectrum, that of 2FSK-t does not change a lot. In the recognition process, it is not necessary to detect that whether the signal spectrum is a single or double peaks for the delayed frequency shift modulation signal, which can be identified from the single-carrier signals.



(e) Random delay 2FSK signal's cyclic spectrum

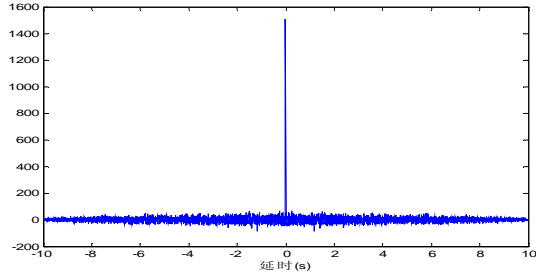


(f) Original 2FSK signal's cyclic spectrum

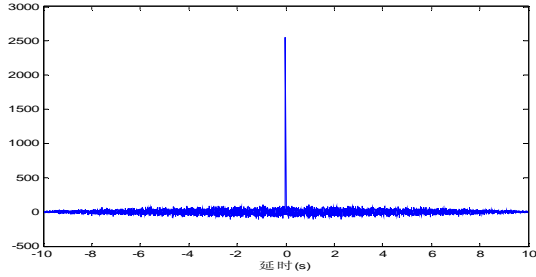
#### IV. Recognition Simulation Results

##### A. Characteristic Parameters

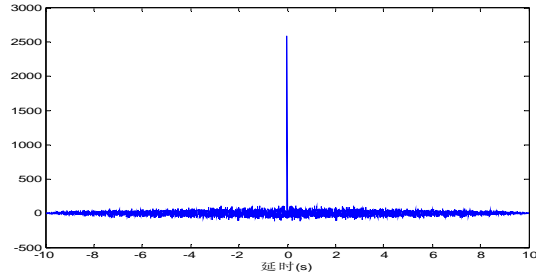
Random delayed signal is stationary that its autocorrelation function is a pulse at  $\tau = 0$ . It is analyzed: the random delayed single autocorrelation functions are shown in Fig.2.



(a) Random delay 2ASK signal autocorrelation



(b) Random delay BPSK signal autocorrelation.



(c) Random delay 2FSK signal autocorrelation

Figure 2. Single carrier delay signals' autocorrelation functions

Through observing the results of autocorrelation functions, it is not difficult to find obvious differences on signal amplitudes of autocorrelation functions, so we choose the maximum value of the autocorrelation function as a characteristic parameter  $d$ ;

$$d = \max(s(i)) \quad (i = 1 \sim N) \quad (9)$$

where  $N$  is the number of sampling,  $s(i)$  is signal's autocorrelation function.

### B. Delayed Signal Recognition Simulation

- The simulation conditions are as above, the measured characteristic parameter  $d$  is graphed in Fig.3 at symbol rate  $R_b = 2$ ,  $R_b = 10$  respectively in the Gaussian channel, which is obtained by Monte Carlo simulation 100 times.

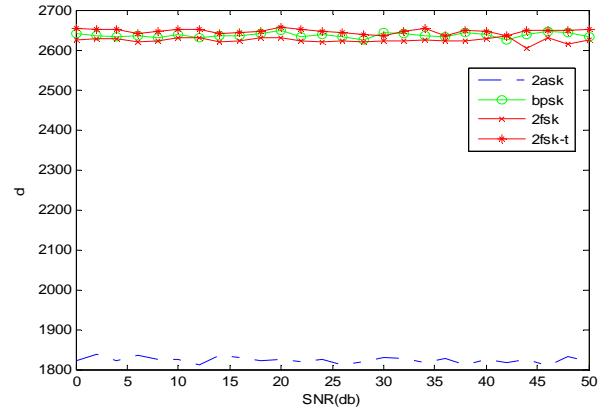


Figure 3. The result of characteristic parameter  $d$

To illustrate the value of parameter  $d$ , it is drew at symbol rate  $R_b = 2$ ,  $R_b = 10$  respectively in Table.1.

