

Electromechanical Coupled Non-linear Dynamics for Micro Plate Simply supported on four Sides

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Abstract—Dynamic characteristic of the micro plate under the micro scale effect shows non-linear features, traditional linear analysis methods can not satisfy the design precision of MEMS system, and non-linear dynamics analysis method provides an effective solution. In this paper, based on the small deflection theory and electrostatics theory, take typical boundary condition of the micro plate simply supported on four sides as example, the basic differential equation of lateral vibration is established, using Linz Ted-Poincare method, displacement and time course of forced vibration, time responses of forced vibration are calculated. The research results provide theoretical basis for optimizing the system parameters, improving the ability to resist interferences and increases the reliability.

Keywords- micro plate; MEMS; non-linear dynamics; small deflection theory; Linz Ted-Poincare method

I. INTRODUCTION

Within the microscopic scale, machinery can be divided into small machinery dimensions of 1~10 mm, micro machinery dimensions of 1 μ m~1mm, and nano machinery of 1nm~1 μ m according to the different characteristics scale. Micro-electromechanical systems is called Micro machine and MEMS system, which generally consists of micro machinery and nano machinery, but not the macro machinery be miniaturized simply. Micro-electromechanical system (MEMS) can be described as micron even smaller order micro-component or micro system made up of electron or mechanical component, and its processing method and driving principle are different from macroscopic machine. Such devices are new technology products which can realize higher accuracy, more convenient and efficient, cheaper and better energy saving micro operation, also, can feel, understand, reform and control objective world. MEMS have potential applications in automatic control, mobile communication, aerospace instrument and biomedical engineering field^[1-3].

Micro components are important parts, which are composed of micro agencies and micro-electromechanical systems. Although, structure of the micro components is generally simple, we can use their displacement, deformation and movement to realize the function of micro sensor and micro actuator. Main form of the micro components are consist of micro plate, micro beam, micro film and micro hinge etc. Among them, as a typical structural element, the

micro plate has been widely used in MEMS technology, such as micro valve, micro pump and micro sensor etc. At present, for the dynamic characteristic of the micro plate, especially under the micro scale effect, the research on the vibration of electromechanical coupled micro plate is still very few. However, these researches would have a positive impact on structural design, optimization, control and improving the stability and reliability of MEMS devices and systems.

In the micro plate electromechanical coupled system, we utilize the electrostatic field which has been applied on fixed driving plate and micro mobile plate to generate electrostatic field driving force. The electrostatic field driving force would cause the mutual attraction between the fixed driving plate and the micro mobile plate, the micro mobile plate would displace relatively due to the attraction, under alternating electric field force, and the high frequency vibration of the micro mobile plate would be generated. Displacement of the micro mobile plate would be changed with the changes of the electrostatic field driving force frequencies, when driving frequency closes to natural frequency of the micro mobile plate, the micro mobile plate would resonate, by this time, the displacement maximum of the micro mobile plate is reached.

In this paper, based on the small deflection theory and electrostatics theory, the electromechanical coupled dynamics model is established, formulas of the static and dynamic electric field force are derived, and moreover, the basic differential equation of lateral vibration is established by using elasticity theory. Take typical boundary condition of the micro plate as example, displacement and time course of forced vibration, time responses of forced vibration are calculated with the aid of computing simulation to achieve the systemic analysis and design, which provides theoretical basis for optimizing the system parameters, improving the ability to resist interferences and increases the reliability.

II. DYNAMIC MODEL OF ELECTROMECHANICAL COUPLED MICRO PLATE

Fig.1 illustrates a dynamic model of electromechanical coupled micro plate, consisting of mechanical system, electrical system, and coupled part. The mechanical system is a micro plate subjected to electric field force which distributes equally on the micro plate. The upper is the micro mobile plate, it is the main object of study, we call it micro plate, the below

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is the fixed driving plate, the DC electrostatic bias voltage is applied between the two plates. The electrical source, resistance and capacitance construct electrical system. Mechanical system and electrical system achieve electromechanical coupled state by electric field force. The micro plates system obey the basic assumptions of elastic thin plate small deflection theory, as follows

- Middle surface normal hypothesis, that is to say, the middle surface normal maintains a line and always be vertical to the middle surface throughout the process of deformation.
- The normal stress of the middle surface is neglected.
- Without the deformation inside the middle surface.
- Inertial forces should be included in but moment of inertia should be ignored^[9].

