The Inverse Solution to Eigenvalue Problem of Milling Machine Spindle based on Neural Network-Optimization Design

Hongyan SHAO
Guiyang College，Guiyang 550005,China

Abstract—**The design of milling machine spindle with respect to dynamic properties can be classified as the solution to inverse eigenvalue problem. In the paper, the solution to the inverse eigenvalue problem based on artificial neural network is studied, the research results show that the direct inverse model of ANN can not derive correct results while the optimization means based on artificial neural network is an effective one of solving inverse eigenvalue problem.**

Key words-Neural network-optimization design; milling machine spindle; eigenvalue; inverse problem solution

I. INTRODUCTION

Milling spindle is a very important part in milling machine, the design of the part related to the overall performance of the milling machine.Therefore, in addition to consider its static properties in design , we still should consider its dynamic characteristics requirement.

The design of milling spindles' dynamic characteristic comes down to the inverse problem solving of eigenvalue.The so-called inverse problem solving of eigenvalue as that the eigenvalue of the structure is given in advance, and then according to the structural generalized characteristics equation solve the structure parameters.The solving methods of the inverse problem of eigenvalue including optimization method, structure matrix perturbation method and sensitivity analysis [1].

The application of the inverse problem solving method of dynamics based on neural network[2~3],The basic idea is using finite element analysis software or measured value attain the training sample that reflects the relationship between the structure and input (structure parameters) output (structural response), then respectively as the output-input of the neural network to train network, so as to realize the nonlinear inverse mapping from output parameters space to input parameter space.But this method can't get the right result in the spindle contains multiple design variables , therefore the usable range is very narrow. Research shows that the neural network-optimization method is a universally applicable method of solving the inverse problem of eigenvalue.

II. THE NEURAL NETWORK MODEL OF THE INVERSE PROBLEM OF EIGENVALUE

A. *The inverse problem of eigenvalue*

The generalized characteristic equations of vibration for :

$$[K]\{q_j\}=\lambda_j[M]\{q_j\}; \qquad j=1,2,\ldots N \qquad (1)$$

Thereinto,[K] for stiffness matrix, [M] for quality array, λ_i for $_i$ order eigenvalue, $\{q_i\}$ for the corresponding feature vector, N for free degrees. The inverse problem of eigenvalue,given the target eigenvalue λ_{obi} of the structure.According to (1) find out the structure size parameters $X = \{x_i\}$, (i= 1,... M), M as the design variables，contained in [K] and [M].

B. *The neural network direct inverse model*

In the basis that about the continuous function indicate law, Kolmogorov has proved a three orders feed forward networks have the ability with arbitrary precision approaching n sphere continuous function defined in tight subset of K[4].Thus multiorder feed forward neural network (BP network) is a kind of general function force implement, can realize the arbitrary the arbitrary nonlinear mapping from Rm to Rn , this is neural network obverse model (as shown in figure 1).Neural network model is used to replace the finite element in proceeding structure similar analysis, realize the nonlinear mapping from structure size to structural response.The obverse model put the cart before the horse becomes direct inverse model (as shown in figure 2),realize the nonlinear mapping from Rn to Rm.Neural network direct inverse model is used to realize the nonlinear mapping from the structure size to structural response , is not always able to get the right result.

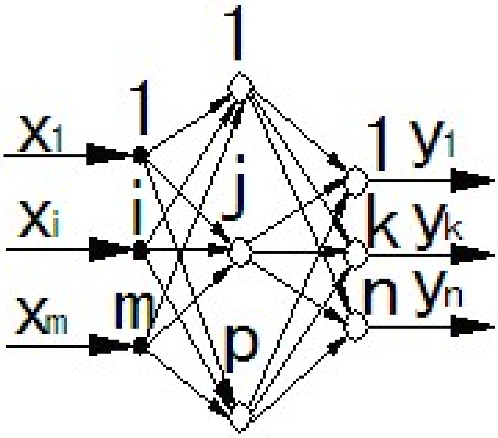

Figure 1: Neural network obverse model

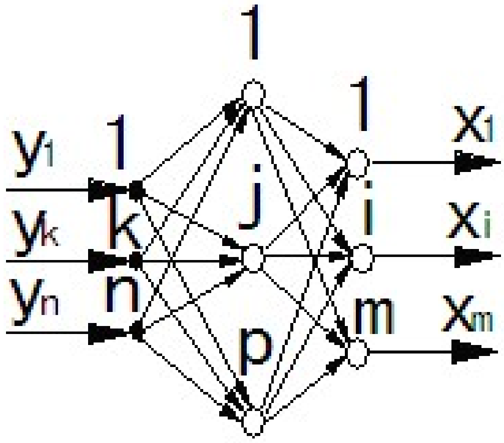

Figure 2: Neural network direct inverse model

C. Neural network-optimization model method

The eigenvalue inverse problem solving boiled down to a no constrained nonlinear programming problem.

