

A note on the Penetration of Semi-infinite Metallic Targets Struck by Long Rods at High Velocities

Y. He

CAS Key Laboratory for Mechanical Behavior and Design of Materials
University of Science and Technology of China
Hefei, Anhui, China
hexiaoyu@mail.ustc.edu.cn

H.M. Wen

CAS Key Laboratory for Mechanical Behavior and Design of Materials
University of Science and Technology of China
Hefei, Anhui, China
hmwen@ustc.edu.cn

Abstract—Analytical equations are presented herein to predict the penetration of semi-infinite metallic targets struck normally by long rods at high velocities. The analysis is based on energy balance where the kinetic energy loss of a long rod is related to the energy dissipated by the plastic deformations in the target, the energy consumed by the long-rod itself and the kinetic energy remaining in the ejected rod debris. The present analytical equation is found to be in good agreement with the experimental data for a wide range of impact velocities.

Keywords-penetration, long rod, semi-infinite metallic target

I. INTRODUCTION

Much effort has been directed towards the research of long rod penetrators penetrating targets for more than half a century. Significant amount of terminal ballistic data and numerical simulation results were obtained for the combination of penetrators and targets made of various materials[1]-3]. Overviews of long rod penetration were presented by Zook et al.[4], Goldsmith[5] and Orphal[6].

A one-dimensional model proposed independently by Alekseevskii[7] and Tate[8] has become the standard reference for long rod penetration into semi-infinite metallic targets and various modifications and extensions of the Alekseevskii-Tate model have been made by many authors[9]-[20].

II. PENETRATION DEPTH

Consider a unitary long rod of length L_0 penetrates into a semi-infinite target at impact velocity v , the initial kinetic energy of the long rod can then be expressed as

$$E_k = \frac{1}{2} \pi r^2 \rho_p L_0 v^2 \quad (1)$$

where r and ρ_p are the radius and density of the long rod, respectively.

To a first approximation, it is assumed that the kinetic energy loss of the penetrator is mainly related to the energy dissipated by plastic deformations of the target, the energy associated with the crushing/erosion of the rod itself and the kinetic energy remaining in the ejected rod debris. The energy for crushing the penetrator (E_{rod}) may be expressed as

$$E_{rod} = \pi r^2 Y_p L_0, \quad (2)$$

where Y_p is the dynamic strength of penetrator material. The energy dissipated by plastic deformations in the target (E_{target}) may be approximated as

$$E_{target} = \pi R_{crater}^2 P S, \quad (3)$$

where P is the total depth of penetration in the target, S is the work expended per unit volume of the crater[21] which can be taken as the static resistive pressure derived from cavity expansion approximation and R_{crater} is the crater diameter in the semi-infinite target which can be evaluated as[19]

$$\frac{R_{crater}}{r} = \sqrt{\frac{Y_p + \rho_p(\varphi+1)(v-u)^2}{S}}, \quad (4)$$

where

$$\varphi = \sqrt{1 - \frac{2Y_p}{\rho_p(v-u)^2}}. \quad (5)$$

The kinetic energy remaining in the ejected rod debris can be expressed as

$$E_{debris} = \frac{1}{2} \pi r^2 \rho_p L_0 v_e^2, \quad (6)$$

where v_e is the velocity of the ejected rod debris with opposite direction to impact velocity determined by[19]

$$V_e = \varphi(v-u) - u \quad (7)$$

Hence, from the energy balance one obtains

$$E_k = E_{rod} + E_{target} + E_{debris} \quad (8)$$

Substituting (1-7) into (8) and rearranging gives

$$\frac{P}{L_0} = \frac{1/2 \rho_p v^2 - 1/2 \rho_p \left(\sqrt{(v-u)^2 - 2Y_p/\rho_p} - u \right)^2 - Y_p}{Y_p + \rho_p(v-u) \left(\sqrt{(v-u)^2 - 2Y_p/\rho_p} + (v-u) \right)} \quad (9)$$

or

$$\frac{P}{L_0} = \frac{I - \left(\sqrt{I(1-u/v)^2 - 2} - \sqrt{I} u/v \right)^2 - 2}{2 + 2(1-u/v) \left(\sqrt{I^2(1-u/v)^2 - 2I} + I(1-u/v) \right)}, \quad (10)$$

