

## Research on the Distribution of the Repeaters

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**Abstract.** This paper discusses the optimization problem of the repeaters' distribution in a circular area with a radius of 40 miles. The hexagonal Cellular model is used to achieve regional coverage. We divide the frequency band of repeaters as well as combining different private line (PL) to prevent the interference between the signals. Considering the situation of 1000 users in this region online at the same time, when the population is uniformly distributed, we combine the maximum number of assumptions of PL and build a mathematical model to get an accurate coverage program; When the number of online users turns into 10,000, we modify the model, introducing the normal distribution to constrain the population distribution. It divides this region into 20 small areas and a reasonable radiation radiuses and the distribution of repeaters can be get respectively.

### Introduction

The VHF radio spectrum is the kind of radio wave whose band is 30 MHz to 300MHz in wireless communication. Due to its higher frequency, the sky-wave will generally penetrate the ionosphere and fire to the space, rather than be reflected back to the ground. As VHF has a limited capacity of diffraction, it transmits by line-of-sight transmission. The propagation distance is restricted not only by sight, but also may by mountains or tall buildings. To spread even further, we must rely on repeaters, for a widen dissemination. Repeaters pick up weak signals, amplify them, and retransmit them on a different frequency. However, repeaters can interfere with one another unless they are far enough apart or transmit on sufficiently separated frequencies. In addition to geographical separation, the "continuous tone-coded squelch system" (CTCSS) can be used to mitigate interference problems. This system associates to each repeater a separate sub audible tone that is transmitted by all users who wish to communicate through that repeater. The repeater responds only to receive signals with its specific PL tone. With this system, two nearby repeaters can share the same frequency pair (for receive and transmit); so more repeaters (and hence more users) can be accommodated in a particular area.<sup>[1]</sup>

Now, for a circular flat area of radius 40 miles radius, we try to calculate the minimum number of repeaters necessary to accommodate 1,000 (or 10000) simultaneous users. Assume that the spectrum available is 145 to 148 MHz, the transmitter frequency in a repeater is either 600 kHz above or 600 kHz below the receiver frequency, and there are 54 different PL tones available.

### Model Preparations

**Effective Coverage in Model.** We use the same regular polygon to cover the plane, and each vertex is focused around by angles of each regular polygon<sup>[2]</sup>, we suppose the number of the regular polygon is  $x$ , the polygon has  $y$  edges. Because the sum of horns should be  $360^\circ$ , so the following equations are established:

$$\begin{cases} \frac{x(y-2)}{y} = 2 \\ x > 2 \\ y > 2 \\ x-2, y-2 \in \mathbb{Z}^* \end{cases} \quad (1)$$

Solution of equations is  $y=3, 4$  or  $6$ . This proves that there are only three cases existing when covering the surface completely with a certain kind of regular polygon. That is covered by the triangle, square or hexagon.

By using the inscribed regular  $N$ -gon of the same radius to cover one plane, the public area of two adjacent circles occupies for rate  $\Phi_y$  for one circle.

$$\Phi_y = \frac{2\pi / y - \sin(2\pi / y)}{\pi} \quad (2)$$

When  $y=6$ ,  $\Phi_y=5.77\%$ , is the minimum. So the largest covered area can be regular hexagon with the minimum node when comes to the inscribed circle with the same radius.

**Repeater's Coverage Radius.** Now we are calculating the coverage radius of a repeater (reference to MOTOTRBO XiR R8200<sup>[3]</sup>) :

The transmit power of a repeater:

$$P_t = 30dBm \quad (3)$$

The free-space loss  $L_{bf}$  is

$$L_{bf} = 32.45 + 20\lg F + 20\lg D \quad [4] \quad (4)$$

In which,  $F$ : frequency (MHz),  $F=145\text{MHz}$ ,  $D$ : distance (km).

Signal sensitivity that users receive  $R_{ss}$  is

$$R_{ss} = P_t + G_t + G_r - LR - L_{bf} \quad (5)$$

The signal sensitivity that users receive  $R_{ss} = -80dBm$ , the gain of transmission  $G_t = 5dBm$ , the gain of reception:  $G_r = 5dBm$ . Flat areas  $LR=17dB$ , according to the mathematical formula, the coverage radius of the repeater  $D_f = 14\text{mile}$ .

**Repeater's Capacity.** In Literature [5] by Wenqiang Wang etc (2011), this issue has been examined. This literature has analyzed and calculated the issue according to Shannon's theory. From the literature, we can get the information that the capacity of a repeater is about 119.

