

## Study on the key buses to Power System Steady-state Voltage Stability

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**Keywords:** Power system, Small signal analysis, Modal Analysis, Participation factor, Load shedding, Steady-state stability.

**Abstract.** With the method of modal analysis, the full Jacobian according to the power system power flow equations is studied and a method to identify the suitable bus for loading shedding is proposed in this paper. The proposed method makes use of the participation factors corresponding to the active power, by which the structural characteristic of the power system nonlinearity is included. As a result, the effective bus where the load shedding should be used can be found. In order to show the effectiveness of the proposed method, the WSCC 9-bus power system is used as an example. The simulation results indicate that with the load shedding at the place where the proposed method decided, the voltage stability indices are considerably increased. The proposed method in this paper is effective.

### Introduction

To the developing country, it is concerned in the power system for the imbalance between the generation and the demand, the large distance of the plant location from the load center, and the lack of transmission capability. Fault happens occasionally in such power system, which results into serious blackouts. When the power system operates at insecurity or the stability domain of the operating point is not enough, preventive and corrective controls will be adopted [1]. When it does not work for the power system stability enhancement after the control of generator, switching capacitor or the OLTC triggered, the emergency control of load shedding will be used as the last measurement to prevent the power system blackouts [2]. The research results in [3] show that control at different bus makes different effect on the power system stability. When the power system stability is maintained or even improved, it is expected to implement the controls at the key buses, in order to minimize the control cost. Reference [4-6] extended the traditional modal analysis method and proposed active power participation factor to study the effect of generator active power output to the transmission capability. The modal analysis was adopted in [7] to determine the suitable location of SVC for purpose of the voltage stability enhancement. In [8] the right eigenvector was studied to optimize the allocation of SVC. In this paper, the modal analysis is adopted to study the effective bus for stability enhancement. The proposed method is used to study the WSCC 9-bus power system, load shedding will improve the steady-state voltage stability effectively.

### Power flow equations and its linear transformation

Voltage stability is a subset of overall power system stability. It can be investigated by the use of the power flow equations. In this paper, the power system is modeled by the following power flow equations.

$$\begin{aligned}
 P_i &= V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad i=1,2..n-1 \\
 Q_i &= V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad i=1,2..m
 \end{aligned}
 \tag{1}$$

where  $V_i$  is the voltage at bus  $i$ ,  $\theta_{ij}$  is the angle difference between buses  $i$  and  $j$ ,  $G_{ij}$  and  $B_{ij}$  are the conductance and susceptance of the line connecting the buses  $i$  and  $j$  respectively,  $P_i$  and  $Q_i$  are the active and the reactive power injections at bus  $i$  respectively, and  $m$  and  $n$  are the number of PQ buses and total buses of the system respectively.

For the sake of simplification, the equation (1) can be rewritten as following.

$$L = g(x) \tag{2}$$

where  $L=(P_1, P_2, \dots, P_{n-1}, Q_1, Q_2, \dots, Q_m)^T$ ,  $\mathbf{x}=(\theta_1, \theta_2, \dots, \theta_{n-1}, V_1, V_2, \dots, V_m)^T$ .

The Taylor's series expansion of equation (2) about an equilibrium point (EP) is given by Eq. (3).

$$L = \mathbf{J}\mathbf{x} + H.O.T \tag{3}$$

where  $\mathbf{J}$  is the Jacobian matrix, and  $H.O.T$  is the high order terms.

Using the similarity transformation  $\mathbf{x}=\mathbf{W}\mathbf{y}$ , where  $\mathbf{W}$  is the matrix of right eigenvectors for  $\mathbf{J}$ , the original system can be represented by the following Taylor expansion with up to first order in the  $y$ -coordinates.

$$\mathbf{W}^{-1}\mathbf{L} = \Lambda\mathbf{y} \tag{4}$$

where  $\Lambda$  is the eigenvalue matrix of  $\mathbf{J}$ .

### Participation factor (PF)

The participation factors provide useful information about the power system performance. The active power participation factors with the interesting mode give the degree of contribution of the buses to the corresponding system. The larger the active power participation factor is, the more influence the bus will contribute to the voltage stability. Consequently, this bus is the appropriate location where load shedding should be used for the voltage stability enhancement of the overall power system. In this paper, the active power participation factor is used as an index to measure this kind of contribution and it can be computed by Eq. (5).

$$p_{ki} = w_{ki} \times v_{ik} \tag{5}$$

where  $w_{ki}$  and  $v_{ik}$  are the element of  $\mathbf{W}$  and  $\mathbf{W}^{-1}$  respectively.  $p_{ki}$  represents a measure of the participation of the  $k$ th state to the  $i$ th mode.

### Ranking buses

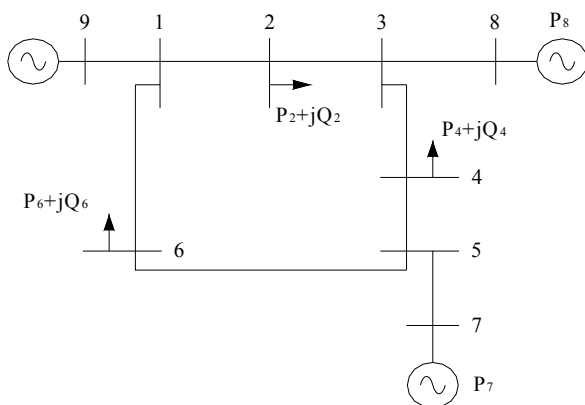
As has been mentioned in the previous section, based on the power flow equations, this paper uses the active power participation factor  $p_{ki}$  as an index to measure the load shedding allocation. The large the active power participation factor associated with the interesting mode indicates more impact of the bus on the voltage stability. Consequently, it is also the most effective installation location of load shedding.