Solving: $X = \{x_i\} \qquad (i = 1,2,...,m)$

$$\min . \quad Er = \frac{1}{2}\sum_{i=1}^{n}(\lambda_i - y_i)^2 \qquad (2)$$

In the formula, λ i (i= 1,..., n) is expected receive i order ,y_i indicate the actual eigenvalue of the structure. When the objective function Er to the given convergence precision , the corresponding design variable value is the solution of the inverse problem's eigenvalue. For the function minimization ,which of quadratic sum form , common optimization method can solve [5].The mathematical model that formulas (2) defined may have a number of solutions, this is because the mapping from eigenvalue to the structure size is not one-to-one, it can be seen in the follow analysis. To make the results sole, mathematical model can be changed to the lightest weight design problem constrainted by frequency [6].

Solving: $X = \{x_i\} \qquad (i = 1,2,...,m)$

$$\min . \; W(X)$$

$$s.t. \; g_j(X) = \lambda_j - y_j = 0 \qquad (j = 1,2,...,n)$$

The lightest weight design constrainted by frequency,if criterion method is adopted to solve, the method will be into the local minimum[7].In recent years, the good properties of genetic algorithm [8] that make it show a good application prospect in some continuous and discrete variable optimization design field .Its main advantage is the strong general optimization ability, do not need gradient information, the continuous function is not necessary, the optimal result is overall , so this paper using genetic algorithm.Genetic algorithm is a kind of probability search method, it needs a considerable number of chromosomes after generations breeding to search the optimal solution, a lot of heavy work for finite element structure analysis are very difficult calculation tasks, so using neural network obverse model instead of finite element for the structure similar analysis .

III. The calculation results and analysis

A. An inverse eigenvalue problem in variables design

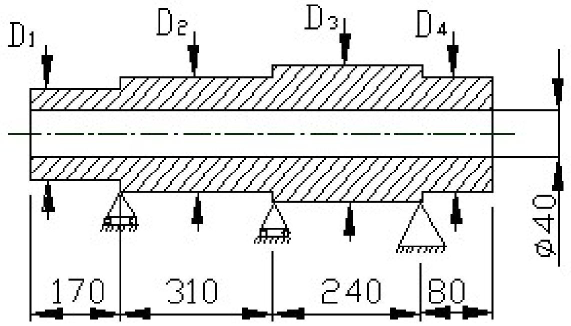

Figure 3 Horizontal milling spindle

Figure 3 indicates horizontal milling spindle,it is known that ρ=7800kg/m^3,E=206x10^{3}Mpa.Now assume that all outside diameters indicated by a design variable, namely: D1 =D2 = D3=D4 = X = {x1}, seek the section size x1 of shaft ,when 1 order frequency λ_1 = 900, 1200, 1500, 1700 Hz.With neural network direct inverse model solving, through ANSYS gained five training samples shown in table 1, using a 1×5×1 three-layer neural network realized the mapping from λ_1 to x . Mapping the results in table 2.

Table 1 Training samples

X_1(mm)	60	80	100	120	140
λ_1 (Hz)	927.7	1138.8	1355.3	1569.4	1783.3

Table 2 Neural network direct inverse model mapping results

λ_1 (Hz)	900	1200	1500	1700
X_1 (mm)	54.3	84.5	112.6	139
The corresponding λ_1 for X_1 solved by finite element(Hz)	869	1187	1494	1768
Reverse error（%）	-3.4	-1.1	-0.4	+4

From table 2 it is known when only a design variable, the results that neural network direct inverse model solve eigenvalue inverse problems are correct, the error generally is small, only in the sample points at both ends is larger.

B. The eigenvalue inverse problem of two design variables

Still take figure 3 for example,Take two design variables, namely x1 = D2, x2 = D3, the rest D1 = D4 = 60 mm, require 1 order frequency λ_1 = 1600 Hz, solve the section size of spindle X = {x1, x2}.

(1) With the neural network direct inverse model solving, through ANSYS get 25 training samples as shown in table 3.To make the design point evenly distributed in design variables space, use the orthogonal table [9] setting points.Firstly tried to adopt a 1×6×2 single hidden layer BP

network to realize the mapping from λ_1 to x1 and x2, but BP network don't convergence. Figure 4 is training error curve.It can see from figure 4, from about 300 times to 20000 times, training error keep the same in 0.6, which indicates that the network can't convergence.When changing learning rate, the times of training the parameters still can't convergence. Of course,it can't come to a conclusions by a practice.Sometimes, the normal mapping occasionally appear the condition of figure 4,the training followed by can be normal; But when a mappings don't exist, no matter how to repeat training, will occur the condition shown in figure 4.

