On the First-Order Markov Model for the PLC Fading Channel

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Abstract- **The power line communications (PLC) channel is noisy one, which can be modeled by the Markov process. The order is the key concerning when used the Markov process. the level of complexity will be incurred from using higher order , while the first-order Markov models may lead to the less accurate channel response. In the paper, the first-order Markov channel is under thoroughly discussion, and it can provide a mathematically tractable model for time-varying channels and uses only the received SNR of the symbol immediately preceding the current one. With the first-order Markov chain, given the information of the symbol immediately preceding the current one, any other previous symbol should be independent of the current one. We show that given the information corresponding to the previous symbol, the amount of uncertainty remaining in the current symbol should be negligible. That means the first-order of Markov process is enough when modeled the PLC channel.**

Keywords: ***PLC, channel model, Markov process, first-order***

I. INTRODUCTION

With the rapid increasing research interesting in the smart grid, the power line communications systems (PLC) has been more and more important and popular[1]. Recently, the focus of the PLC community has widened and, in fact, shifted back to areas innate to PLC: PLC for control and automation systems[2]. One reason for this development has been the need for a powerful communication infrastructure that enables the idea of smart grid[3]. Power utilities have been native users of PLC technology, and it is expected that PLC will play a role in the blend of technologies composing the smart grid communication infrastructure[4].

In order to achieve top performance for numerous communication applications such as data, audio, and video transmissions with prescribed delay, data rate, and quality of service (QoS), the channel knowledge of PLC must be very accurate. Therefore, there is growing interest in power line communication modeling[5]. Due to the nature of impendence mismatch and switch on/off of the electrical devices in the power networks, esp. the LV networks, the PLC channel shows a strong fading and noisy characteristic, which is quite similar with the wireless/mobile communications channels [6][7].

The noisy channel may possess certain time-varying memory content that causes channel quality to vary with time, depending on the precious channel condition, or in other words, it is a memory channel. In many communications systems, the memory channel can be modeled as the Markov chain or Markov process[8,9,10]and [11]. Although the noisy channel can be modeled as the Markov one, however, there are still arguments. Some papers proposed to use the first-order Markov models, because any other previous state should be independent of the current state given the information of the state immediately preceding the current one[12][13], while some ones argue that second- or higher-order Markov processes should provide a more accurate model [14][15].

For the PLC systems, there are some works trying to model the channel with the Markov chain [16]. The order of the Markov chain, however, is not yet considered when modeled the PLC Markov model. In this paper, we answer this question by showing that given the information corresponding to the previous symbol, the amount of uncertainty remaining in the current symbol should be negligible. The mathematical channel model is presented in Section I, followed by analytical derivation of the joint probability density function of the received envelope amplitudes in Section III. In Section IV, results from numerical evaluation and computer simulation are presented to show the relative importance of previous channel symbols to the current one. Conclusions are drawn in Section V.

II. SYSTEM MODEL

We assume that a signal at frequency ω transmitted, and that there are M waves with propagation delay times T_m. The received signal can be written as

$$r(\omega,t)=A_0\sum_{m=1}^{M}C_m e^{i(\omega t-\omega T_m)} \tag{1}$$

The amplitude coefficients C_n are defined as

$$C_m^2 = p(T_m)dT \tag{2}$$

Representing the power associated with each individual wave. With the approximated exponential distribution of the delay spreads [17], the function $p(T)$ can be written as

$$p(T)=\frac{1}{2\pi T_D}e^{-\frac{T}{T_D}} \tag{3}$$

Where T_D is a measure of the time delay spread..
Equation (2) can be rewritten as

$$r(\omega,t)=x_I(t)\cos(\omega t)+x_Q(t)\sin(\omega t) \tag{5}$$

Where

$$x_I(t)=\left[A_0\sum_{m=1}^{M}C_m\cos(\omega T_m)\right] \tag{6}$$

$$x_Q(t)=\left[A_0\sum_{m=1}^{M}C_m\sin(\omega T_m)\right]$$

By the central limit theorem, $x_I(t)$ and $x_Q(t)$ are Gaussian random processes and are jointly Gaussian distributed for large number of M .

