Planning Strategy for the Omni-directional Movement Path of the Four-wheel Differential Robot

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Abstract. The four-wheel differential omni-directional mobile robot has shown a strong research value and application prospect. This paper explored the kinematics and dynamics model of the omni-directional mobile robot by former differential and rear differential features, with a particular focus on its path planning strategy. The design includes path planning reference point, whilst analysis was made on the temporary path design strategy for linear and circular two features, with the resulting temporary path being optimal and most valid. This paper concludes with the research results that are available for providing the necessary preliminary studies and theoretical basis for the control of the mobile robot of this type.

Introduction

Among the mobile robots, Wheeled Mobile Robot (WMR) is perfect for efficient operation in a smooth, flat ground, due to its fast, stable, simple design, and therefore the WMR has been listed in one of the main directions of mobile robot technology development.

According to the drive and steering, the WMR can be roughly divided into the following types [1~3]: two-wheel drive, three-wheel drive, four-wheel drive and omni-directional drive. 4-wheel differential omni-directional mobile robot (4WDR) is an omni-directional mobile robot, which consists of two the differential robots, with the formed differential unit able to be rotated around the robot body. 4WDR features high flexibility and controllability, able to complete the translational motion, rotational motion, and the combination of movement in any direction, with great value and a good prospect.

Kinematics and dynamics analysis of 4DWR

4WDR drive part is composed of four casters and two differential units: the caster is distributed in the four corners of the robot, only playing a secondary support role; differential unit is composed of two coaxial driving wheels that are independently driven by a motor. Differential units are connected to the robot body via a revolute pair, which is located at the midpoint of the two wheels. Due to this, the differential units are available for 360 ° rotation around the robot body. The two sets of differential units work as the power source for the robot movement. Below is the 4WDR diagrammatic illustration:
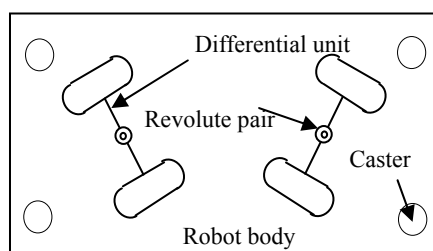

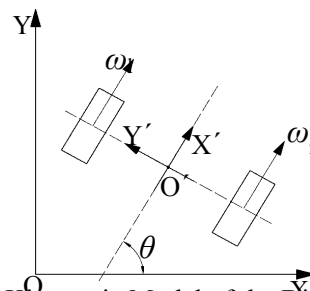Fig. 1 Diagrammatic Illustration of 4WDR

Fig. 2 Kinematic Model of the Differential Unit

The caster only plays a secondary support role on 4WDR, not acting a binding role in the robot plane. This makes it possible for kinematic analysis to ignore the caster, merely focus on two differential units. During the kinematic analysis, we make the following assumptions:

①Robot moves in the plane, and always in the point contact with the wheel and the motion plane;

②Wheels only have pure rolling on a flat surface without slipping;

③The robot body, the differential unit, wheels and running surface are all the rigid bodies.

4WDR is driven by two identical differential units, so this section is first to establish the kinematic model of the differential unit. The differential unit is provided with the kinematic model shown in Figure 2, here, XOY - The world coordinate system, X´O´Y´ - solid joint coordinate system, O´ is the midpoint of the two center connection, i.e. projection point of the rotation pair in the plane, ωl and ωr respectively, the angular velocity of the left and right driving wheel, θ - heading angle of the differential unit, i.e. the angle between O´X´ and OX.

Let O´ in the absolute coordinate system has the coordinates (x, y), then the position and posture of the differential unit in the absolute coordinate system can be expressed as (x, y, θ). According to the point speed decomposition theorem, we have:

$$\begin{cases} x = x_0 + \int_{t_0}^{t} v(t)\cos\theta(t)\mathrm{d}t \\ y = y_0 + \int_{t_0}^{t} v(t)\sin\theta(t)\mathrm{d}t \\ \theta = \theta_0 + \int_{t_0}^{t} \omega(t)\mathrm{d}t \end{cases} \tag{1}$$

In(1),(x_0, y_0, θ_0) is the position and posture of the differential unit at the moment t_0.

Strategy for path planning

Selecting the path reference point. During the robot path planning and control, the usual practice is to select the geometric center of the robot as a reference point [4~8]. Due to the particularity of the robot structure in this study, where the two differential units work independently of each other, the choice of the reference point can be considered to have the geometric center of the differential unit as a reference point. In this paper, the dual reference point is set to achieve the robot path planning and the follow-up control, i.e. the geometric center of the front and rear differential units are taken equally as the robot path planning point of reference, enabling the two points to follow the same path. In the path following, the rear reference point should have a certain time of delay than the front reference point. Such an approach will greatly reduce the complexity of the control system, making it possible to have a simple control over the robot.