## The Model

Moreover, by calculation we know that the average relative error of least squares regression model is 3.96%, which is less precise than that of PLSR. So this article chooses the relatively most precise PLSR as the preliminary model. Feed the performance data of C919 (see Table 1) into linear PLSR equation (3) and we get preliminary value of forecast price, namely 74.97 million dollars per aircraft.

**Group Spectrum.** Here we divide the radio spectrum into 4 parts in table 1.

Table 1 Troup Spectrum

| Receiver Frequency     | Transmitter Frequency |
|------------------------|-----------------------|
| A: 145.0MHz---145.6MHz | 145.6MHz---146.2MHz   |
| B: 145.6MHz---146.2MHz | 146.2MHz---146.8MHz   |
| C: 146.2MHz---146.8MHz | 146.8MHz---147.4MHz   |
| D: 146.8MHz---147.4MHz | 147.4MHz---148.0MHz   |

**The Cellular Model.** When we get the information above, we can draw the cellular. Firstly, draw numbers of connecting hexagons which length of side is 15 miles. Then picture a circle with the radius of 40 miles which represents the entire area above the cellular. Get the numbers of the hexagons covered by the circle. For the hexagons in boundary, if the center of it is not covered by that circle, this one is not counted. This will cause a problem that a few of hexagons, some parts of which

are covered by the circle, near the boundary are without repeaters. Fortunately, the unsaturated repeaters in the nearby hexagons can also cover this area due to the large coverage of a repeater. (The coverage radius of a repeater has more than 2 times than that of hexagon)

Now, with the cellular model, we can get the distribution of the repeaters in figure 1.

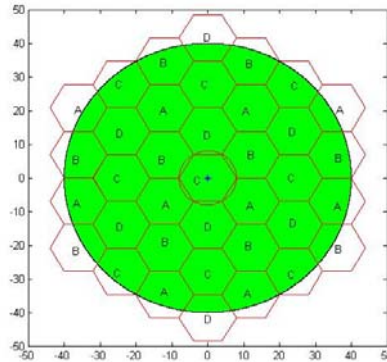


Fig .1 The distribution of the repeaters

In this picture, every repeater is located in the center of every hexagon. 'A, B, C, D' represent the 4 types of repeaters. With different frequency bands and different PL, the repeater can distribute in different. The repeater in different will not be interfered even at the same place.

Then, set a different set of radius values to the repeaters to, the number are required as follows.

Table 2 Distribution of the Number of Repeaters

| Radius of the Repeaters | Number of Repeaters |
|-------------------------|---------------------|
| 4                       | 121                 |
| 5                       | 85                  |
| 6                       | 55                  |
| $\vdots$                | $\vdots$            |
| 13                      | 13                  |
| 14                      | 7                   |

**Model Verification.** Giving the population 1000, thus the minimum number of repeaters  $N_s$  is:

$$N_s = \frac{1000}{119} = [8.4] = 9 < 13 \quad (6)$$

Meet the situation when the radius of the repeaters is 13mile.

When the population is 10000, the area per capita  $S_p$  is:

$$S_p = \frac{S}{10000} = 0.5024 \text{mile}^2 / p \quad (7)$$

Area occupied by single repeater  $S_s$  is:

$$S_s = N_p \times S_p = 119 \times 0.5024 = 59.79 \text{mile}^2 \quad (8)$$

The actual radius of a repeater  $R_s$  is

$$R_s = \sqrt{\frac{S_s}{\pi}} = [4.36] = 5 \text{mile} \quad (9)$$

The maximum area occupied by repeaters  $S'_s$  is:

$$S'_s = \pi R^2 = 615.75 \text{mile}^2 \quad (10)$$

The minimum number of repeaters  $N_s$  is:

$$N_s = \frac{S}{S_s} = 84 < 85 \quad (11)$$

From table I, we can get the required number of the repeaters is 85. So the result is close to the Cellular model, so the Model is feasible.

## The Improvement of the Model

In reality, some people live in neither urban centers nor suburbs. Therefore, the uniform distribution is not rational. Among the known distribution, the normal distribution is the most suitable choice here. We use the normal distribution which is influenced by the radius in a way to describe the distribution of population. Then the whole region is divided into 20 districts, each district is two-miles wide.

