Integrated Control of Electric Power Steering and Active Suspension System Based on LQG Theory

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Abstract. The vehicle dynamics models which have integrated electric power steering (EPS) and active suspension system (ASS) were built, the integrated LQG (Linear-Quadratic Gaussian) controller which adopts centralized structure was designed, computer simulations were carried out with the MATLAB/Simulink software. The simulation results showed that the LQG controller could not only apparently filter sensor measurement noise influencing the integrated systems, but also effectively improve vehicle handing stability and steering portability.

Introduction

The integrated control for vehicle chassis system has become a research hot point in the field of vehicle control [1]. EPS and ASS have a direct impact on vehicle's handing stability and ride performance respectively, these two systems were considered as not related dynamics systems to research according to traditional method[2]. This method usually has better control effect on separate subsystem control; however, it has poor control effect due to neglecting the coupling among the subsystems during the process of the vehicle's actual moving.

Integrated control research of EPS and ASS at present was focused on control structure and control algorithm. The control structure was usually divided into multi-layer and centralized [3]. Multi-layer control structure kept the original subsystem controller and added a coordinating controller that calculated the required control force or torque of global optimization according to current vehicle driving conditions and apply to subsystem controller compensation or correction control command when subsystem coupled, this structure was usually used in the coordination and matching among multiple subsystems. Centralized control structure which had the highest degree of integration and best effect of control gave all subsystems control input by a global controller, it generally applied to two subsystems (such as the steering and suspension systems) integration that closely related and seriously coupled [4]. Generally in the control algorithm, EPS used PID control and ASS used optimal control which respectively used for SISO system and MIMO system [5].

In this paper, a centralized control structure was used to create the whole vehicle mathematical model that integrated EPS and ASS, taking into account the controller feedback signals must be measured and the sensor measurement noise interference in the practical engineering application, the LQG controller based on optimal control theory on integrated systems was designed. The simulation results show that although the designed centralized controller has limited effect to improve ride performance, but improve vehicle handing stability and steering portability effectively.

The system mathematical models

A. Whole vehicle dynamics model. According to the Fig.1 and Fig.2, The whole vehicle dynamics model that contained vehicle's vertical, lateral , pitch, roll, yaw motion and four wheels lateral movement was established in order to comprehensively analyze the coupling between steering and suspension systems under steering condition.

Sprung mass vertical movement:

$$m_s \ddot{Z}_s = F_1 + F_2 + F_3 + F_4 \tag{1}$$

Sprung mass pitch movement:

$$I_{ys}\ddot{\theta} = -(F_1 + F_2)(l_f - c_s) + (F_3 + F_4)(l_r + c_s) \quad (2)$$

Sprung mass roll movement：

$$I_{xu}\ddot{\varphi} - m_s h\dot{v} - m_s g h\varphi = \frac{d_f}{2}(F_1 - F_2) + \frac{d_r}{2}(F_3 - F_4) \quad (3)$$

Whole vehicle lateral movement:

$$m(\dot{v} + u\omega) - m_s h\ddot{\varphi} = (F_{y1} + F_{y2})\cos\delta + F_{y3} + F_{y4} \quad (4)$$

Whole vehicle yaw movement:

$$I_z\dot{\omega} - I_{zx}\ddot{\varphi} = l_f(F_{y1} + F_{y2}) - l_r(F_{y3} + F_{y4}) \quad (5)$$

Four wheels vertical movement:

$$m_{ui}\ddot{Z}_i = K_{ui}(Z_{0i} - Z_i) - F_i \qquad (i = 1, 2, 3, 4) \quad (6)$$

Suspension forces：

$$F_i = K_{si}(Z_{1i} - Z_{2i}) + C_{bi}(\dot{Z}_{1i} - \dot{Z}_{2i}) + f_i \quad (i = 1, 2, 3, 4) \quad (7)$$