Assume that the path equation $y=f(x)$, then at the moment t, the position and posture equation of the robot body (with geometric center of the robot body as a reference point) as follows:

$$\begin{cases} x = (x_\mathrm{f} + x_\mathrm{r})/2 \\ y = (f(x_\mathrm{f}) + f(x_\mathrm{r}))/2 \\ \theta = \arctan(\dfrac{f(x_\mathrm{f}) - f(x_\mathrm{r})}{x_\mathrm{f} - x_\mathrm{r}}) \end{cases} \tag{2}$$

In(2), x_f and x_r are, respectively, the coordinate x of the front and rear reference points at the moment t, and both meeting the following relationship:

$$(\mathrm{f}(x_\mathrm{f}) - \mathrm{f}(x_\mathrm{r}))^2 + (x_\mathrm{f} - x_\mathrm{r})^2 = \mathrm{L}^2 \tag{3}$$

L is the distance between the front and rear reference points.

Provisional path planning. During the path following control, the starting point of the robot is not set in the desired path to be followed. When the starting point of the robot is away from the desired path, no matter what control method might be employed, all easily lead to excessive control, failing to achieve the ideal effect. Therefore, it is required to design a temporary transition path from a starting point to the desired path.

At present, the geometric method is most common to generate a temporary path. In this paper, two reference points are combined with the manual driving idea to propose a new method that is used to generate a temporary path, where two cases are included to cover the straight line and circular desired paths (the angle between the fixed joint coordinate systems of the front and rear is restricted to a range of [-π/2, π/2]).
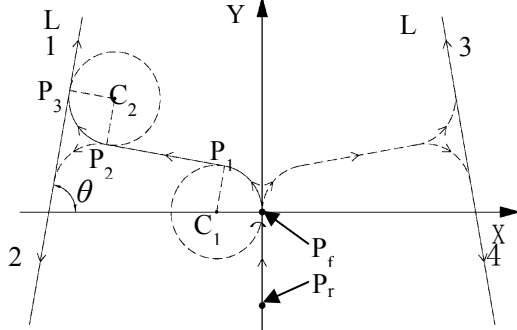
a)Expected route as a straight line

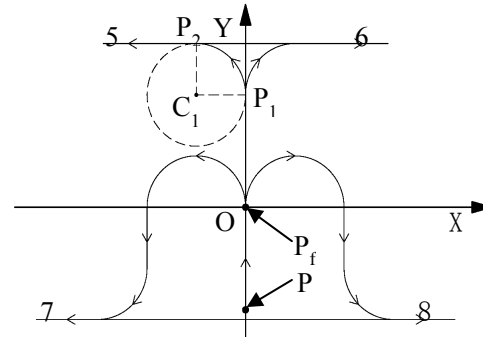Fig. 3 Temporary paths for Path No. 1~4

Fig. 4 Temporary paths for Path No. 5~8

As shown in Fig.3, P_f and Pr are, respectively, the selected reference points. With P_f as the origin of the coordinates, the directed line segment P_rP_f as the Y axis, we established the coordinate system XOY. It is defined that the straight line to be followed is crossed with the X axis at the coordinate (x_0, 0), with the Y axis at the coordinate (0, y_0); As the path vector has a positive projection on the Y axis, the directivity of the defined path is positive, otherwise it is negative; If the path is parallel to the X axis, the projection of the path vector on the X axis is then used to define the directivity. Based on the crossing point between the desired path and the X axis and the Y axis, as well as the path directivity, the desired path can be divided into eight categories, as shown in Table 1, where path 1~4 are indicated in Fig.3, and path 5~8 are shown in Fig.4.

Table 1 Categories of the desired path as straight line

Coordinates	$x_0<0$		$x_0>0$		x_0 N/A, $y_0>0$		x_0 N/A, $y_0<0$	
Directivity	Positive	Negative	Positive	Negative	Negativz e	Positiv e	Negative	Positive
Code	1	2	3	4	5	6	7	8

①Generated temporary paths for the desired path 1~4:

Take path 1 for analysis, and the temporary path is generated as indicated in Fig.3: first is to let the reference point travel along a period of counterclockwise arc to the straight line vertically pointing at the desired path L_1. And then along the line, it comes towards the point P_2, which has a distance R from the straight line L1. Finally, it is followed by 1/4 counterclockwise arc to reach P_3 that is tangent to the desired path. Set the coordinates of the points in the figure as: $P_r(xr,yr)$, $P_f(x_f,y_f)$, $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, $P_3(x_3,y_3)$, $C_1(xC_1,yC_1)$, $C_2(xC_2,yC_2)$.

It is known that the first segment of the arc has the equaltion as:

$$\begin{cases} x = R\cos\alpha - R \\ y = R\sin\alpha \end{cases} (0 \le \alpha \le \theta) \tag{3}$$

The equation of the straight line segments that connects the first and the second segments of the arc is:

$$y = -\cot\theta \cdot x + R(\csc\theta - \cot\theta), x_1 < x < x_2 \tag{4}$$

The second segment of the arc has the equation:

$$\begin{cases} x = R\cos\alpha + x_{C2} \\ y = R\sin\alpha + y_{C2} \end{cases} (\theta - \pi < \alpha \le \theta - \frac{\pi}{2}) \tag{5}$$

Meanwhile, as the rear reference point lags behind the front one, temporary path should be added with the line segment PrPf, that is, the ultimately generated temporary path is ($P_r \to P_f \to P_1 \to P_2 \to P_3$).

