The Model and Algorithm of the Traffic Flow Assignment in Road Network

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Abstract. the article studies the limit cases of the traffic flow in road network, such as the problem of the Biggest Traffic Flow & the Shortest Time Flow and the Optimal Flow in road network. The model and algorithm of the Biggest Traffic Flow & the Shortest Time Flow are presented, and an explorative study about the Optimal Flow is made in this paper.

Introduction

With road traffic being more and more crowded, the control of the traffic flow from 'disorder' to 'order' becomes a hot topic in traffic study. The traffic flow assignment is one of the main measures of controlling the traffic flow. The article studies the limit cases of the traffic flow in road network: how to assign the traffic flow to reach the biggest traffic flow, and also the shortest time flow; how to assign the traffic flow reach the optimal flow under the limit cases.

In order to simplify the problem, assuming that the traffic flow has features of being directive, fine timeliness and controllable, the traffic flow in road networks can be seen as a peculiar traffic flow that has only one Origin and only one Destination in road network.

The Model and Algorithm of the Biggest Traffic Flow & the Shortest Time Flow

To Construct Model. Assuming 'f'is an available traffic flow in the road network 'D', 'p' is a path from starting point 'v_s' to terminal point 'v_t', so the time that 'f' flows from starting point 'v_s' to terminal point 'v_t' in the road network 'D' can be presented:

$$T(f) = \max_p T(p) = \max_p \left\{ \sum_{(v_i, v_j) \in p} t_{ij} \left(q_{(v_i, v_j)} \right) \right\} \tag{1}$$

t_{ij} —— the time that the traffic flow travels in the path (v_i, v_j)

$T(p)$ —— the time that the traffic flow travels in the path'p' from starting point 'v_s' to terminal point 'v_t'.

$q_{(v_i, v_j)}$ —— the number of vehicles in the path (v_i, v_j)

So the objective function of the Biggest Traffic Flow & the Shortest Time Flow can be shown:

$$\begin{cases} \max\limits_{f} v(f) \\ \min\limits_{f} \max\limits_{p} \left\{ \sum\limits_{(v_i,v_j)\in p} t_{ij}\left(q_{(v_i,v_j)}\right) \right\} \\ s.t. \begin{cases} 0 \le f_{ij} \le c_{ij} \\ \sum\limits_{j} f_{ij} - \sum\limits_{j} f_{ji} = 0 \quad v_i \ne v_s, v_t \\ \sum\limits_{j} f_{sj} - \sum\limits_{j} f_{js} = v(f) \quad v_i = v_s \\ \sum\limits_{j} f_{tj} - \sum\limits_{j} f_{jt} = -v(f) \quad v_i = v_t \end{cases} \end{cases} \tag{2}$$

Incremental Assignment Method. Incremental Assignment Method is an approximate equilibrium traffic assignment method. The idea of the method is to divide the traffic flow into N parts, then assigns each part in the road network, and assigns it in the corresponding shortest path of the road network each time.

Algorithm Analysis.

Initial conditions can be assumed as the following：

- the capacity of paths in the road network is $C_a, \forall a$;
- the number of vehicles be assigned to the path cannot more than the capacity of the path ;
- the wait time is negligible in joint positions;
- the travel time function uses the BPR function.

The algorithm steps：

Step 0: select the right unit q , and make $n=1, x_a^0 = 0, \forall a$, set $A = \phi$;

Step 1: $t_a^n = t_a(x_a^{n-1})$, $\forall a$, calculate the shortest path;

Step 2: assigning ‘q’ into the shortest path，then $x_a^n = x_a^{n-1} + q, \forall a$;

Step 3: judge $x_a^n \ge C_a, \forall a$, if it is ‘yes’, then $B = \{a\}, A = A \cup B$, at the same time, delete the path ‘a’ in the road network，gains a simplified road network. If it is ‘no’, go to step1;

Step 4: are there the path from starting point ‘v_s’ to terminal point ‘v_t’ ?，if it is ‘yes’，then $n = n+1$，go to Step 1；If it is ‘no’，then return the assignment number of vehicles of each path, i.e. the shortest time flow f^*, the capacity of the road network $C = nq$ and the set A of saturated paths.

The Model of the Optimal Flow

The traffic flow’s speed will be slow when the number of vehicles reach the capacity of the road network.

According to the travel time function (Fig.1), the travel time in the path t_a is a monotonically increasing function of the number of vehicles in path a . $q_a \uparrow$ then $t_a \uparrow$, but t_a isn’t a linear function of q_a . The change of t_a isn’t proportionate to the change of q_a .

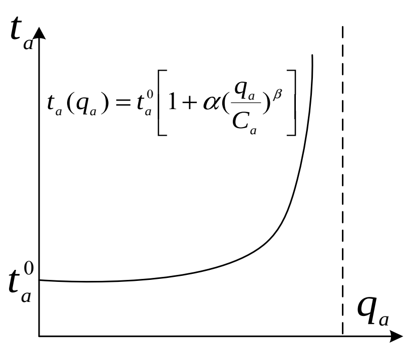

Fig.1

Although the number of vehicles reaches the capacity of the road network, the efficiency of the road network doesn't reach the maximum value. In order to get the best efficiency, it is necessary to develop an Optimal Flow.

Given the Time Scope, the Optimal Flow.

