Multi-objective Genetic Algorithm based on Game Theory and its Application

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Abstract. Mufti-objective optimization has been a difficult problem and focus for research in fields of science and engineering. There already have a lot of classical methods for solving mufti-objective optimization problems before evolutionary algorithms were introduced in 1985. Classical mufti-objective optimization methods have been thoroughly developed, but there are still Lots of shortcomings in solving high dimension, multimodal problems. GAs can handle large space of problem and get a lot of trade-of fronts (possible solutions) in one evolution. A GA does not need much information about the problem before starting the optimization process, also it is not sensitive to the convex of the defined fields of the objective functions. So using GAs in solving mufti-objective optimization problems is the most important research direction in the future. We import knowledge of immune, co-evolution and game theory into genetic algorithm to improve the performance on solving the mufti-objective optimization problems. The results of the riments show that all of them can get better results than the original algorithm.

Introduction

In M. Sefrioui and J. Periaux article *Nash Genetic Algorithms: examples and applications* proposed Nash GAs, in solving the multi-objective optimization problem, only once get a Pareto optimal solution, instead of Pareto optimal solution front surface, this is obviously not enough.

Based on the Nash GAs in the foundation, the union NSGA algorithm, the paper proposes a kind of can get more Pareto optimum solution of the multi-objective optimization algorithm, Nash GAs with the combination of the NSGA algorithm, at the same time also played the role of accelerated NSGA algorithm convergence.

The basic idea of the algorithm

Algorithm can optimize object of every target, corresponding to the game of a participant, and all the independent variables in these between participators distribution, in which each participant distribution is a subset of the independent variables. In the optimization process, each player at the same time the use of genetic algorithm operation must be algebra optimize their corresponding target, until all goals are unable to optimize so far.

Algorithm will Nash equilibrium method embedded NSGA, in NSGA generation evolution ended, remove preserved the Pareto optimum solution of the external set, the use of Nash equilibrium thought optimization, the method is as follows: first take out the optimal solution of i-th Pareto $(x_{i1}, x_{i2}, ..., x_{in})$, its distribution to the first objective function (participant), a set of corresponding argument is X_1, random extraction subset X'_1 of X_1 (shall meet the requirements $when |X_1| = 1 or 2, |X'_1| = 1; others \frac{|X_1|}{4} \le |X'_1| \le \frac{|X_1|}{2}$), $X_1 \setminus X'_1$ will set the argument with the first i-th Pareto optimal solutions corresponding value is fixed, the independent variable as a new objective function arguments. Will the new objective function, using the genetic algorithm optimization, this will be a single objective optimization problem, the use of SGA able to quickly solve, the final result will get a optimal solution, using the optimal solution update first Pareto. The optimal solution of the corresponding position of the value, will get a new Pareto optimal solution. Use the same method will be the Pareto optimal solution assigned to *2, 3,... , k* target function, and eventually add existing external set will get *M (k + 1)* solution (M for the current external set scale). In these solution, eliminate the bad solution, if the number of final solution or less $\overline{N}$ ($\overline{N}$ for external set scale), retains all the solution, as the current Pareto optimal solution set, will enter the next generation of population to cross and mutation operation: if the last solution quantity $\geqslant \overline{N}$, then use crowded distance criterion, retain crowded distance larger than before a individual as a new Pareto optimal solution set.

Algorithm test and result analysis

In order to facilitate comparison, selecting the following functions for test, that is to say, the problem is described as follows:

$$Min(f_1(x_1), f_2(x_1, x_2)) \tag{1}$$

$$f_1(x_1) = 4x_1 \tag{2}$$

$$f_2(x_1, x_2) = g(x_2) \cdot h(f_1(x_1), g(x_2)) \tag{3}$$

$$g(x_2) = \begin{cases} 4 - 3\exp(-(\frac{x_2 - 0.2}{0.02})^2) & if\ 0 \le x_2 \le 0.4 \\ 4 - 2\exp(-(\frac{x_2 - 0.7}{0.2})^2) & if\ 0.4 \le x_2 \le 1 \end{cases} \tag{4}$$

$$h(f_1, g) = \begin{cases} 1 - (\frac{f_1}{g})^a & if\ f_1 \le g \\ 0 & otherwise \end{cases} \tag{5}$$

$$\alpha = 0.25 + 3.75(g(x_2) - 1) \quad x_1 \in [0,1], x_2 \in [0,1] \tag{6}$$

This question has a concave local Pareto optimal solution set and a convex global optimal solution set this problem have a local not convex Pareto front end and a convex global Pareto front end.

Objective function were used respectively to linear weighted method, the NSGA and the proposed algorithm is presented to solve this problem, the results are as follows:

The linear weighting method

Using linear weighted method 50 times results:

72% convergence in $\begin{pmatrix} x_1 = 0.235033 \\ x_2 = 0.200288 \end{pmatrix}$, the corresponding values of f_1 and f_2 are

$\begin{pmatrix} f_1 = 0.940134 \\ f_2 = 0.0156209 \end{pmatrix}$,

This point is located in global Pareto front end.

28% convergence in $\begin{pmatrix} x_1 = 0.503219 \\ x_2 = 0.683019 \end{pmatrix}$, the corresponding values of f_1 and f_2 are

$\begin{pmatrix} f_1 = 2.01288 \\ f_2 = 0.006432 \end{pmatrix}$,

This point is located in local Pareto front end.

The solution process using real number coding method, population scale is 20, the biggest evolution algebra is 2000.

NSGA

Using real number coding, based on the Pareto sort and sharing mechanism of the NSGA solving this problem, population scale is 50. Table 1 shows different evolutionary algebra, global Pareto optimal solution for the proportion of all solutions.

Table 1. The NSGA algorithm to solve the results

Evolution algebra	40	100	200	300	400
The proportion of global Pareto optimal solution	60%	60%	90%	90%	100%

From the above we can see that when the population scale is 50, NSGA need to run 400 generation evolution to guarantee 100% convergence to the optimal solution.

The algorithm of this paper

In this paper two goals, two independent variables function optimization problem, we will first objective function as the first player, optimize shut; The second objective function as the second player, fixed gas, optimization of the people. In order to facilitate comparison, the population scale as 50, separately using linear weighting method, NSGA and the proposed algorithm is presented to solve the problem. Get the result is as follows:

Table 2. The comparison of the three kinds of algorithm

Evolution algebra	40	100	200	300	400
The proportion of weighted Pareto optimal solution	26%	32%	35%	35%	54%
The proportion of NSGA Pareto optimal solution	60%	60%	90%	90%	100%
The proportion of this paper algorithm Pareto optimal solution	62%	70%	100%	100%	100%

Visibly, the proposed algorithm, the convergence speed of the optimal, only need 200 generation that can be 100% convergence to global Pareto optimal solution. The NSGA embedded in the process of the Nash equilibrium, so the algorithm of time is NSGA doesn't improve.

Conclusions

Based on the Nash equilibrium in the foundation, the union NSGA algorithm, and put forward a kind of solving multi-objective optimization problem of the genetic algorithm. Algorithm through the use of Nash equilibrium thoughts in the NSGA have obtained Pareto optimal solution based on the fixed part of the argument, optimize the other independent variable, continuously optimize Pareto optimal solution method, to speed up the convergence of the algorithm. Experiments show that the proposed algorithm has fast convergence characteristics, but the running time of the algorithm is NSGA and without too much improved, this is also put forward in this paper to improve the algorithm of a direction.

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