As for the path 2~4, path 2 and path 1 are symmetrical against the line P_1P_2, path 3 and path 1 against the Y axis, path 4 and path2 against the Y axis. Thus, path 2~4 can also be obtained easily.

②The desired path 5~8 with the generated temporary path

Take path 5 for analysis, and the Temporary path is generated as indicated in Fig.4: first is to let the reference point travel to P_2, followed by rotating counter-clockwise by a $\pi/2$ arc to reach P2 that is tangent to the desired path.

The straight line segment P_fP_c has the equation as：

$$x = 0 \quad (0 < y \le y_0 - R) \tag{6}$$

The arc segment equation:

$$\begin{cases} x = R\cos\alpha - R \\ y = R\sin\alpha + y_0 - R \end{cases} \quad (0 < \alpha \le \frac{\pi}{2}) \tag{7}$$

Like path 1,it is required to add a straight line segment P_rP_f,that is, the ultimately generated temporary path is$(P_r \to P_f \to P_1 \to P_2)$.

b)Desired route as a circle

As shown in Fig.5, Pf and Pr are, respectively, the selected reference points. With P_f as the origin of the coordinates, the directed line segment P_rP_f as the Y axis, we established the coordinate system XOY.

Based on the positional relationship between the starting point and desired circle, the circle following can be divided into two cases: the first one is that the starting point is outside the desired circle, the second, inside. The following discussion will be given separately.
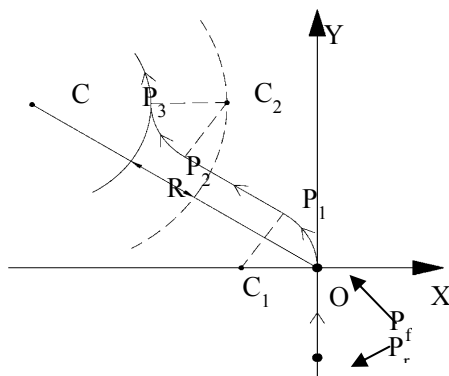
①The starting point outside the desired circle

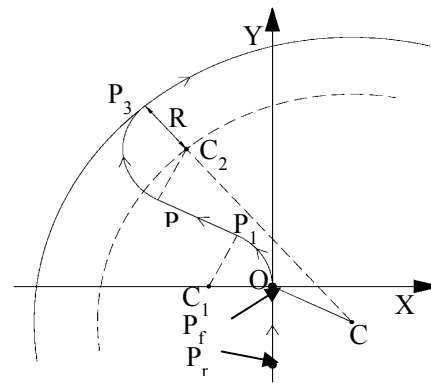Fig. 5 Path of the following circle round   Fig. 6 Temporary path of the following

As shown in Fig. 5,assume that the desired circle has its center C in the second quadrant of the XOY coordinate system, with the radius RC and the desired path in counterclockwise rotation. At this moment, the planned temporary path：it starts by rotating counterclockwise the reference point to P_1,which requires the tangent line that crosses the point to be parallel to OC;And then, it moves along a straight line to P_2;Finally, it clockwise rotates to P_3. Here, the second segment arc has the center with a distance R from P_1P_2,and RC+R from the center C. So the ultimate temporary path is $(P_r \to P_f \to P_1 \to P_2 \to P_3)$. As the desired path rotates clockwise,the center of the second arc is positioned below the straight line P_1P_2,subject to the same calculation method.

②The starting point inside the desired circle

As shown in Fig.6 ,assume that the desired circle has its center C in the fourth quadrant of the XOY coordinate system, with the radius RC and the desired path in counterclockwise rotation. At this moment, the planned temporary path:it starts by rotating counterclockwise the reference point to P_1 (like the case of the starting point outside),which requires the tangent line that crosses the point to be parallel to OC;And then, it moves along a straight line to P_2;Finally, it clockwise rotates to P_3. Under the case where the starting point is outside the circle, here the second segment arc has the center with a distance R from P_1P_2,and RC-R from the center C. So the ultimate temporary path is

($P_r \rightarrow P_f \rightarrow P_1 \rightarrow P_2 \rightarrow P_3$). As the desired path rotates counterclockwise,the center of the second arc is positioned below the straight line P_1P_2,subject to the same calculation method.

Similarly, when the desired center is in the other quadrants, regardless of the starting point that is inside or outside the circle, the temporary path can be generated according to the principle of the mirror.

Conclusions

This paper made the kinematics and dynamics analysis of four-differential omni-directional mobile robot and provided its motion model. Based on the model, the robot path tracking problem is explored by designing the path reference points. Targeting at the desired route as a straight line and a circle, planning methods and strategies are proposed to enable the robot to moves along an optimum temporary path with the highest timeliness.

Acknowledgements

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