Request descriptions: there are enormous relief goods which should be sent to the strichen area from the supply bases. It is known of the nature of the road network. How to make the transport project, the efficiency of the road network is the biggest during the time scope 'T'.

If the following are assumed:

- equilibrium traffic assignment;
- the wait time is negligible in joint positions;

the model can be constructed as:

$$Q_{\max} = \max\left[(T - \frac{\sum_i \sum_j x_{ij} t_{ij}}{Q^*}) Q^* \right] \quad (3)$$

s.t. $Q^* < Q$

$Q_{\max}$ —— maxinum quantity can be sent during the time scope 'T';

Q^* —— assign the Optimal flow each hour;

$Q^{'}$—— capacity of the road network;

x_{ij} —— the number of vehicles when it reach equilibrium traffic assignment in the path (v_i, v_j);

t_{ij} —— the travel time when it reach equilibrium traffic assignment in the path (v_i, v_j).

This is an approximate model by step-by-step testing according to different values of Q^*. It is difficult to get the exact solution; however, the approximate solution has good practicability. Example:

$$Q^* = 0.1Q^{'},\ 0.2Q^{'},\ 0.3Q^{'},\ 0.4Q^{'},\ 0.5Q^{'},\ 0.6Q^{'},\ 0.7Q^{'},\ 0.8Q^{'},\ 0.9Q^{'},\ 1.0Q^{'} \quad (4)$$

According to different values of the time scope 'T', the returns of model testing may be shown in the following two cases:

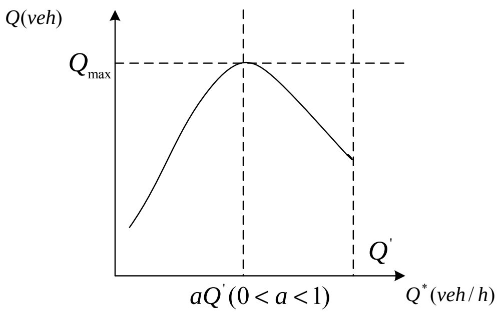

Fig.2

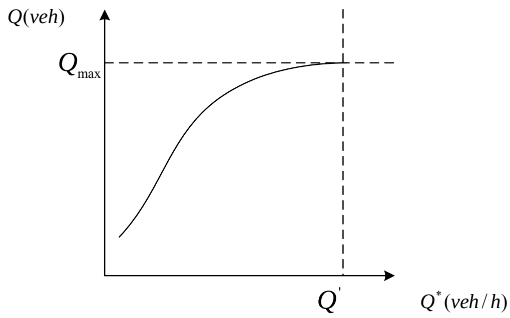

Fig.3

When the time scope 'T' is smaller, it gets the best efficiency not in the point of capacity of the road network(Fig.2); when the time scope 'T' is bigger, it gets the best efficiency in the point of capacity of the road network (Fig.3).

Given Transportation Amounts, the Optimal Flow.

Request descriptions: there are enormous relief goods which should be sent to the strichen area from the supply bases. It is known of the nature of the road network. How to make the transport project, sending these goods to the strichen area in the shortest time .

Assuming the conditions are ibid

construct model:

$$T_{\min} = \min\left[\frac{\sum_i \sum_j x_{ij} t_{ij}}{Q^*} + \frac{Q}{Q^*} \right] = \min\left[\frac{\sum_i \sum_j x_{ij} t_{ij} + Q}{Q^*} \right] \tag{5}$$

s.t. $Q^* < Q$

$T_{\min}$ is the shortest time of Q across the road network。 The other parameters ibid. This is also an approximate model by step-by-step testing according to different values of Q^*. For example：

$$Q^* = 0.1Q'，0.2Q'，0.3Q'，0.4Q'，0.5Q'，0.6Q'，0.7Q'，0.8Q'，0.9Q'，1.0Q' \tag{6}$$

According to different values of relief goods ‘Q’ , the returns of model testing may be shown as the following two cases:

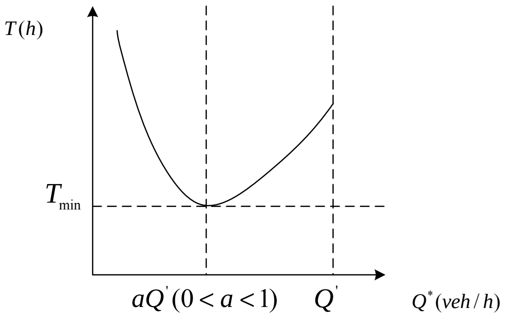

Fig.4

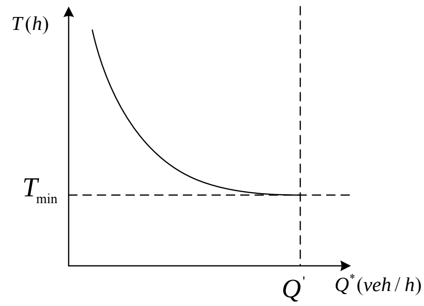

Fig.5

When ‘Q’ is smaller, it gets the shortest time not in the point of capacity of the road network(Fig.4); when ‘Q’ is bigger, it gets the shortest time in the point of capacity of the road network (Fig.5).

Conclusion

The traffic flow assignment is one of main measures of controlling the traffic flow. This article studies the limit cases of the traffic flow in road network, such as the problem of the Biggest Traffic Flow & the Shortest Time Flow and the Optimal Flow in road network. The research is useful for the problem about the limit cases of the traffic flow.

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