

# The Research of Tire Pressure Monitoring Method Based Model Between Force and Slip

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**Abstract**—In order to prevent severity traffic accident on freeway caused by tire blowout, it is necessary to monitor the tire pressure in real time. This paper established a linear model between the force and the slip to estimate the tire radius through least square, where only ABS wheel speed sensors were used. The result of the simulation about this method is perfect, it is a effective method for indirect tire pressure monitoring.

**Keyword**-slip; tire radius; tire pressure

## I. PREFACE

The Automobiles get touch with the ground by tires, the tire pressure has an direct influence on the safety, operation stability, fuel economic character and comfortability of the vehicles. In China, 70% of the traffic accidents on freeway are caused by tire blowout at present, comparing 80% in the United States. According to the investigation of SAE, the number of the traffic accidents caused by shortage of tire pressure is 260,000 in United States annually, and 75% of the tire malfunction is caused by shortage or leak of tire pressure [1]. The cause of the tire recall of Bridgestone in the United States is that the desquamation of tires caused by shortage of tire pressure results in severe traffic accidents. The shortage of tire pressure will increase the tire distortion, causing the heightening of tire temperature, and destroy the internal structure of tires. Especially when the car is driving at a high speed, the low tire pressure will easily engender standing wave and cause the tire bursting, which will bring on severe traffic accidents. Therefore, the technique of intelligence tire that can monitor the pressure and temperature of tire is being attached importance to increasingly.

The tire pressure monitoring system (TPMS) can be thought a kind of simple intelligent tire technique, and it contains two types: direct TPMS and indirect TPMS. The direct TPMS monitors the tire pressure by the pressure sensors installed in each tire, so it needs new hardware and the cost is higher. The indirect TPMS makes use of the sensors and software that the vehicles have installed, so it doesn't need to increase the new hardware and the cost is lower. This paper presents an effectively method basing on the wheel speed sensors of ABS. It estimates the change of tire radius so as to monitor the tire pressure by establishing tire model, and it will bring a better effect.

## II. TIRE MODEL

There are many tire models based on the slip of wheels to forecast the tire longitudinal force, and these models origin from experience data usually. They contact the slip with tire longitudinal force under the condition of limit of loading, road, character of tire and other factors. The Magic Formula [2] is a typical model of them, it uses the combination formula of trigonometric function to calculate the data of experimented tire, and show fully the longitudinal force, transverse force and moment by some formulas of same form. The general expression is as follows:

$$Y = D \sin(C \arctan(B\Phi)) + \Delta S_r \tag{1}$$

$$\Phi = (1 - E)(X + \Delta S_h) + (E/B) \arctan(B(X + \Delta S)) \tag{2}$$

Where  $D$  is the factor of peak value,  $B$  is the factor of stiffness,  $C$  is the factor of curvature,  $E$  is the factor of curve shape,  $S_h$  is horizontal excursion of curve and  $S_r$  is vertical excursion of curve.

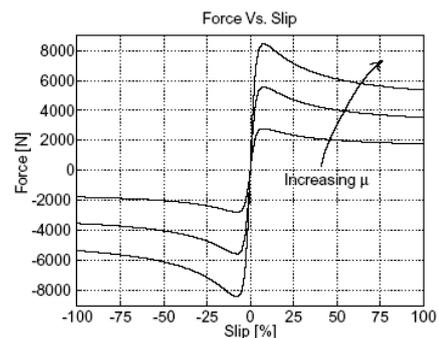


Figure 1: Force-slip curves of the Magic Formula

Figure 1 shows typical force-slip curves of the Magic Formula for different values of road surface to tire friction. We can see that although the curves of different friction coefficients are different, the relation between force and slip is approximately linear when the tire slip is less than 3%. During ordinary driving, however, the tire slip rarely exceeds 2%. By linearizing the model in this small region, the force-slip can be characterized as:

$$F = C_x S \quad (3)$$

Where  $F$  and  $C_x$  and  $S$  are the force and longitudinal stiffness and slip of the tires transmitting the force.