Since we are interested in the correlation of channel quality between three consecutive symbols, derivation of the joint probability density function (j .p.d.f.) of the corresponding envelope amplitudes is important. Let

$$\begin{array}{lcl} x_1(t) = x_I(t) & & y_1(t) = x_Q(t) \\ x_2(t) = x_I(t+\tau) & \text{and} & y_2(t) = x_Q(t+\tau) \\ x_3(t) = x_I(t+2\tau) & & y_3(t) = x_Q(t+2\tau) \end{array} \quad (7)$$

Then $x_1(t), x_2(t), x_3(t)$ and $y_1(t), y_2(t), y_3(t)$ are jointly Gaussian distributed, and without loss of generality, zero mean. Their second order statistics can be derived as

$$E[x_1^2] = E[x_2^2] = E[x_3^2] = \sigma^2$$

$$E[y_1^2] = E[y_2^2] = E[y_3^2] = \sigma^2 \quad (8)$$

At the same time, we can also get

$$E[x_1x_2] = E[x_2x_3] = E[y_1y_2] = E[y_2y_3] = \sigma^2$$

$$E[x_1x_3] = E[y_1y_3] = \sigma^2 \quad (9)$$

$$E[x_iy_j] = 0 \qquad i, j = 1,2,3$$

Thus, we can get the joint probability density function of $\bar{x} = [x_1, x_2, x_3]$ and $\bar{y} = [y_1, y_2, y_3]$:

$$f(\bar{x}, \bar{y}) = \frac{1}{(2\pi)^3 \sqrt{\det[C_{\bar{x},\bar{y}}]}} e^{\frac{1}{2}[\bar{x},\bar{y}]^T C_{\bar{x},\bar{y}} [\bar{x},\bar{y}]} \quad (10)$$

from equation (9), we can conclude that the sets $\bar{x}$ and $\bar{y}$ are independent. Therefore equation (10) can be further simplified as

$$f(\bar{x}, \bar{y}) = f(\bar{x}) f(\bar{y})$$

$$= \frac{1}{(2\pi)3\sqrt{\det[C_{\bar{x},\bar{x}}] * \det[C_{\bar{y},\bar{y}}]}} e^{\frac{1}{2}\left[\bar{x}^T C_{\bar{x},\bar{x}} \bar{x} + \bar{y}^T C_{\bar{y},\bar{y}} \bar{y}\right]} \quad (11)$$

JOINT P.D.F. OF ENVELOPE AMPLITUDES

The relationship between $[\bar{x}, \bar{y}]$ and the envelope amplitudes and phase is

$$x_i = r_i \cos\theta_i$$

$$y_i = r_i \sin\theta_i \quad (12)$$

where

$$r_i = \sqrt{x_i^2 + y_i^2}$$

$$\theta_i = \arctan\frac{y_i}{x_i} \quad (13)$$

By the above relationship, we perform polar transformation to obtain the j.p.d.f. of r_i and θ_i .

$$\begin{aligned} & f(r_1, r_2, r_3, \theta_1, \theta_2, \theta_3) \\ &= \frac{r_1 r_2 r_3}{(2\pi)^3 \sqrt{\det(C_{\bar{x},\bar{y}})}} \\ &* e^{-\frac{1}{2}\left[c_{11}r_1^2 + c_{22}r_2^2 + c_{22}r^3\right]} \\ &* e^{-r_{12}r_1r_2\cos(\theta_1-\theta_2)} \\ &* e^{-r_{23}r_2r_3\cos(\theta_2-\theta_3)} \\ &* e^{-r_{13}r_1r_3\cos(\theta_1-\theta_3)} \end{aligned} \quad (14)$$

III. SIMULATION RESULTS

In order to answer the question that whether the first-order is better than second- or higher order Markov model when modeled the PLC channel, we will show that given the information corresponding to the previous symbol, the amount of uncertainty remaining in the current symbol should be negligible. From the point of the information theory, it is the mount of the measurement of the mutual information. Let R_i be the received amplitude of the i th channel symbol, the information of R_i provided by the joint ensemble $R_{i-1}\ R_{i-2}$ can be quantified by the average mutual information $I(R_i; R_{i-1}R_{i-2})$ and can be decomposed as

$$\begin{aligned} & I(R_i; R_{i-1}R_{i-2}) = \\ & I(R_i; R_{i-1}) + I(R_i; R_{i-2} \mid R_{i-1}) \end{aligned} \quad (14)$$

It is then clear that, given R_{i-1}, the significance of R_{i-2} in providing the information for R_i can be measured by the ratio :

$$\gamma = \frac{I(R_i; R_{i-2} \mid R_{i-1})}{I(R_i; R_{i-1}R_{i-2})} \quad (15)$$

Where

$I(R_i;R_{i-2} \mid R_{i-1})$ is the average conditional mutual information and $I(R_i;R_{i-1}R_{i-2})$ is the average mutual information.

