Image Denoising Using Gaussian Scale Mixtures in Lifting Stationary Wavelet Coefficient

Dongmei Zhang[1,a], Minzhi Wang[1,a], Jianhua Liu[2,b] , Zhong Xiao[3,c]

[1]Science Department, Zhangjiakou Education College, Zhangjiakou 075000, China

[2] College of Mechanical Engineering, Donghua Unverisity, Shanghai 200000, China

[3] Shijiazhuang Tiedao University, Shijiazhuang, 050043, China

[a]sunxiuming_075000@163.com, [b]liujianhuasjz@163.com

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Abstract: A new method for image denoising is proposed in this paper. Firstly, apply the Lifting Stationary Wavelet transform on the denoised image. Secondly, Gaussian scale mixtures (GSM) is combined with the marginal distributions of neighbor coefficients in the nonsub-sampled contourlet domain are modeled. The Bayes least square estimation is adopted to evaluate high pass coefficient to remove additive white Gaussian noise. Finally, inverse Lifting Stationary Wavelet transform is applied on the denoised coefficients to reconstruction the denoised image.

Introduction

It is well known that statistical modeling in the wavelet domain is favorable for many image processing applications, such as denoising and compression, because of wavelet's capability of analyzing and representing images [1]. However, wavelet has its own drawbacks. It is time and memory consuming, which impede its real-time application. Lifting scheme is put forward by Sweden [2], which is a kind of wavelet construction method do not rely on the Fourier transform in the 1990s. Compared with traditional WT (wavelets transform), LWT (Lifting Wavelet Transform) possesses several advantages, including possibility of adaptive and nonlinear design, in place calculations, irregular samples and integral transform. It can be seen as an alternate implementation of traditional wavelet transform. Unfortunately, the original LWT lack shift-invariance and cause pseudo-Gibbs phenomena around singularities, which will reduce the resultant image quality. Relative to the wavelet transform and LWT, Lifting Stationary Wavelet Transform (LSWT) can overcome the shortcomings of wavelet because its a fully shift-invariant. In this paper, we propose an image denoising algorithm based on Gaussian scale mixtures in Lifting Stationary Wavelet domain.

Lifting Stationary Wavelet transform

The main feature of the lifting wavelet transform is that it provides an entirely spatial domain interpretation of the transform, in contrast to the traditional frequency domain based constructions lifting wavelet algorithm realization is divided into three steps: division, prediction and update.

P_l and U_l denote the prediction and update operator of the lifting wavelet at level l, respectively. a is l+1 level decomposition by LWT of the input signal. d_l+1 and a_l+1 respectively are the detail and approximate coefficients after LWT decomposition of the al [3].

In the lifting scheme, the lifting wavelet transform Is lack of shift-invariance because there exists the split step(odd sample and even sample) . However, the shift-invariance is vital in many image applications such as image enhancement, image denoising and image fusion. For example, Pseudo gibbs phenomenon will appear in the image denoising. We can obtain the lifting stationary wavelet transform which possesses the shift-invariance by canceling the split step (odd sample and even sample) and insert some zero to realize continuation of the filter [4]. In the lifting stationary wavelet transform, the split step is discarded.

Gaussian scale mixtures

Consider an image decomposed into oriented subbands at multiple scales. We denote as $x_c^{s,o}(n,m)$ the coefficient corresponding to a linear basis function at scale s, orientation o, and centered at spatial location $(2^s n, 2^s m)$ [5]. We assume the coefficients within each local neighborhood around a reference coefficient of a subband are characterized by a Gaussian scale mixture (GSM) model. A random vector x is a GSM [13] if it can be expressed as the product of two independent random variables: $x=\sqrt{z}u$, where z is a positive scalar and u is a zero-mean Gaussian vector [6]. We model the desired spatially variant behavior by defining $C_u^{j(n,m)}$ as the (spatially variant) covariance of $u^{j(n,m)}$. The density of x is determined uniquely by $p_z(z)$ and $C_u^{j(n,m)}$ as follows:

$$p^{j(n,m)}(x) = \int p^{j(n,m)}(x \mid z) p_z(z) dz$$
$$= \int \frac{\exp(-x^T (zC_u^{j(n,m)})^{-1} x/2)}{(2\pi)^{N/2} \mid zC_u^{j(n,m)} \mid^{1/2}} p_z(z) dz \quad (1)$$

Where N is the dimension of x. $x^{j,(n,m)}$ is conditionally Gaussian for a given z. Without loss of generality, one can assume $E\{z\}=1$, which implies $C_x = C_u$ [7]. We aim to solve the classical denoising problem, where an image is corrupted by additive zero-mean independent Gaussian noise of known (but arbitrary) spectral density. As many other methods, we perform the denoising in the transform domain and obtain the image estimation by reconstructing the image from the contourlet coefficients [8-9]. A vector y corresponding to a neighborhood of N observed coefficients of the pyramid representation can be expressed as:

$$y = x + w = \sqrt{z}u + w \quad (2)$$

Where W is Independent additive Gaussian white noise vector.