TABLE 3 SECTION SIZE AND 1\2ORDER NATURAL FREQUENCY (HZ)

X1 (mm)	X2(mm)				
	60	80	100	120	140
60	927/1785	969 / 2170	985 / 2349	992 / 2447	995 / 2507
80	1271/1526	1320/2059	1354/2383	1372/2595	1380/2743
100	1272/1491	1514/1801	1539/2167	1556/2434	1566/2635
120	1820/1615	1572/1623	1633/1939	1643/2222	1650/2447
140	938/1677	1386/1679	985/2349	1687/2020	1715/2275

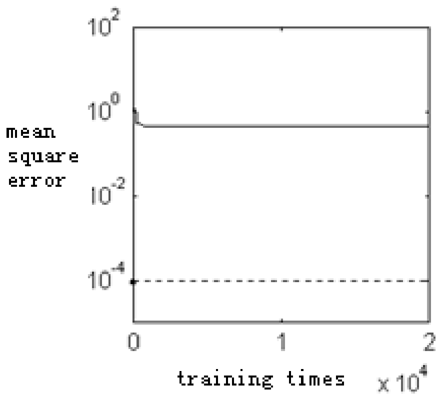

Figure 4 Obverse model training error curve

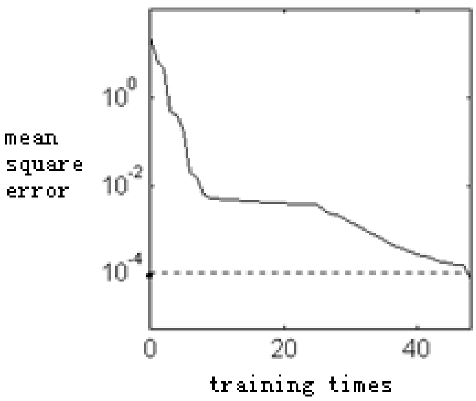

Figure 5 Inverse model training error curve

This phenomenon can be seen from size-frequency grid figure (figure 6, figure 7), λ_1 and λ_2 are multi-modal function, therefore x1 x2→λ_1 and x1 x2→λ_2 are not one-to-one relationships.In other words, a λ_1 or λ_2 may correspond to multiple x1, x2, the contours of figure 6 or figure 7 , are the combination of all x1 x2 in a frequency.From the function definition, X = {x1, x2} = f (λ_1) is not a function, so it can't adopt neural network to approach to this relationship.

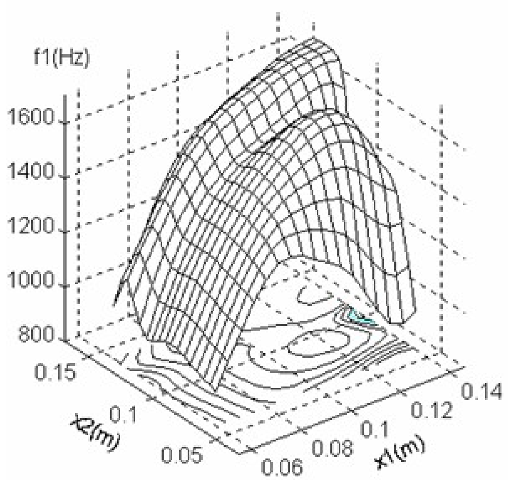

Figure 6 X-f1 grid chart

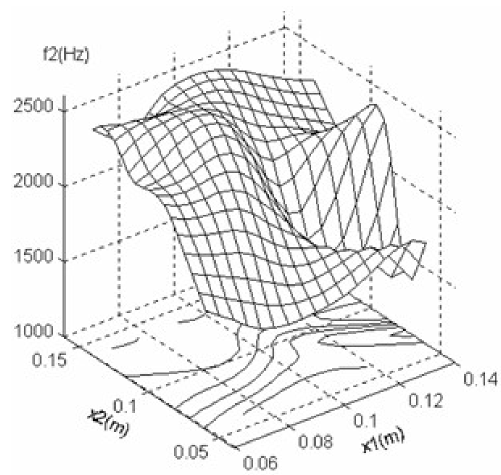

Figure 7 X-f2 grid chart

Therefore, when more than one design variable, the eigenvalue inverse problem of designed object horizontal milling spindle can't use neural network direct inverse model solving.

Neural network-optimization model solving.Adopting a 2 ×8×1 BP network to realize the mapping from X = [x1, x2] to λ_1.Training network used the BP algorithm that join a momentum and learning rate self-adjusting, the largest number of training step ep = 5000, allow error e = 0.001, learning rate η = 0.01, incremental learning rate η_1 = 1.05 adopted, reducing learning rate η_2 = 0.75, momentum α = 0.7, training error curve as shown in figure 5.In genetic algorithm,the initial population size is 80, cross rate P_C = 0.7,mutation rate P_M=0.01,biggest genetic algebra epoch=500.The results obtained by the neural network + genetic algorithm inverse model solving are given in table 4, the two are results are very close to, show that the genetic algorithm can converge to the most advantage points with a probably rule.

IV. CONCLUSION

The research in this paper shows that:

(1) In the solving method of milling spindle eigenvalue inverse problem, only in the condition eigenvalue and design variables have a one-to-one to mapping relationship,can it adopt neural network direct inverse model, and the using range is largely restricted;

(2) The solving model based on neural network and optimization method ,is a universally applicable method in eigenvalue inverse problem. To make the results sole, the solution of eigenvalue inverse problem can change to the lightest weight design problem constrainted by frequency; In this model, the neural network is used as obverse model instead of finite element proceeding structure similar analysis;

(3) Using the genetic algorithm to solve the optimization mathematical model is expected to get the optimal solution and is a good optimization method.

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