

Study on GM(1,1) Boundary Model based on Surveying Engineering

Yonghe Deng

Department of Civil Engineering, Lishui College, Lishui 323000, China

lsxydengyonghe@sina.cn

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Abstract. GM(1,1) models are widely applied to many fields including surveying engineering. Their researches are growing, and study on boundary conditions of GM(1,1) models is an important work. Based on least squares method, two sorts of new and better methods about boundary condition of GM(1,1) models were put forward in this paper. Finally, author proved that some models are equals, and proposed some advices. These results are important to GM(1,1) models and distortion monitoring of surveying engineering.

Introduction

Because GM (1,1) model is simple, and it has higher prediction accuracy based on a few samples, so, it has been widely used in surveying engineering such as high slope safety monitoring, building settlement prediction, prediction of dam, landslide monitoring, etc [1-10], and has been continually developed. Currently, we watched researches of GM(1,1) are ascending, and boundary condition of GM(1,1) is an important research direction.

At first, boundary condition of GM (1,1) has been selected as $\hat{X}^{(1)}(1) = X^{(1)}(1)$ [10], but prediction accuracy of GM(1,1) is not high, because $X^{(1)}(1)$ is the oldest data, so, some people have selected the newest data as boundary value and $\hat{X}^{(1)}(n) = X^{(1)}(n)$ as boundary condition [11,12,16]. But people found $\hat{X}^{(1)}(n) = X^{(1)}(n)$ as boundary condition is still not good, so they have selected the best condition among $\hat{X}^{(1)}(m) = X^{(1)}(m)$ ($m=1,2,\dots,n$) as boundary condition [1,13,14]. For error of the boundary condition $\hat{X}^{(1)}(1) = X^{(1)}(1)$, someone selected $\hat{X}^{(1)}(1) = X^{(1)}(1) + b$ as boundary condition to improve accuracy [15]; but literature [2] directly selected least squares method to improve accuracy.

The results of these studies undoubtedly enriched and developed GM (1,1) boundary condition theory. Based on previous studies, author proposed two new and better boundary models of GM(1,1), verified validity of these new models, proved some models are equals, these works have a positive significance of theory and practice for GM (1,1) model and deformation monitoring of surveying engineering.

Two new and improved boundary models of GM(1,1) put by author

It is obvious that, because of considered least squares method, model $\hat{X}^{(1)}(1) = X^{(1)}(1) + b$ is superior to model $\hat{X}^{(1)}(1) = X^{(1)}(1)$ [15]; and because of considered the newest information, model $\hat{X}^{(1)}(n) = X^{(1)}(n)$ is superior to model $\hat{X}^{(1)}(1) = X^{(1)}(1)$ [11,12,16]. But literatures [15,11,12,16] only considered one aspect, so they are undoubtedly not perfect. If we take into account both the newest information and least squares method, we can get a new and better model such as $\hat{X}^{(1)}(n) = X^{(1)}(n) + b$ based on least squares method in this paper.

Literatures [1,13,14] thought the best boundary conditions model among $\hat{X}^{(1)}(m) = X^{(1)}(m)$ ($m=1,2,\dots,n$) is better than that of $\hat{X}^{(1)}(n) = X^{(1)}(n)$, but they didn't consider least squares

method, so, it is more valid that I select the best boundary conditions model among $\hat{X}^{(1)}(m) = X^{(1)}(m) + b_m$ ($m=1,2,\dots,n$) based on least squares method in this paper.