LSWT-GSM denoised approach

Firstly, Perform the lifting stationary wavelet transform decomposition on the noisy image, and obtain all coefficients including a low-pass subband and a series of high-pass subbands. Secondly, For each high-pass subband. (a)Compute neighborhood noise covariance C_w. (b) Estimate noisy neighborhood covariance C_y. (c) Estimate C_u from C_w and C_y. (d) Simplify E{E|Y,z} as the Local wiener estimate. (e) Calculate every center coefficient xc using the BLS method. Finaly, Execute the lifting stationary wavelet reconstruction with the lowpass residual and denoised above highpass subbands.

Experiment

Three sets of images such as Lena, Barbara and House are used to evaluate the proposed denoised algorithm. The proposed image denoised approach was compared with several state-of-the-art image denoised methods including the soft-threshold in traditional discrete wavelet domain(Soft-Wavelet), GSM method in traditional wavelet domain(GSM-Wavelet), GSM method in Contourlet domain(GSM-Contourlet), GSM method in Lifting Stationary Wavelet domain(GSM-LSWT). For the traditional wavelet domain based methods including GSM-Wavelet and Soft-Wavelet, the available “db1” wavelet is used and four decomposition levels are used for image decomposition. In the Contourlet method, four decomposition levels are used for image decomposition. In LSWT, four decomposition levels and “db1” wavelet are used in the image decomposition. Table 1 shows the performance results from different image denoised algorithms. The results presented in this example can show that our approach can denoised images while retaining much more detail information than that of the other tree approach. Fig.1 shows the denoised result by above four methods.

Tab.1 PSNR of different denoising methods

Image	Variance	Soft-Wavelet	GSM-Wavelet	GSM-Contourlet	GSM-LSWT
Lena	20	30.72	31.42	32.12	32.92
	30	28.37	29.14	30.67	31.18
	40	26.29	28.31	28.98	29.80
Barbara	20	25.18	26.82	27.64	28.22
	30	22.99	23.93	24.07	25.68
	40	23.67	24.82	25.13	25.93
House	20	30.02	31.26	31.97	32.71
	30	28.68	29.95	31.14	31.97
	40	27.55	28.51	29.69	30.23

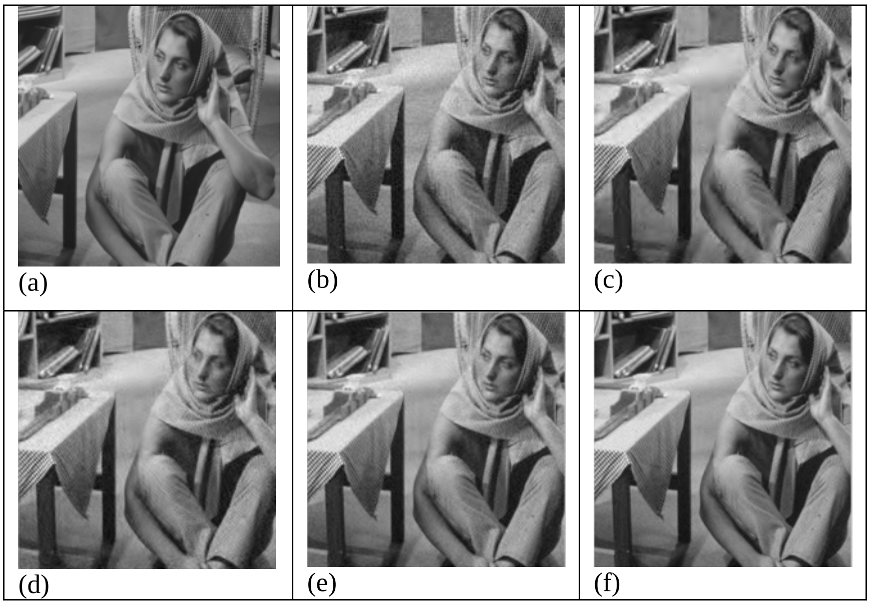

Fig. 1 (a)Original image, (b)Noisy image (noise variance is 30), (c) Soft-Wavelet (d) GSM-Wavelet, (e) GSM-Contourlet, (f) GSM-LSWT

Conclusion

In this paper, we propose a method for removing noise from digital images, based on GSM in lifting stationary wavelet coefficient. The statistical model is then used to obtain the denoised coefficients from the noisy image decomposition by Bayes least squares estimator. The comparison of the denoising results obtained with our method, and with the state-of-the-art denoising method, shows the efficiency of our approach which gave the best output PSNRs for most of the images. The visual quality of our denoised images is moreover characterized by fewer artifacts than the other methods.

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