When the roll angle φ and pitch angle θ changes in a small range, the displacement of the sprung mass four endpoints expressed as follows:

$$\begin{cases} Z_{2i} = Z_s - c_f\theta \pm \frac{1}{2}d_f\varphi & (i = 1, 2) \\ Z_{2j} = Z_s + c_r\theta \pm \frac{1}{2}d_r\varphi & (j = 3, 4) \end{cases} \quad (8)$$

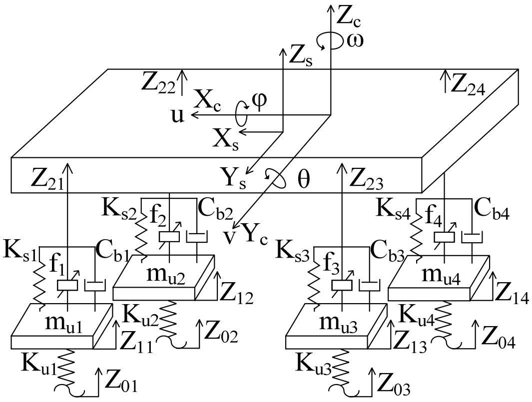

Fig.1 Whole vehicle dynamics model and its reference frame

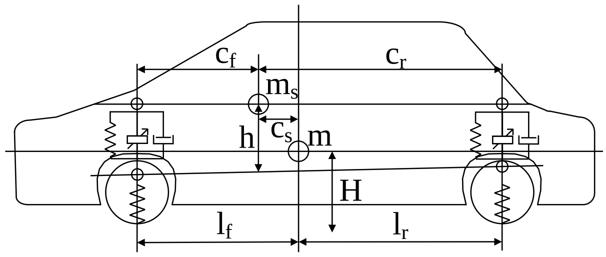

Fig.2 The whole vehicle side view

C_{bi}—Shock absorber damping coefficient, c_f—Dsitance from sprung mass centroid to front axle, c_r—Distance from sprung mass centroid to rear axle, c_s—Distance from sprung mass centroid to vehicle mass centroid, d_f—Front wheel spacing, d_r—Rear wheel spacing, f_i—Active suspension forces, F_i—Suspension forces, h—Vertical distance from sprung mass centroid to roll axis, I_{xu}—Sprung mass around the vehicle centroid vertical axis moment of inertia, I_{ys}—Sprung mass around its centroid transverse axis moment of inertia, K_{si}—Suspension spring stiffness, K_{ui}—Unsprung mass equivalent stiffness, l_f—Dsitance from vehicle mass centroid to front axle, l_r—Dsitance from vehicle mass centroid to rear axle, m—Whole vehicle mass, m_s—Sprung mass, m_{ui}—Unsprung mass, u—Longitudinal velocity of vehicle centroid, v—Lateral velocity of vehicle centroid, Z_{0i}—The tire ground displacement input, Z_{1i}—Unsprung mass vertical displacement, Z_{2i}—The vertical displacement of sprung mass endpoint, Z_s—Sprung mass vertical displacement, φ—Sprung mass roll angle, θ—Sprung mass pitch angle, ω—The vehicle centroid yaw rate.

B. Tire model. In case of a small rotational angle, the tire characteristic can be considered to be linear, linear tire model can be found in ref.[6].

C. Road input model. The filtered white noise can be a true reflection of road spectrum approximate horizontal within a low frequency range of actual situation [7]. This method was used to establish the road input model as follows:

$$\dot{Z}_{0i} = -2\pi f_0 Z_{0i} + 2\pi\sqrt{G_0 u}\,w(t) \quad (9)$$

D. EPS dynamics model. The shaft assist EPS system is used as the research subject, rack and pinion transmission is used as the form of steering transmission. The EPS dynamics equations were established according to Ref.[8].