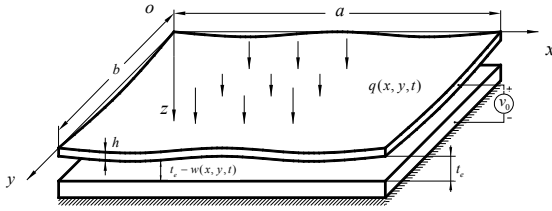


Fig.1 Dynamic model of electromechanical coupled micro plate

The micro plate simply supported on four sides is widely used in micro pump membrane; fig.2 illustrates the Cartesian coordinate system of micro plate simply supported on four sides.

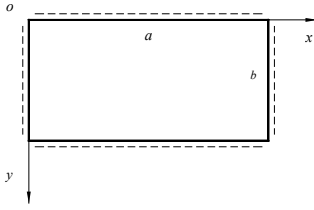


Fig.2 Cartesian coordinate system of micro plate simply supported on four sides

III. DYNAMIC DIFFERENTIAL EQUATIONS

Based on the thin plate deflection theory, the control differential equation of the deflection surface subjected to electric field force is given by

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{\rho h}{D} \frac{\partial^2 w}{\partial t^2} = \nabla^2 \nabla^2 w + \frac{\rho h}{D} \frac{\partial^2 w}{\partial t^2} = \frac{q}{D}(x, y, t) \quad (1)$$

Here w is transverse displacement of the micro plate, D is the flexural rigidity of the micro plate material, (e.g. $D = E_t h^3 / 12(1 - \nu^2)$, where E_t and ν are the elastic modulus and Poisson ratio of the micro plate material respectively, and h is the thickness of the micro plate), $\nabla^2 \nabla^2$ is biharmonic operator, $\nabla^2 \nabla^2 = \partial^4 / \partial x^4 + 2\partial^4 / \partial x^2 \partial y^2 + \partial^4 / \partial y^4$, ρ is the mass density of the micro plate, and q is the load per unit area above the micro plate. When $q(x, y, t)$ is equal to 0, the free vibration differential equation of the micro plate is obtained.

When the sinusoidal alternating voltage is applied on the system, (e.g. $\Delta v_{is} = U_0 \cos \omega_e t$, U_0 is excitation voltage amplitude, ω_e is excitation voltage frequency), because of positive and negative values canceling each other, the voltage only has effect on the dynamic responses, but not the static displacement. According to reference [10], the dynamic electrostatic force per unit area caused by the voltage fluctuation can be calculated as follows

$$\Delta q = U_0 \varepsilon_0 A \cos \omega_e t / (t_0 - w_0 + d_c / \varepsilon_r)^2 \quad (2)$$

Here ε_0 is the permittivity of free space, equal to $\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2}$, ε_r is the relative dielectric constant of the upper micro plate coating, d_c is the thickness of the upper micro plate coating, t_0 is the initial clearance between two micro plates, and w_0 is the static average displacement along z -axis of micro plate.

The basic equation of the forced vibration for micro plate under the harmonic voltage exciting is

$$D \nabla^2 \nabla^2 w + \rho h \frac{\partial^2 w}{\partial t^2} = U_0 \varepsilon_0 A \sin \omega_e t / (t_0 - w_0 + d_c / \varepsilon_r)^2 \quad (3)$$