Forecasting or fitting models and their equivalence among different boundary conditions of GM (1,1)

Forecasting or fitting model based on $\hat{X}^{(1)}(n) = X^{(1)}(n) + b$ and least squares method

According to literature[13], based on boundary condition as $\hat{X}^{(1)}(n) = X^{(1)}(n) + b$, we can obtain

$$\hat{X}^{(1)}(k+1) = \left[X^{(1)}(n) + b - \frac{u}{a} \right] e^{-a(k-n+1)} + \frac{u}{a} \quad (1)$$

According to Eq.1 and the minimum of $\sum_{k=0}^{n-1} [\hat{X}^{(1)}(k+1) - X^{(1)}(k+1)]^2$, we can get

$$b = -X^{(1)}(n) + \frac{u}{a} + \frac{(1-e^{-2a})e^{a(1-n)}}{(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(1)}(k+1)e^{-ak}] - \frac{u}{a} \frac{e^{a(1-n)}(1+e^{-a})}{(1+e^{-an})} \quad (2)$$

If we bring Eq.2 to Eq.1, forecasting or fitting model is shown as Eq.3.

$$\hat{X}^{(1)}(k+1) = \left\{ \frac{(1-e^{-2a})}{(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(1)}(k+1)e^{-ak}] - \frac{u}{a} \frac{(1+e^{-a})}{(1+e^{-an})} \right\} e^{-ak} + \frac{u}{a} \quad (3)$$

There is another method to compute forecasting or fitting model as follow. First, according to Eq.1, we can obtain

$$\hat{X}^{(0)}(k+1) = (1-e^a) \left[X^{(1)}(n) + b - \frac{u}{a} \right] e^{-a(k-n+1)} \quad (4)$$

Then, according to Eq.4 and the minimum of $\sum_{k=0}^{n-1} [\hat{X}^{(0)}(k+1) - X^{(0)}(k+1)]^2$, we can get

$$b = -X^{(1)}(n) + \frac{u}{a} + \frac{(1-e^{-2a})e^{a(1-n)}}{(1-e^a)(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(0)}(k+1)e^{-ak}] \quad (5)$$

If we bring Eq.5 to Eq.4, forecasting or fitting model is shown as Eq.6.

$$\hat{X}^{(0)}(k+1) = \frac{(1-e^{-2a})e^{-ak}}{(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(0)}(k+1)e^{-ak}] \quad (6)$$

Forecasting or fitting model based on the best model among $\hat{X}^{(1)}(m) = X^{(1)}(m) + b_m$ ($m=1,2,\dots,n$) and least squares method

According to the best boundary conditions among $\hat{X}^{(1)}(m) = X^{(1)}(m) + b_m$ ($m=1,2,\dots,n$), we can obtain

$$\hat{X}^{(1)}(k+1) = \left[X^{(1)}(m) + b_m - \frac{u}{a} \right] e^{-a(k-m+1)} + \frac{u}{a} \quad (7)$$

where: m of Eq.7 is selected the best value among $1,2,\dots,n$ to make model be best.

According to Eq.7 and the minimum of $\sum_{k=0}^{n-1} [\hat{X}^{(1)}(k+1) - X^{(1)}(k+1)]^2$, we can get

$$b_m = -X^{(1)}(m) + \frac{u}{a} + \frac{(1-e^{-2a})e^{a(1-m)}}{(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(1)}(k+1)e^{-ak}] - \frac{u}{a} \frac{e^{a(1-m)}(1+e^{-a})}{(1+e^{-an})} \quad (8)$$

If we bring Eq.8 to Eq.7, forecasting or fitting model is shown as Eq.9.

$$\hat{X}^{(1)}(k+1) = \left\{ \frac{(1-e^{-2a})}{(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(1)}(k+1)e^{-ak}] - \frac{u}{a} \left(\frac{1+e^{-a}}{1+e^{-an}} \right) \right\} e^{-ak} + \frac{u}{a} \quad (9)$$

There is still another method to compute forecasting or fitting model as follow. First, according to Eq.7, we can obtain

$$\hat{X}^{(0)}(k+1) = (1-e^a) \left[X^{(1)}(m) + b_m - \frac{u}{a} \right] e^{-a(k-m+1)} \quad (10)$$

Then, according to Eq.10 and the minimum of $\sum_{k=0}^{n-1} [\hat{X}^{(0)}(k+1) - X^{(0)}(k+1)]^2$, we can get

$$b_m = -X^{(1)}(m) + \frac{u}{a} + \frac{(1-e^{-2a})e^{a(1-m)}}{(1-e^a)(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(0)}(k+1)e^{-ak}] \quad (11)$$