According to concept of longitudinal stiffness,  $C_x$  has relation to not only the longitudinal force, but also the structure of tire and the length of tire imprint with road, especially the tire pressure.

The SAE definition for wheel slip is:

$$S = -\frac{(V - R\omega)}{V} \quad (4)$$

Where  $V$  is the velocity of the center of the tire,  $R$  is the effective radius of the tire and  $\omega$  is angular velocity of the tire.

Then equation (3) can be written as:

$$F = C_x \left( \frac{R\omega - V}{V} \right) \quad (5)$$

### III. TIRE RADIUS ESTIMATION

When the automobile is driving, its equation of driving [2] is:

$$F_t = F_f + F_w + F_i + F_j \quad (6)$$

Where  $F_t$  is the driving force,  $F_f$  is the rolling resistance,  $F_w$  is the air resistance,  $F_i$  is the grade resistance and  $F_j$  is the accelerating resistance.

Here we first discuss the relation between the driving force and resistance. The driving velocity influences the rolling resistance coefficient greatly, we can see from figure 2 that the car's change of rolling resistance is not obvious when the velocity of car is less than 110km/h. But it increases more quickly when the velocity of car exceeds 140km/h. While the velocity reaches 110km/h, the rolling resistance increases rapidly. Commonly, the rolling resistance of a car is below 100N when the velocity of car doesn't exceed 140km/h. Comparing to the driving force, it is very little, so we can neglect it.

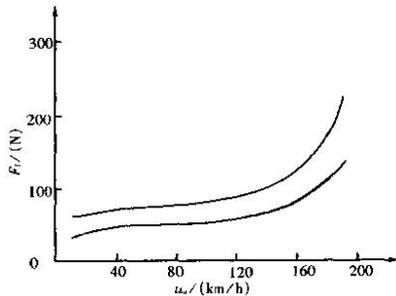


Figure 2: The changing curve of rolling resistance following the change of velocity

Suppose the automobile is driving on flat road and the

driving velocity is within certain speed, therefore we can neglect the grade resistance of driving[2]. In order to estimate further the tire parameter of automobile, we can predigest the equation and neglect the rolling resistance of driving equation in the linear area of force-slip. When the automobile is driving,

the air resistance of car can be written as  $F_w = \frac{C_D A V^2}{21.15}$ . As

the factor of air resistance  $C_D A$  is a fixed value, so air resistance is determine only by  $V^2$ . So we can make  $L = \frac{C_D A}{21.15}$ , then  $F_w = L \cdot V^2$ .

So the equation (6) can be predigested as follows:

$$F_t = F_w + F_j = F_w + ma$$

$$F_t = F_w + ma \quad (7)$$

Where  $m$  and  $a$  are the mass and acceleration of the car.

We can come to a conclusion from equation (5) and equation (7) as follows:

$$ma + LV^2 = C_x \left( \frac{R\omega - V}{V} \right)$$

$$a = \frac{1}{m} \left[ C_x \left( \frac{R\omega - V}{V} \right) - L \cdot V^2 \right] \quad (8)$$

Where  $a$  and  $V$  and  $\omega$  can be directly measured and calculated by the wheel speed sensors of ABS.

In the moment of  $k$ , we suppose  $a(k)$  and  $V(k)$  and  $\omega(k)$  as the acceleration and velocity and angular velocity of automobile, then :

$$a(k) = \frac{1}{m} \left[ C_x \left( \frac{R\omega(k) - V(k)}{V(k)} \right) - L \cdot V(k)^2 \right]$$