IV. NON-LINEAR FORCED VIBRATION

When apply the periodic exciting voltage between the micro plate and the fixed driving plate, the dynamic electrostatic force per unit area caused by the voltage fluctuation can be calculated by using Eq. (2). Introducing the dimensionless small parameter ε , as the static average displacement \bar{w}_0 is small, $\bar{w}_0 / (t_0 + d_c / \varepsilon_r)$ can be taken as small parameter ε , and the parameter is assumed to be small, $\varepsilon = \bar{w}_0 / (t_0 + d_c / \varepsilon_r)$, ($0 < \varepsilon < 1$). Let the exiting force amplitude engendered by exciting voltage and small parameter ε is the same order of magnitude. Then, Eq. (2) can be defined in Taylor series form (neglect high-order terms) as follows

$$\Delta q = \frac{U_0^2 \varepsilon_0 \Delta w}{(t_0 - \bar{w}_0 + d_c / \varepsilon_r)^3} + \frac{3U_0^2 \varepsilon_0 \Delta w^2}{(t_0 + d_c / \varepsilon_r)^3 \bar{w}_0} \varepsilon + \frac{6U_0^2 \varepsilon_0 \Delta w^3}{(t_0 + d_c / \varepsilon_r)^4 \bar{w}_0} \varepsilon^2 + \frac{U_0 \varepsilon_0 E_0 \cos \omega_e t}{(t_0 - \bar{w}_0 + d_c / \varepsilon_r)^2} \quad (4)$$

Substituting equation (4) into equation (1), neglect high-order terms, and let $\Delta w(x, y, t) = \phi(x, y)q(t)$, the non-linear dynamical equation can be obtained

$$\ddot{q}(t) + q(t) \left[\omega_0^2 - \frac{b_1}{\rho h} \varepsilon q(t) - \frac{b_2}{\rho h} \varepsilon q^2(t) \right] = F_0 \cos \omega_e t \quad (5)$$

Where $b_1 = 3U_0^2 \varepsilon_0 \bar{\phi} / [(t_0 + d_c / \varepsilon_r)^3 \bar{w}_0]$,

$b_2 = 6U_0^2 \varepsilon_0 \bar{\phi}^2 / [(t_0 + d_c / \varepsilon_r)^4 \bar{w}_0]$,

$b_3 = U_0 \varepsilon_0 / [(t_0 - \bar{w}_0 + d_c / \varepsilon_r)^2 \bar{\phi}]$, $F_0 = b_3 E_0 / (\rho h)$,

$\bar{\phi} \approx \frac{1}{ab} \int_0^a \int_0^b \phi(x, y) dx dy$, $\bar{\phi}^2 \approx \frac{1}{ab} \int_0^a \int_0^b \phi^2(x, y) dx dy$.

When $\varepsilon = 0$, the Eq. (5) degrades to the linear dynamical equation of derivation system.

$$\ddot{q}(t) + \omega_0^2 q(t) = F_0 \cos \omega_e t \quad (6)$$

According to reference [11], using the Linz Ted-Poincare method, the solution of the equation can be given as follows

$$q(t, \varepsilon) = q_0(t) + \varepsilon q_1(t) + \varepsilon^2 q_2(t) + \dots \quad (7)$$

The initial conditions are

$$\begin{cases} q_0(0) = Q_0, \dot{q}_0(0) = 0 \\ q_1(0) = 0, \dot{q}_1(0) = 0 \\ q_2(0) = 0, \dot{q}_2(0) = 0 \\ \dots \end{cases} \quad (8)$$

Substituting Esq. (7) and (8) into Eq.(5), and then setting the coefficients with same power of ε to zero, we obtain the linear equations, then using the initial condition (8) to calculate repeatedly, we get the time solution which has enough precision as follows

$$q(t, \varepsilon) = B_0 \varepsilon + C_0 \varepsilon^2 + (Q_0 + B_1 \varepsilon + C_1 \varepsilon^2) \cos \omega_e t + (B_2 \varepsilon + C_2 \varepsilon^2) \cos 2\omega_e t + (B_3 \varepsilon + C_3 \varepsilon^2) \cos 3\omega_e t + C_4 \varepsilon^2 \cos 4\omega_e t + C_5 \varepsilon^2 \cos 5\omega_e t \quad (9)$$