If we bring Eq.11 to Eq.10, forecasting or fitting model is shown as Eq.12.

$$\hat{X}^{(0)}(k+1) = \frac{(1-e^{-2a})e^{-ak}}{(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(0)}(k+1)e^{-ak}] \quad (12)$$

Because Eq.9 and Eq.12 do not contain variable m , so, whether m selects any value among $1, 2, \dots, n$, corresponding boundary conditions models are equivalent, their precisions are same.

Forecasting or fitting model based on $\hat{X}^{(1)}(k+1) = -\frac{c}{a}e^{-ak} + \frac{u}{a}$ and least squares method

According to Eq.1, we can hypothesize

$$\hat{X}^{(1)}(k+1) = -\frac{c}{a}e^{-ak} + \frac{u}{a} \quad (13)$$

According to Eq.13 and the minimum of $\sum_{k=0}^{n-1} [\hat{X}^{(1)}(k+1) - X^{(1)}(k+1)]^2$, we can get

$$c = -a \frac{1-e^{-2a}}{1-e^{-2an}} \sum_{k=0}^{n-1} [X^{(1)}(k+1)e^{-ak}] + u \left(\frac{1+e^{-a}}{1+e^{-an}} \right) \quad (14)$$

If we bring Eq.14 to Eq.13, forecasting or fitting model is shown as Eq.15.

$$\hat{X}^{(1)}(k+1) = \left\{ \frac{(1-e^{-2a})}{(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(1)}(k+1)e^{-ak}] - \frac{u}{a} \left(\frac{1+e^{-a}}{1+e^{-an}} \right) \right\} e^{-ak} + \frac{u}{a} \quad (15)$$

There is another method to compute forecasting or fitting model as follow. First, according to Eq.13, we can obtain

$$\hat{X}^{(0)}(k+1) = -\frac{c}{a}(1-e^a)e^{-ak} \quad (16)$$

Then, according to Eq.16 and the minimum of $\sum_{k=0}^{n-1} [\hat{X}^{(0)}(k+1) - X^{(0)}(k+1)]^2$, we can get

$$c = -a \frac{1-e^{-2a}}{(1-e^{-2an})(1-e^a)} \sum_{k=0}^{n-1} [X^{(0)}(k+1)e^{-ak}] \quad (17)$$

If we bring Eq.17 to Eq.16, forecasting or fitting model is shown as Eq.18.

$$\hat{X}^{(0)}(k+1) = \frac{(1-e^{-2a})e^{-ak}}{(1-e^{-2an})} \sum_{k=0}^{n-1} [X^{(0)}(k+1)e^{-ak}] \quad (18)$$

In this paper, because Eq.9, Eq.15 and Eq.3 are equals, Eq.12, Eq.18 and Eq.6 are equals, so, forecasting or fitting models based both on $\hat{X}^{(1)}(n) = X^{(1)}(n) + b$ and least squares method, or based both on the best model among $\hat{X}^{(1)}(m) = X^{(1)}(m) + b_m$ ($m=1, 2, \dots, n$) and least squares method, or

based both on $\hat{X}^{(1)}(k+1) = -\frac{c}{a}e^{-ak} + \frac{u}{a}$ and least squares method, are equals.

Then, boundary conditions of literature[2] and [15] should bring same result, but literature[2] published in 2009 year thought that it improved boundary condition of literature[15] published in 2001 year before 8 years, this is wrong.

Conclusion

1) These two new and better models of GM (1,1) boundary condition proposed by author in this paper are feasible and effective.

2) This paper revealed, enriched and developed theoretical relationship of some boundary conditions of GM(1,1) model.

3) In practice of surveying engineering, this paper can help us reduce much computation work because some methods are equals.

4) This paper pointed out there is mistake in literature[15].

5) These results in this paper can be used to all fields of GM(1,1) application.

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