The results of reference [4] pointed out that the eigenvectors of the full Jacobian matrix are always equal to those of the reduced Jacobian matrix and the information of the full Jacobian matrix eigenvectors is all included in the eigenvectors of the reduced Jacobian matrix at the singularity point. However, at the non-singularity points, the eigenvector of the full Jacobian matrix do not match those of the reduced Jacobian matrix. Furthermore, the larger the operating condition is away from the

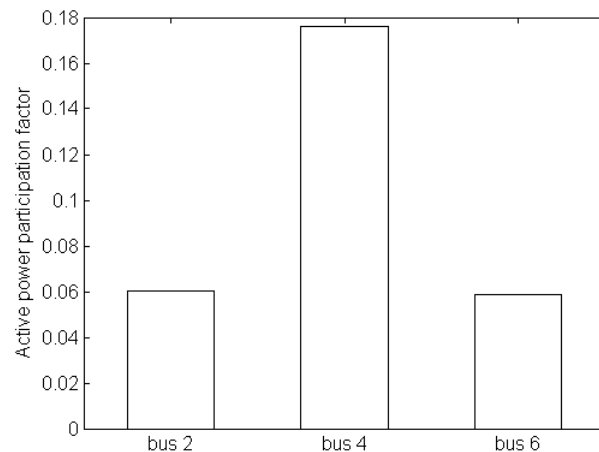
singularity point, the larger the difference between the eigenvectors is. The conclusion that the reduced matrices contain all information of the full Jacobian is only valid at the singularity point. Therefore, in order to get correct information, this paper analyzes the full power flow equations before the singularity point.

### Case study

The WSCC 9-bus power system shown in Fig. 1 is used in this paper to verify the efficiency of the method proposed in this paper. The data in this paper are all in per unit. Bus from number 1 to number 6 are PQ buses, bus 7 and bus 8 are PV buses, bus 9 is slack bus. The standard operation condition is adopted for the case study. The mode with least eigenvalue is selected as the interesting mode. The active power participation factors corresponding to the interesting mode are calculated and shown in Fig. 2. In the WSCC 9-bus power system, the loads are distributed at bus 2, bus 4 and bus 6, then participation corresponding to these buses are analyzed. It is shown in Fig. 2 that the active power participation factor corresponding to bus 2, bus 4 and bus 6 is 0.060, 0.176 and 0.059 respectively. The participation factor corresponding to bus 4 is the largest one, which means that bus 4 is the most appropriate location for load shedding in order to improve the stability.



**Fig.1 WSCC 9-bus system**



**Fig. 2 Active power participation factors**

Many analysis methods of voltage stability determination have been developed on static analysis techniques based on the power flow model since they are simple, fast and convenient to use. The eigenvalue  $\lambda$  of the power flow Jacobian is one of the indices to preliminarily scale the degree of the power system voltage stability [9]. In order to verify the result, the largest eigenvalue  $\lambda_{\max}$  is used as an index to measure the steady-state voltage stability. That all of the eigenvalues of the power flow Jacobian are negative indicates the power system is steady-state voltage stable. The larger distance between the  $\lambda_{\max}$  and the imaginary axis, i.e., the larger the  $\lambda_{\max}$ , the more stable the power system is. When the  $\lambda_{\max}$  is equal or larger than 0, the power system will lose stability.

For the simulation case, the stable index  $\lambda_{\max}$  is calculated respectively when there is no load shedding, adopted load shedding at the bus 2, bus 4 and bus 6 in the system. The amount of load shedding at different bus is 0.03 p.u.. The eigenvalues  $\lambda_{\max}$  are shown in the Table 1. From this table, it can be decided that the load shedding at all of the bus 2, bus 4 and bus 6 can enhance the steady-state voltage stability greatly. Moreover, the most enhancement of voltage stability can be gotten by load shedding at bus 4, the second one is bus 2, and the last one is bus 6. The sequence is the same as that of active power participation factor shown in Fig. 2. Therefore, the active power participation factor is accurate and can be used to estimate the correct load shedding location.

**Table 1 voltage stability indices for different situations**

	load shedding at bus 2	load shedding at bus 4	load shedding at bus 6	no load shedding
$\lambda_{\max}$	0.93618	0.93620	0.93615	0.93586

## Conclusions

The method of modal analysis is adopted to study the power system power flow equations, and the full Jacobian according to the power system power flow equations is analyzed. The active power participate factor is used to determine the location of load shedding for power system steady-state voltage stability enhancement. The simulation results show that by using active power participate factor to determine the location of load shedding, the steady-state voltage stability will be improved. The active power participate factor can be used to estimate the key buses of the power system for security.

## Acknowledgments

We would like to thank the project supported by Science and Technology Foundation of Guizhou Province([2009]2236). This work is also supported by the Talent Foundation of Guizhou University ( (2008)011), the Science and Technology Funds for Excellent Young Talents of Guizhou Province([2009]14).

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