According to the information theory,

$$I(R_i;R_{i-2} \mid R_{i-1}) = \iiint_{r_i,r_{i-1},r_{i-2}} f(r_i,r_{i-1},r_{i-2})\log\frac{f(r_i,r_{i-2} \mid r_{i-1})}{f(r_i \mid r_{i-1})f(r_{i-2} \mid r_{i-1})}dr_i dr_{i-1} dr_{i-2} \quad (16)$$

And

$$I(R_i;R_{i-2}R_{i-1}) = \iiint_{r_i,r_{i-1},r_{i-2}} f(r_i,r_{i-1},r_{i-2})\log\frac{f(r_i,r_{i-2},r_{i-1})}{f(r_i)f(r_{i-1})f(r_{i-2})}dr_i dr_{i-1} dr_{i-2} \quad (17)$$

In our simulation, the number of waves M is equal to 100. The range of received envelop is partitioned into S intervals and each state of the channel corresponds to one of these intervals, which can be used by the Markov process.

In Fig.1, simulation results with $S = 100,80$ are presented. It is noticed that the ratio $\gamma = \frac{I(R_i;R_{i-2} \mid R_{i-1})}{I(R_i;R_{i-1}R_{i-2})}$ is less than 1% for most values of SNR, which correspond to the Markov channel states. As the SNR increases, the received signal envelope fluctuates slowly and the information about the current channel symbol that can be provided by the previous symbols is limited. In general, the importance of R_{i-2} given R_{i-1} is negligible, which means that the PLC channels can be modeled as the first-order Markov process.

IV. CONCLUSION

In this paper, we derive the joint probability density function of the received envelope amplitudes at time $t, t+\tau$ and $t+2\tau$ for the PLC channels. To investigate the correlation between the current channel symbol and the previous ones, the ratio between $I(R_i;R_{i-2} \mid R_{i-1})$ and $I(R_i;R_{i-1}R_{i-2})$ is calculated through computer simulations. We conclude that as for the current channel symbol, the effect of channel symbols other than the immediately preceding one can be negligible. Therefore, the first-order Markovian model can be used when modeled the PLC channels.

REFERENCES

[1] W. Liu, M. Sigle, and K. Dostert, "Channel characterization and system verification for narrowband power line communication in smart grid applications," IEEE Communications Magazine, vol. 49, no. 12, pp. 28–35, 2011.

[2] B. Adebisi, A. Treytl, A. Haidine, A. Portnoy, R. U. Shan, D. Lund, H. Pille, and B. Honary, "Ip-centric high rate narrowband plc for smart grid applications," IEEE Communications Magazine, vol. 49, no. 12, pp. 46–54, 2011.

[3] S. Kim, E. Y. Kwon, M. Kim, J. H. Cheon, S. ho Ju, Y. hoon Lim, and M. seok Choi, "A secure smart-metering protocol over power-line communication," vol. 26, no. 4, pp. 2370–2379, 2011.

[4] M.-Y. Zhai, "Transmission characteristics of low-voltage distribution networks in china under the smart grids environment," IEEE Transactions on Power Delivery, vol. 26, no. 1, pp. 173–180, 2011.

[5] S. Barmada, A. Musolino, M. Raugi, R. Rizzo, and M. Tucci, " A wavelet based method for the analysis of impulsive noise due to switch commutations in power line communication (plc) systems," IEEE Transactions on Smart Grid, vol. 2, no. 1, pp. 92−101, 2011.

[6] M. Nassar, J. Lin, Y. Mortazavi, A. Dabak, I. H. Kim, and B. L. Evans, "Local utility power line communications in the 3−500 khz band: Channel impairments, noise, and standards," IEEE Signal Processing Magazine, vol. 29, no. 5, pp. 116−127, 2012.