where $I = \frac{\rho_p v^2}{Y_p}$ and u is the penetration velocity which can

be determined by the following equations, viz. [18]

$$\frac{1}{2} \rho_t [u - \delta(u)]^2 + S + C \rho_t \delta(u)^2 = \frac{1}{2} \rho_p (v - u)^2 + Y_p \quad \text{for} \quad u \geq U_{F0} \quad (11)$$

or

$$S + C \rho_t u^2 = \frac{1}{2} \rho_p (v - u)^2 + Y_p \quad \text{for} \quad u \leq U_{F0} \quad (12)$$

where

$$\delta(u) = U_{F0} \exp \left[- \left(\frac{u - U_{F0}}{n U_{F0}} \right)^2 \right], \quad n \geq 2\sqrt{C} \quad (13)$$

and ρ_t is target density, C is the constant of the dynamic resistive pressure and for incompressible materials, $C = 3/2$, U_{F0} is the critical penetration velocity above which a flow region appears in the target near the penetrator-target interface, otherwise there are only plastic and elastic regions and $U_{F0} = \sqrt{HEL / \rho_t}$ where HEL is the Hugoniot Elastic Limit of the target material[18].

If the impact velocity is allowed to go to very high or the dynamic strength of rod and target material are allowed to go to zero, the dimensionless number I is allowed to become very large and then (10) gives the same penetration depth with hydrodynamic theory.

III. COMPARISONS AND DISCUSSION

Figure 1 shows comparison of (10) with the experimental data for the penetration into semi-infinite armor steel targets struck by tungsten alloy long rods with $L/D = 23$ [22]. In the theoretical calculation, the dynamic strength of the tungsten

alloy is taken be 2082.5 MPa[18] and the density is 17300 kg/m³. The material properties of the armor steel targets used in the calculation are given in Table 1. It is clear from Figure 1 that (10) is in good agreement with the experimental data[22] over a wide range of impact velocities. The discrepancy between (10) and the experimental data for impact velocities higher than 3000m/s may be due to the so-called “after-flow” effect which has not been taken into account in the present formulation. It is also can be seen in Fig. 1 that the penetration depth from (10) approaches an asymptote predicted by hydrodynamic theory at high impact velocities.

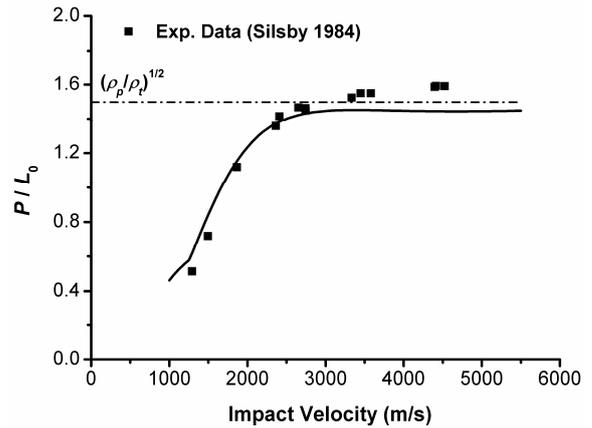


Figure 1. Comparison of equation (10) with the experimental data[22] for semi-infinite armor steel targets impacted by tungsten alloy long rods

TABLE I. MATERIAL PROPERTIES OF THE TARGETS EXAMINED.

Target Materials	ρ (kg/m ³)	E_1 (GPa)	E_2 (MPa)	λ	Y_0 (MPa)	Y (MPa)	HEL (MPa)	S (MPa)
Armor Steel[18]	7850	205	177.2	0.3	--	1130	1925	4395.2
GerArmor Steel[18]	7850	205	500	0.3	--	988.6	1730.1	4262.9
RHA[23]	7850	205	177.2	0.3	--	1370	2397.5	5242

where ρ is the density, E_1 is the elastic modulus, E_2 is the plastic modulus in a bilinear material model, λ is the Poisson's ratio, Y_0 is the yield stress or proof stress in a true stress-strain curve, Y is the yield stress in a bilinear material model, and HEL is Hugoniot Elastic Limit calculated by $HEL = Y(1-\lambda)/(1-2\lambda)$.