Where

$$\begin{aligned} B_0 &= b_1 Q_0^2 / 2\rho h \omega_0^2 \\ B_1 &= 3b_2 Q_0^3 \omega_0^2 (4\omega_e^2 - \omega_0^2) (9\omega_e^2 - \omega_0^2) / 4\rho h M \\ B_2 &= b_1 Q_0^2 \omega_0^2 (\omega_e^2 - \omega_0^2) (9\omega_e^2 - \omega_0^2) / 2\rho h M \\ B_3 &= b_2 Q_0^3 \omega_0^2 (\omega_e^2 - \omega_0^2) (4\omega_e^2 - \omega_0^2) / 4\rho h M \\ M &= \omega_0^8 - 14\omega_0^6 \omega_e^2 + 49\omega_0^4 \omega_e^4 - 36\omega_0^2 \omega_e^6 \end{aligned}$$

$$\begin{aligned} C_0 &= [Q_0 \omega_e^{10} (-57600 B_1 b_1 - 43200 Q_0 B_2 b_2 - 86400 Q_0 B_0 b_2) + \\ & Q_0 \omega_e^6 \omega_0^4 (-30580 B_1 b_1 - 22935 Q_0 B_2 b_2 - 45870 Q_0 B_0 b_2) + \\ & Q_0 \omega_e^4 \omega_0^6 (4092 B_1 b_1 + 3069 Q_0 B_2 b_2 + 6138 Q_0 B_0 b_2) + \\ & Q_0 \omega_e^2 \omega_0^8 (-220 B_1 b_1 - 165 Q_0 B_2 b_2 - 330 Q_0 B_0 b_2) + \\ & Q_0 \omega_0^{10} (4 B_1 b_1 + 3 Q_0 B_2 b_2 + 6 Q_0 B_0 b_2)] / \rho h N \end{aligned}$$

$$\begin{aligned} C_1 &= [Q_0 \omega_e^8 \omega_0^2 (129600 Q_0 B_1 b_2 + 43200 Q_0 B_3 b_2 + 115200 B_0 b_1 + 57600 B_2 b_1) + \\ & Q_0 \omega_e^6 \omega_0^4 (-60084 Q_0 B_1 b_2 - 20028 Q_0 B_3 b_2 - 53408 B_0 b_1 - 26704 B_2 b_1) + \\ & Q_0 \omega_e^4 \omega_0^6 (8721 Q_0 B_1 b_2 + 2907 Q_0 B_3 b_2 + 7752 B_0 b_1 + 3876 B_2 b_1) + \\ & Q_0 \omega_e^2 \omega_0^8 (-486 Q_0 B_1 b_2 - 162 Q_0 B_3 b_2 - 432 B_0 b_1 - 216 B_2 b_1) + \\ & Q_0 \omega_0^{10} (9 Q_0 B_1 b_2 + 3 Q_0 B_3 b_2 + 8 B_0 b_1 + 4 B_2 b_1)] / \rho h N \end{aligned}$$

$$\begin{aligned} C_2 &= [Q_0 \omega_e^8 \omega_0^2 (21600 Q_0 B_2 b_2 + 21600 Q_0 B_0 b_2 + 14400 B_1 b_1 + 14400 B_3 b_1) + \\ & Q_0 \omega_e^6 \omega_0^4 (-26214 Q_0 B_2 b_2 - 26214 Q_0 B_0 b_2 - 17476 B_1 b_1 - 17476 B_3 b_1) + \\ & Q_0 \omega_e^4 \omega_0^6 (4914 Q_0 B_2 b_2 + 4914 Q_0 B_0 b_2 + 3276 B_1 b_1 + 3276 B_3 b_1) + \\ & Q_0 \omega_e^2 \omega_0^8 (-306 Q_0 B_2 b_2 - 306 Q_0 B_0 b_2 - 204 B_1 b_1 - 204 B_3 b_1) + \\ & Q_0 \omega_0^{10} (6 Q_0 B_2 b_2 + 6 Q_0 B_0 b_2 + 4 B_1 b_1 + 4 B_3 b_1)] / \rho h N \end{aligned}$$