[7] M.-Y. Zhai, "Signal recovery in power-line communications systems based on the fractals," IEEE Transactions on Power Delivery, vol. 26, no. 3, pp. 1864−1872, 2011.

[8] W. Song and W. Zhuang, " Performance analysis of probabilistic multipath transmission of video streaming traffic over multi-radio wireless devices," IEEE Transactions on Wireless Communications, vol. 11, no. 4, pp. 1554−1564, 2012.

[9] D. Rey Iglesias and M. Garcia Sanchez, "Semi-markov model for low-elevation satellite−earth radio propagation channel," IEEE Transactions on Antennas and Propagation, vol. 60, no. 5, pp. 2481−2490, 2012.

[10] L. E. Braten and T. Tjelta, "Semi-markov multistate modeling of the land mobile propagation channel for geostationary satellites," IEEE Transactions on Antennas and Propagation, vol. 50, no. 12, pp. 1795−1802, 2002.

[11] L. Lei, J. Zhou, X. Chen, L. Qi, and S. Cai, "Modelling and analysing medium access delay for differentiated services in ieee 802.11s wireless mesh networks," IET Networks, vol. 1, no. 2, pp. 91−99, 2012.

[12] S. Yuksel, "Random-time, state-dependent stochastic drift for markov chains and application to stochastic stabilization over erasure channels," IEEE Transactions on Automatic Control, to be published, early Access.

[13] M. Riediger and E. Shwedyk, " Communication receivers based on markov models of the fading channel, " IEE Proceedings-Communications, vol. 150, no. 4, pp. 275−279, 2003.

[14] F. Wang, J. Huang, and Y. Zhao, "Delay sensitive communications over cognitive radio networks, " IEEE Transactions on Wireless Communications, vol. 11, no. 4, pp. 1402−1411, 2012.

[15] K. Fukawa, H. Suzuki, and Y. Tateishi, "Packet-error-rate analysis using markov models of the signal-to-interference ratio for mobile packet systems," IEEE Transactions on Vehicular Technology, vol. 61, no. 6, pp. 2517−2530, 2012.

[16] M.-y. ZHAI and Z. Qing-an, "Markov characterization of lv power line communications systems channels," Proceedings of the CSEE, vol. 27, no. 22, pp. 116−121, 8 2007.

[17] S. Galli, A. Scaglione, and Z. Wang, "For the grid and through the grid: The role of power line communications in the smart grid," Proceedings of the IEEE, vol. 99, no. 6, pp. 998−1027, 2011.

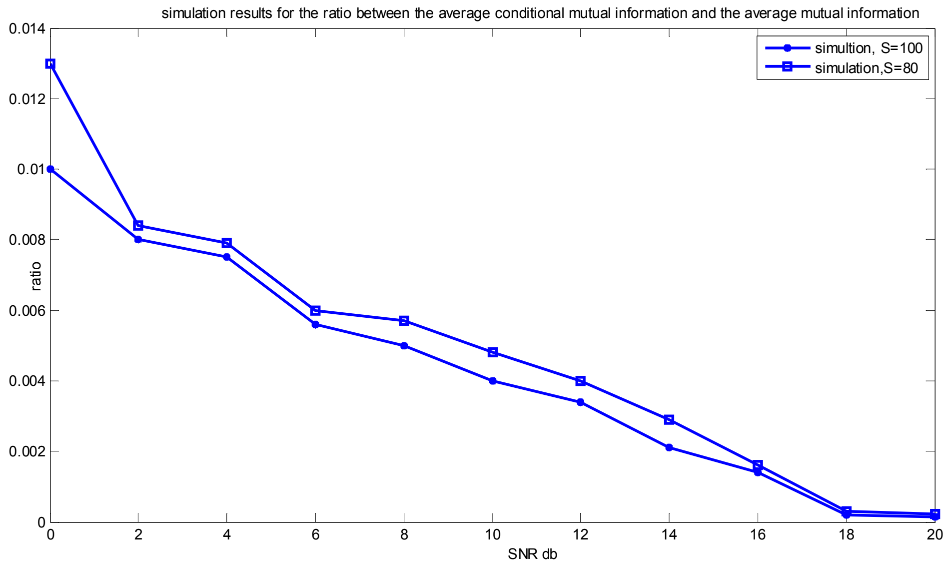

Figure 1. The simulation for the ratio between the average conditional mutual information and the average mutual information