Fig. 2 shows comparison of (10) with the experimental data for Ger Armor Steel targets struck by C110W1 steel long rod penetrators with $L/D = 10$ [1]. In the theoretical calculation, the yield stress of C110W1 was given by Hohler and Stilp as 762 MPa (BHN230)[1]. The dynamic strength of C110W1 is taken to be 1334 MPa[19]. The other material properties of Ger Armor Steel are also listed in Table 1. It can be seen from Fig. 2 that (10) is in good agreement with the test data for impact velocities greater than 1500m/s approximately and for impact velocities less than 1500m/s the depth of penetration is a little bit overpredicted by (10) compared to the experimental results.

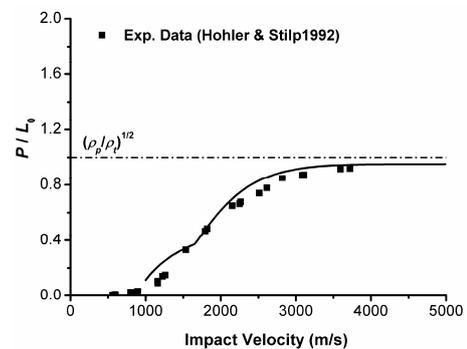


Figure 2. Comparison of equation (10) with the experimental data[1] for semi-infinite armor steel targets impacted by C110W1 steel long rods.

Equation (10) can also be applied to jacketed rod penetration when co-erosion occurs so long as Y_p and ρ are replaced with Y_{eq} and ρ_{eq} which are determined by the following equations[20]

$$\begin{cases} Y_{eq} = Y_j + (Y_c - Y_j) \frac{r_{c0}^2}{r_{j0}^2} \\ \rho_{eq} = \rho_j + (\rho_c - \rho_j) \frac{r_{c0}^2}{r_{j0}^2} \end{cases} \quad (14)$$

where r_{c0} and r_{j0} are core and jacket radii respectively, ρ_c and ρ_j are respective core and jacket material densities, Y_c and Y_j are the dynamic strengths of core and jacket materials, respectively. The relationship of $v-u$ for a jacketed rod penetration when co-erosion occurs can also be determined by those of unitary long rod penetration of the same core material from (11) and (12) where $Y_p = Y_c$ and $\rho_p = \rho_c$.

Figure 3 shows comparison of (10) with the experimental data reported by Cullis et al [23] for EN24 steel jacketed tungsten alloy long-rod penetrators. The dynamic strength of the core and jacket material were taken to be 3.5 GPa and 1.49 GPa by WEN HeMing, HE Yu et al[20]. The RHA(Rolled Homogeneous Armour) target yield stress (i.e. Y in Table 1) is 1.37 GPa[23] and the other constants are taken to be the same as given in Table 1. It can be seen from Figure 3 that good agreement is obtained between (10) and the test data.

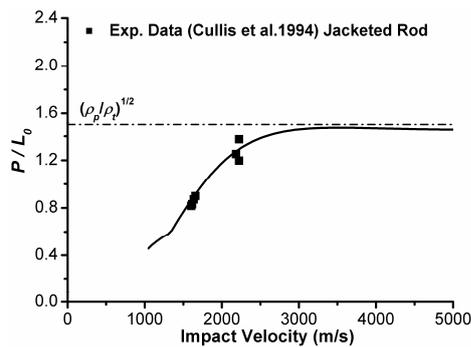


Figure 3. Comparison of equation (10) with the experimental data for EN24 steel jacketed tungsten alloy long rods and tungsten unitary long rods penetrating semi-infinite armor steel targets [23]

IV. CONCLUSIONS

Analytical equations have been given in this paper to predict the penetration of semi-infinite metallic targets struck normally by high speed long rods. The analysis is based on energy balance where the kinetic energy of a long rod is equated to the energy dissipated by the plastic deformations in the target, the energy consumed by the long-rod itself and the kinetic energy remaining in the ejected rod debris. It is shown that the present analytical equation is in good agreement with the experimental data for a wide range of impact velocities.

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