Supposed that  $P(k) = \frac{\omega(k)}{V(k)}$ , then:

$$a(k) = \frac{1}{m} C_x [RP(k) - 1] - \frac{1}{m} L \cdot V(k)^2$$

$$a(k) = \frac{C_x R}{m} P(k) - \frac{C_x}{m} - \frac{1}{m} L \cdot V(k)^2 \quad (9)$$

$$\text{Supposed that } b = \frac{C_x R}{m} \quad (10)$$

$$c = -\frac{C_x}{m} \quad (11)$$

$$d = -\frac{1}{m} L \quad (12)$$

$$\text{Then: } a(k) = bP(k) + c + dV(k)^2 \quad (13)$$

If we sample for  $N$  times, then the square sum of error is:

$$J = \sum_{k=1}^N [a(k) - bP(k) - c - dV(k)^2]^2 \quad (14)$$

We can try to get the differential coefficient of  $J$  if we want the minimum of  $J$ , and make them zero, then:

$$\frac{\partial J}{\partial b} = 0 \quad (15)$$

$$\frac{\partial J}{\partial c} = 0 \quad (16)$$

We can estimate  $b$  and  $c$  as  $\hat{b}$  and  $\hat{c}$  from the above equations (15) and (16), so equations (11) and (12) can be written as:

$$C_x = -\hat{c}m \quad (17)$$

$$\hat{R} = -\hat{b}/\hat{c} \quad (18)$$

#### IV. SIMULATION AND ANALYSIS

The Matlab is the software of strong function which is developed by the Math Work company in the United States, and simulink toolbar can be used to simulate system model in real time. Here we simulate the established model by the simulink toolbar. Figure 3 shows the setting of the velocity and the angular velocity of the tire and the acceleration, and we can get the result of simulation  $C_x$  and  $R$  as Figure 4. From figure 4 we can see that the estimated value of tire radius has a good consistency with the true value, so the change of estimated tire radius can reflect better the change of tire pressure.

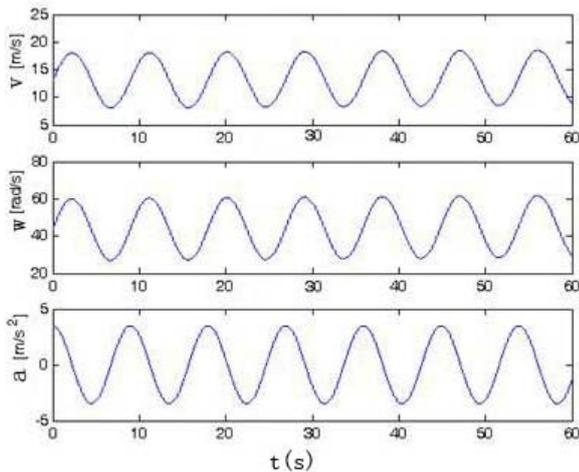


Figure 3: The setting of the velocity and the angular velocity of the tire and the acceleration

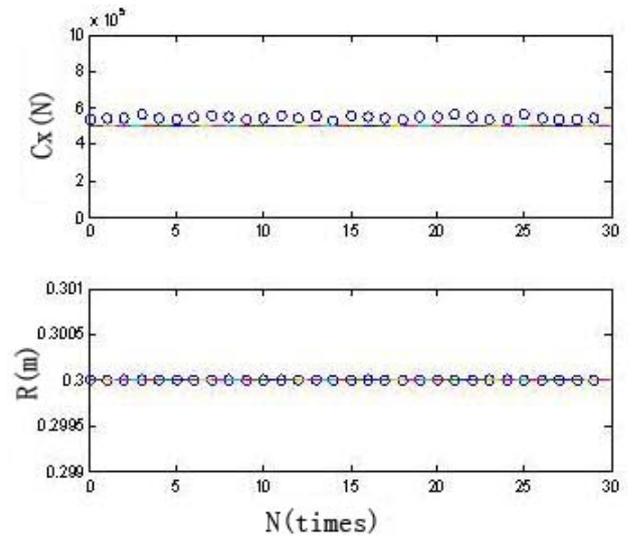


Figure 4: The result of simulation of  $C_x$  and  $R$

#### V. CONCLUSIONS

The slip of tire has not only a close connection with longitudinal force, but also a direct connection with tire radius. We can get the force-slip model of low slip from the Magic Formula Model, estimate the tire radius through least square, and the result of the simulation about this method is perfect. It is an effective method for indirect tire pressure monitoring by only ABS wheel speed sensors.

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