$$\begin{aligned} C_3 &= [Q_0 \omega_e^8 \omega_0^2 (4800 Q_0 B_1 b_2 + 9600 Q_0 B_3 b_2 + 6400 B_2 b_1) + \\ & Q_0 \omega_e^6 \omega_0^4 (-6492 Q_0 B_1 b_2 - 12984 Q_0 B_3 b_2 - 8656 B_2 b_1) + \\ & Q_0 \omega_e^4 \omega_0^6 (1827 Q_0 B_1 b_2 + 3654 Q_0 B_3 b_2 + 2436 B_2 b_1) + \end{aligned}$$

$$\begin{aligned} & Q_0 \omega_e^2 \omega_0^8 (-138 Q_0 B_1 b_2 - 276 Q_0 B_3 b_2 - 184 B_2 b_1) + \\ & Q_0 \omega_0^{10} (3 Q_0 B_1 b_2 + 6 Q_0 B_3 b_2 + 4 B_2 b_1)] / \rho h N \\ C_4 &= [Q_0 \omega_e^8 \omega_0^2 (2700 Q_0 B_2 b_2 + 3600 B_3 b_1) + Q_0 \omega_e^6 \omega_0^4 (-3783 Q_0 B_2 b_2 - 5044 B_3 b_1) + \\ & Q_0 \omega_e^4 \omega_0^6 (1197 Q_0 B_2 b_2 + 1596 B_3 b_1) + Q_0 \omega_e^2 \omega_0^8 (-117 Q_0 B_2 b_2 - 156 B_3 b_1) + \\ & Q_0 \omega_0^{10} (3 Q_0 B_2 b_2 + 4 B_3 b_1)] / \rho h N \\ C_5 &= Q_0^2 B_3 b_2 (1728 \omega_e^8 \omega_0^2 - 2460 \omega_e^6 \omega_0^4 + 819 \omega_e^4 \omega_0^6 - 90 \omega_e^2 \omega_0^8 + 3 \omega_0^{10}) / \rho h N \\ N &= 4\omega_0^{12} - 220\omega_0^{10} \omega_e^2 + 4092\omega_0^8 \omega_e^4 - 30580\omega_0^6 \omega_e^6 + 84304\omega_0^4 \omega_e^8 - 57600\omega_0^2 \omega_e^{10} \end{aligned}$$

Where ω_0 is the natural frequency of the system, ω_e is the exciting frequency.

V. ANALYSIS AND DISCUSSIONS OF TYPICAL EXAMPLE

Take typical boundary condition of the micro plate simply supported on four sides as example, the parameters for the electromechanical coupled system are shown in Table I. Fig.3 shows the time responses of forced vibration. Fig.4 shows the displacement and time course of forced vibration.

TABLE I

The system parameters	a /mm	b/mm	h / μ m	t_0 / μ m
Values	2	1	5	2
The system parameters	d_c / μ m	ε_r	E /GPa	ρ /kg·m ⁻³
Values	0.5	8.4	68	2.7×10^3