Here we define the normal distribution to be  $N(20, 7)$ , the equation is

$$f(x) = \frac{1}{7\sqrt{2\pi}} e^{-\frac{(x-20)^2}{98}}, x \geq 0 \quad (12)$$

The area of the region  $i$  is

$$S_i = \pi(R_i + 2)^2 - \pi R_i^2, i = 1, 2, 3, 4, \dots, 20 \quad (13)$$

And,

$$R_i = 2i - 2 \quad (14)$$

The population of the region  $i$  is

$$N_i = 1.99\rho_i S_i, i = 1, 2, 3, 4, \dots, 20 \quad (15)$$

After calculation, the results of population distribution in each region are showed in the table 3. The form of the distribution of population density and the discrete changes in population density histogram is showed by figure 2.

Table 3 Population Distribution in each Region and the Amount of District Repeaters

| District Number | District Population | Amount of District Repeaters |
|-----------------|---------------------|------------------------------|
| 1               | 30.51220332         | 0                            |
| 2               | 62.70671269         | 1                            |
| 3               | 118.9446036         | 1                            |
| $\vdots$        | $\vdots$            | $\vdots$                     |
| 19              | 62.70671269         | 1                            |
| 20              | 30.51220332         | 0                            |

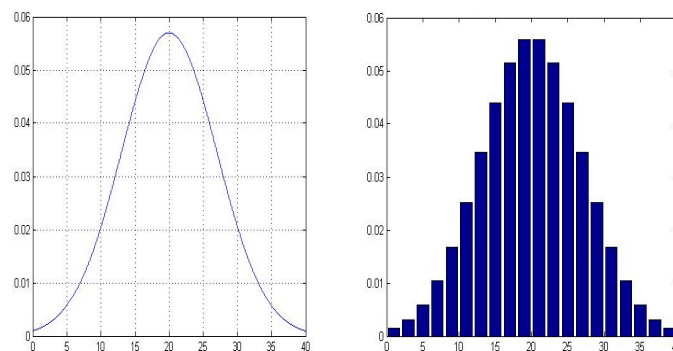


Fig .2 The form of the distribution of population density and the discrete changes in population density histogram.

In flat areas, we change the data, and move the repeaters to the edge as far as possible. The reason is that the same frequency repeaters may interfere with each other when the distance between them is too close. The amount of the repeaters is listed by the table 3.

Therefore, in flat area, the minimum amount of repeaters is 84. The distribution of their location is showed by the following figures.

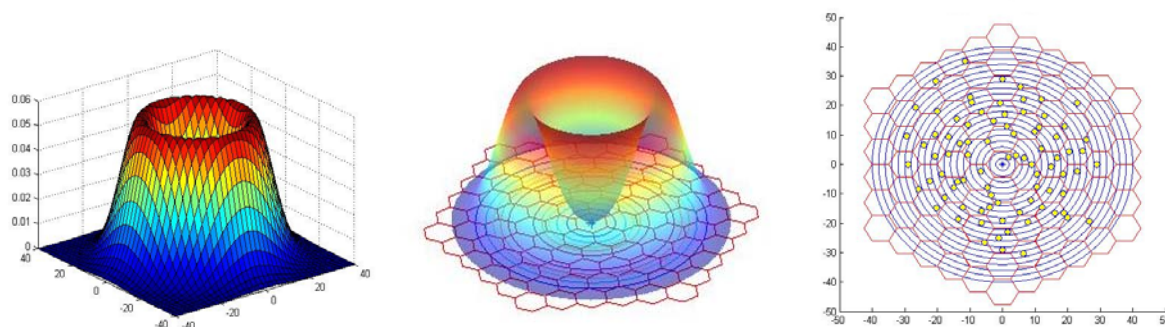


Fig .3 The form of the distribution of population density (3 dimensions), Population density and regional structure diagram and Repeaters distribution.

Yellow dots in figure 3 (the right one) represent repeaters.

## Conclusion

This paper makes the 1000 online users evenly distributed in a circular area with a radius of 40 miles, fills the region with the repeaters of different radiation radius. According to the number of people repeaters accommodate, the radiation radius, and the required number, it is found that when the radiation radius is 13 miles, the cellular distribution of 13 repeaters in the circular area can meet the requirements.

When the number of online users turns into 10,000, it can be found that when the radiation radius is 5 miles, 85 repeaters are needed. Then, because the population distribution is not necessarily well-proportioned, the normal distribution is introduced to constrain. We modify the model and divide this region into 20 small areas to solve respectively. The final number of the repeaters is 84. The both results are very similar, so the model is reasonable.

This research provides a positive idea to the optimization problem of repeaters' distribution and it also helps to reduce the cost of the hardware facilities.

## References

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