Dynamics equation of steering shaft is expressed as:

$$\begin{cases} J_s\ddot{\theta}_s + B_s\dot{\theta}_s + T_s = T_d \\ T_s = K_s(\theta_s - \frac{x_r}{r_s}) \end{cases} \quad (10)$$

Dynamics equation of rack and pinion is expressed as:

$$\begin{cases} m_r\ddot{x}+B_r\dot{x}+F_r=\dfrac{K_s}{r_s}(\theta_s-\dfrac{x_r}{r_s})+\dfrac{K_mG}{r_s}(\theta_m-\dfrac{x_r}{r_s}G) \\ F_r=\dfrac{K_{kp}}{G_r}(\dfrac{x_r}{G_r}-\delta) \\ \delta=\dfrac{\theta_s}{i_s} \end{cases} \tag{11}$$

Kinetic equation of assist motor is expressed as:

$$\begin{cases} J_m\ddot{\theta}_m+B_m\dot{\theta}_m+K_m(\theta_m-\dfrac{x_r}{r_s}G)=K_ai \\ L\dot{i}+Ri+K_v\dot{\theta}_m=U_m \end{cases} \tag{12}$$

E. Mathematical model of integrated systems. According to the above models, the state vector isconstructed as $X=[Z_1\ \dot{Z}_1\ Z_2\ \dot{Z}_2\ Z_3\ \dot{Z}_3\ Z_4\ \dot{Z}_4\ Z_s\ \dot{Z}_s\ \theta\ \dot{\theta}\ \varphi\ \dot{\varphi}\ v\ \omega\theta_s\ \dot{\theta}_s\ x_r\ \dot{x}_r\ \theta_m\ \dot{\theta}_m\ i\ Z_{01}\ Z_{02}\ Z_{03}\ Z_{04}]^T$, the control input vector is $U=[f_1\quad f_2\quad f_3\quad f_4\quad U_m]^T$, interference input vector is $U^{'}=[T_d]$, model noise vector is $W=[w_1\quad w_2\quad w_3\quad w_4\quad w_5]^T$, output vector is $Y=[\ddot{Z}_s\ \theta\ \varphi\ \omega\ \dot{v}\ Z_5-Z_1\ Z_6-Z_2\ Z_7-Z_3\ Z_8-Z_4\ T_s\ \theta_s]^T$ measurement noise vbector is $\xi=[\varsigma_1\ \varsigma_2\ \varsigma_3\ \varsigma_4\ \varsigma_5\ \varsigma_6\ \varsigma_7\ \varsigma_8\ \varsigma_9\ \varsigma_{10}\ \varsigma_{11}\ \varsigma_{12}\ \varsigma_{13}\ \varsigma_{14}]^T$,The state space equation of integrated systems can be obtained via the simplification as follows:

$$\begin{cases} \dot{X}=AX+BU+B^{'}U^{'}+FW \\ Y=CX+\xi \end{cases} \tag{13}$$

Where A is 27×27 order system matrix，B is 27×5 order control input matrix，$B^{'}$ is 27×1 order interfere input matrix，F is 27×5 order model noise input matrix, C is 11×27 order output matrix.

LQG controller design

According to the separation principle of LQG controller design[9], optimal estimation and optimal control can be solved separately，then combine the solution of these two parts together.

A. Kalman filtering optimal state estimator design. In Eq.13 model noise W and measurement noise ξ were zero mean Gaussian white noise and independent signals, their statistical characteristics were: $E\{W(t)\}=E\{\xi(t)\}=E\{W(t)\xi(t)^T\}=E\{\xi(t)W(t)^T\}=0$; $E\{W(t)W(t)^T\}=Q_0$;

$E\{\xi(t)\xi(t)^T\}=R_0$. Where Q_0 is model noise covariance matrix, R_0 is measurement noise covariance matrix. Kalman filter optimal state estimator can be constructed according to the system state space equations are represented as follows:

$$\dot{\hat{X}}=A\hat{X}+BU+L(Y-C\hat{X}) \tag{14}$$

$\hat{X}$ is the state estimate value in the above equation, $L=P_0C^TR_0^{-1}$ is the Kalman filter optimal state estimator feedback gain matrix, matrix P_0 was obtained by the algebraic equation as follows:

$$AP_0+P_0A^T+FQ_0F^T-P_0C^TR_0^{-1}CP_0=0 \tag{15}$$

B. LQR optimal controller design. LQG is an output feedback problem, the output vectors of integrated systems should be designed for control target to make the vehicle has high handing stability, ride performance and steering portability. It should reduce the vehicle's vertical acceleration, pitch angle, roll angle, lateral acceleration, yaw rate, suspension travel, steering shaft measuration torque and angle, in addition should make the minimum control vectors for easy control. Performance index foctional was designed based on the above consideration as follows:

$$J=\frac{1}{2}\int_0^\infty[q_1\ddot{Z}_s^{\ 2}+q_2\theta^2+q_3\varphi^2+q_4\dot{v}^2+q_5\omega^2+q_6(Z_5-Z_1)^2+q_7(Z_6-Z_2)^2+q_8(Z_7-Z_3)^2 \\ +q_9(Z_8-Z_4)^2+q_{10}T_s^{\ 2}+\ q_{10}\theta_s^{\ 2}+r_1f_1^{\ 2}+r_2f_2^{\ 2}+r_3f_3^{\ 2}+r_4f_4^{\ 2}+r_5U_m^{\ 2}]dt \tag{16}$$

Where q_i are weighting coefficients of corresponding control targets, r_i are weighting coefficients of control vectors. The Eq.16 was rewritten in a matrix form:

$$J=\frac{1}{2}\int_0^\infty[Y^TQY+U^TRU]dt \tag{17}$$

Where Q is state weighting matrix, R is control weighting matrix. It can be found by substitute $Y=CX$ into Eq.17:

$$J=\frac{1}{2}\int_0^\infty[X^T\hat{Q}X+U^TRU]dt \tag{18}$$

Where $\hat{Q}=C^TQC$. According to the optimal control theory known LQR optimal control law is $U=-KX$, $K=R^{-1}B^TP$ is the optimal control law feedback gain matrix, matrix P was obtained by the algebraic equation as follows:

$$A^TP+PA-PBR^{-1}B^TP+\hat{Q}=0 \tag{19}$$

Simulation results and analysis

Ode45 was used as the simulation algorithm and automatically step was adopt, model noise power are 20dB, measurement noise power are 1e-5dB, the input torque of steering is 2N.m step input and the vehicle speed is 10m/s. Considering the overall performance of the systems, weighting coefficients were taken as: $q_1=100$, $q_2=50$, $q_3=q_4=80$, $q_5=100$, $q_6\sim q_9=20$, $q_{10}=80$, $q_{11}=120$, $r_1\sim r_5=0.0005$, $q_{0i}=10^{-3}$, $r_{0i}=10^{-7}$.

Integrated systems performance parameters were shown in Tab.1, the simulation comparison results for the LQR and LQG centralized control that added measurement noise and separate control of EPS and ASS were shown in Fig.3~Fig.9. The system disturbance caused by measurement noise was great and the whole system performance was deteriorated obviously when LQR centralized control was used for integrated systems with measurement noise, however, noise filtering effect is significant and system response tends to the steady-state when LQG centralized control was used.

Tab.1 Performance parameters

RMS value	Other	LQR	LQG
θ (rad)	0.0107	0.0154	0.0058
φ (rad)	0.1336	0.1231	0.1153
$\dot{v}$ (m/s^2)	2.4743	2.3186	2.2312
ω (rad/s^2)	0.0407	0.0392	0.0384
$Z_{21}-Z_{11}$ (m)	0.0312	0.0271	0.0256
$\dot{Z}_s$ (rad/s)	0.9732	2.7813	0.9657
θ_s (°)	129.35	122.81	122.43