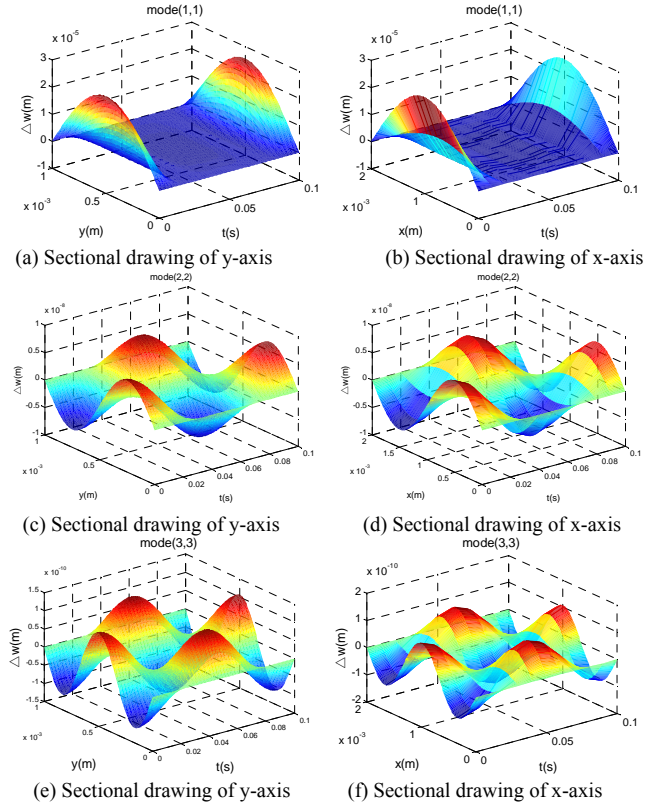


Fig.3 Time responses of forced vibration

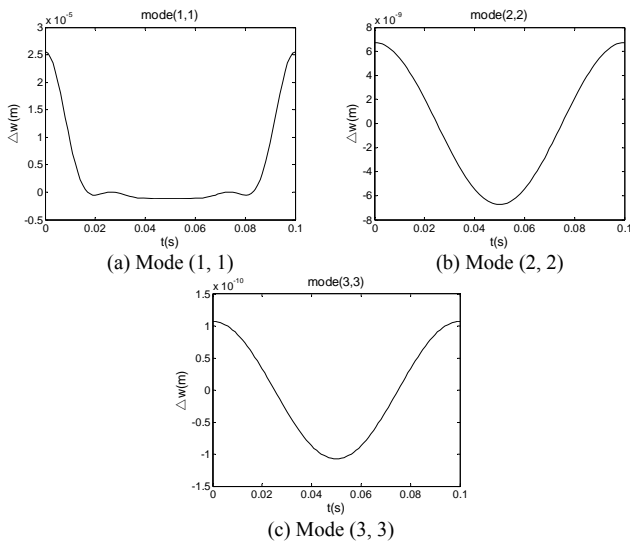


Fig.4 Displacement and time course of forced vibration

Eq. (9) is utilized for the analysis of the non-linear forced response of the electromechanical coupled micro plate. The forced response corresponding to the first three modes are given in Fig.3 and Fig.4, the working voltage between two micro plates $U_0=7.0\text{v}$, the exciting frequency $\omega_e = 20\pi\text{rad/s}$. The maximum dynamic displacement occurs at point ($x=1\text{mm}$, $y=1\text{mm}$). From Fig.3 and Fig.4, following observations are worth noting:

- The region surrounded by displacement-time curve of mode 1 is the largest, and frequencies of mode 2 and mode 3 increase very significant, the regions surrounded by displacement-time curve are smaller.
- For different modes, the peak position and the vibration period are different, and the natural frequencies of higher modes are relatively large, the vibration periods decrease.
- The dynamic responses of non-linear dynamics appear asymmetric, and the displacement-time curve is no longer symmetrical about the system equilibrium position, this phenomenon is most obvious in the first mode. This is due to considering the non-linear factor of electrostatic force in the non-linear solution, and which causes the DC and harmonic component to the vibration response.
- For the micro plate simply supported on four sides, the boundary condition and material geometry are

symmetry, the time response curve is symmetry too, the point of maximum displacement occurs at the center of the moving micro plate.

VI. CONCLUSIONS

In this paper, non-linear dynamic responses of electromechanical coupled micro plate simply supported on four sides are analyzed. Control differential equation and vibration mode function for electromechanical coupled micro plate simply supported on four sides are presented. Defining the dynamic electrostatic force per unit area in Taylor series form, the forced non-linear dynamic equation is obtained. These works can provide theoretical basis for the parameters calculation of dynamic design and manufacture of MEMS typical components driving by electrostatic force.

The time dynamic responses are under the condition of no damp, which is the analysis result of ideal condition, and this work can provide a theoretical basis for the dynamic analysis of damping system.

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