Table.1. the value of parameter  $d$

SNR(dB) 0-50	the value of parameter $d$			
	2ASK	BPSK	2FSK (2FSK-t)	Threshold ( $s_1$ )
$R_b=2$	1817-1835	2639-2651	2630-2660	1830-2610 Recognize ASK
$R_b=10$	1820-1830	2620-2650	2610-2660	

Because the delay signal's autocorrelation function is independent of the carrier frequency, the delay 2FSK-t's and the delay 2FSK's signal autocorrelation functions are the same, their characteristic parameter  $P$  values are substantially the same. Moreover, the amplitude values of delay 2FSK signal's and the delay BPSK signal's the autocorrelation functions are approximately the same, which are consistent with the theoretical results. From Table.1, adopt  $s_1$  (1830-2610) as threshold value to separate 2ASK from BPSK, 2FSK. Through lots of tests, when the SNR is 0-50dB, the recognition rate can reach 98% at  $s_1=2100$ . Next, discuss the identification between 2FSK and BPSK.

- Extract parameter  $q$  to identify delay BPSK and 2FSK.  $q$  can be given as:

$$q = \max |fft[s_{cn}(i)]^2 / N| - \max |fft[\varphi_{cn}(i)]^2 / N| \quad (10)$$

where  $s_{cn}(i)$  is called zero center normalized instantaneous amplitude as follows:

$$s_{cn}(i) = s_n(i) - 1 \quad s_n(i) = \frac{s'(i)}{\text{mean}[s'(i)]} \quad (11)$$

where  $s'(i)$  is signal's instantaneous amplitude. Similarly,  $\varphi_{cn}(i)$  is zero center normalized instantaneous phase as:

$$\varphi_{cn}(i) = \varphi_n(i) - 1 \quad \varphi_n(i) = \frac{\varphi(i)}{\text{mean}[\varphi(i)]} \quad (12)$$

where  $\varphi(i)$  is signal's instantaneous phase.

Simulation conditions are the same, parameter  $q$  is simulated in Fig.4, which is obtained by Monte Carlo simulation 100 times.

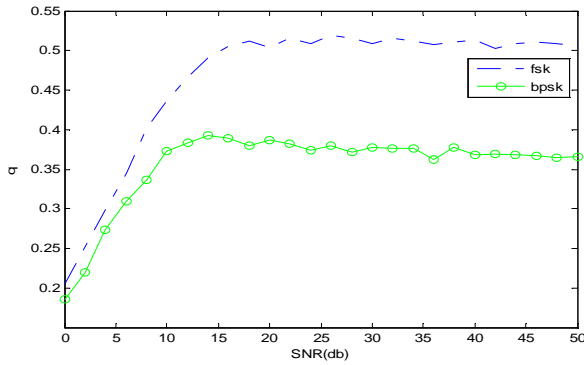


Figure 4. The result of characteristic parameter  $q$

It, threshold selected is correct or not, will greatly affect the performance of the modulation recognition algorithm. In Gaussian or multi-path channels, threshold should be adjusted accordingly based on channel information, this article uses adaptive threshold. It is given that how  $q$  threshold  $s_2$  changes with 0-50dB in AWGN channel in Fig.5.

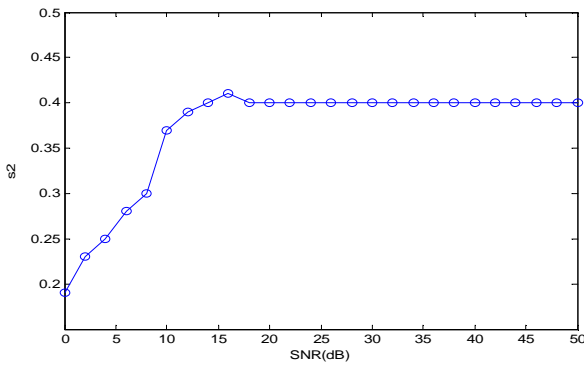


Figure 5. The result of threshold  $s_2$

When  $SNR < 4\text{dB}$ ,  $s_2 = 0.25$ , recognition rate only reaches 75%; and When  $SNR$  is 4-50dB,  $s_2 = 0.39$ , recognition rate can reach 98%.

## V. Conclusion

In the process of signal recognition, cyclic spectral plays an important role. The signal must be synchronized at first, otherwise, if there exists a random delay variable, it will make the second order cyclostationary signal into stationary signal. Cyclic spectrum will lose significance. Meanwhile, when there is a random variable delay, it can reach a high recognition rate through extraction of characteristic parameters in the Gaussian channel, This article comes up with some references to make the signal identification based on cyclic-spectrum more efficient, more accurate.

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