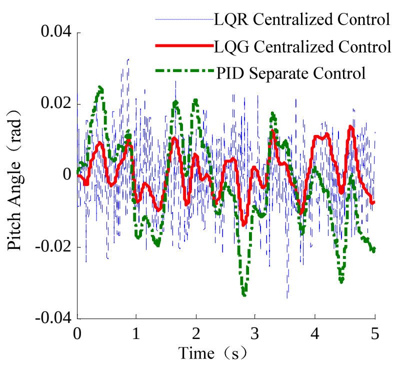

Fig.3 Response comparison of pitch angle

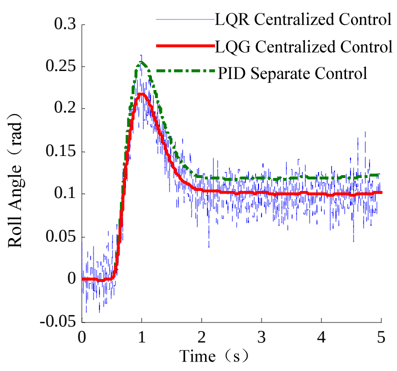

Fig.4 Response comparison of roll angle

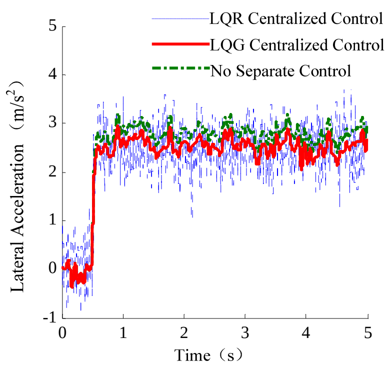

Fig.5 Response comparison of lateral acceleration

It can be found in Fig.3~Fig.6, Sprung mass pitch angle, roll angle, lateral acceleration were reduced by different levels, so that the vehicle handing stability was improved under the LQG centralized control. Suspension travel response overshoot and steady-state value were less than that of LQG separate control indicate the probability of impact buffer block was reduced effectively.

The sprung mass vertical acceleration under LQG centralized control was not improved significantly compared whit that of LQR separate control, which were shown in Fig.7, it indicates that the effect of improving vehicle ride performance under LQG centralized control was limited. System transient response time and overshoot were significantly reduced under LQG centralized control than that of PID separate control, which were shown in Fig.8, the steering portability was improved because the RMS value decreased from 129.35° to 122.43°.

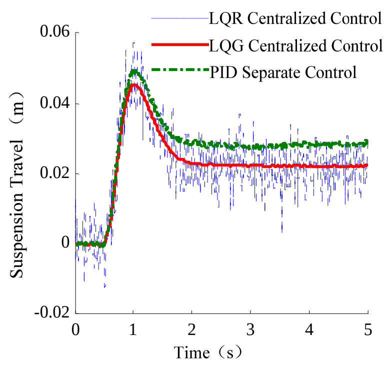

Fig.6 Response comparison of left front suspension's travel

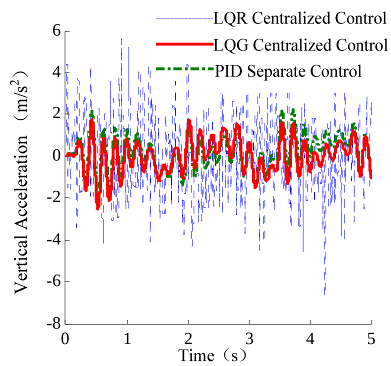

Fig.7 Response comparison of vertical acceleration

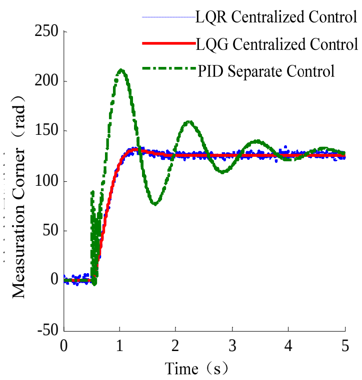

Fig.8 Response comparison of steering shaft measuration

Conclusion

The whole system performance was deteriorated obviously when LQR centralized control was used for integrated systems with measurement noise, however, noise filtering effect is significant and system steady-state response is well when LQG centralized control was used.

The LQG controller could effectively improve vehicle handing stability, steering stability and steering portability.

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