Kalman/Particle Filter for Integrated Navigation System

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Abstract—**Particle filter is introduced. Since the particle filter would bring hard computation, a new Kalman/Particle mixed filter used on SINS/GPS integrated navigation system was proposed. The new method divides the system into two sub-models, one is linear, the other one is nonlinear, and then implement Kalman filter and particle filter separately. The simulation results show that their performance is almost equal, but the computation complexity of the Kalman/particle filter is much lower than traditional particle filter.**

Keywords- integrated navigation system; particle filter; kalman/particle filter

I. INTRODUCTION

During warfare, the start-up time of navigation system needs to become less and less. The complicated environment, such as electronic interference, and instable GPS signal, is also needed to be overcome. To SINS/GPS integrated navigation system, when the SINS system starts, the azimuth error margin is big, and the non-linear factor during navigating can not be neglected. Furthermore, when the navigation equipment works in complicated environment, the noise model can not be made certain.。

As the calculation speed is quickening, and the memory cost reducing, a new non-parameter modeling particle filter based on Bayes theorem is being attention gradually[1]. Particle Filter is put forward by Gordon and Salmond at 1993. The optimal solution of Physical model can be obtained via this method. And the filter can be applied to non-linear and non-Gaussian noise system. Much attention is also paid to this method in integrated navigation system[2,3]. To the ordinary Particle Filter algorithm, when the number of particle is tended to endless, particle distribution is constringency to real posterior probability density. When the number of particle is large, the calculation burden of ordinary particle filter algorithm aggravates. Therefore, when the number of state parameter is larger, the particle filter algorithm will be more complicated.

Considering the characteristic of particle filter algorithm and Kalman filter algorithm, a new Kalman/Particle mixed filter used on SINS/GPS integrated navigation system was proposed. The new method divides the system into two sub-models, one is linear, the other one is nonlinear, and then implement Kalman filter and particle filter separately. This method can make the best use of dynamic properties of

system. Further more, the state dimension of particle filter can be reduced, so that the dimension disaster would be avoided. With the large misalignment SINS error for model, we use the two filter method to simulate the SINS/GPS integrated navigation system. The simulation results show that, in the circumstances of the large misalignment angle, the Kalman/Particle mixed filter performs as well as the particle filter, but it is briefer in computation than particle filter.

II. PARTICLE FILTER ALGORITHM

A. Non-linear and Non-Gaussian System Model

Non-linear and non-Gaussian system can be described by the following dynamic state space model:

$$x_k = f_k\left(x_{k-1}, e_{k-1}\right) \qquad (1)$$

$$y_k = h_k\left(x_k, v_k\right) \qquad (2)$$

x_k ($x_k \in R^{n_x}$) is the state of system. y_k ($y_k \in R^{n_y}$) is the measurement. e^k and v^k are system noise and measurement noise respectively. And they are separate. $f_k : R^{n_x} \times R^{n_e} \to R^{n_x}$, $h_k : R^{n_x} \times R^{n_v} \to R^{n_y}$, and they are bounded nonlinear maps. n_x, n_e, n_v, n_y are the state vector , noise vector, measurement noise and measurement vector of the system respectively.

Based on the measurement data alignment, we use Bayesian estimation algorithm to estimate the conditional posterior probability distribution $p(x_{1:k}|y_{1:k})$ of the state, especially the marginal probability distribution $p(x_k|y_{1:k})$. $x_{0:k} = \{x_0, x_1, \cdots x_k\}$ and $y_{1:k} = \{y_1, y_2, \cdots y_k\}$ are the initial marginal probability distributions. Here, we suppose that the initial marginal probability distribution function of the system state is $p(x_0|y_0) = p(x_0)$. We can get the posterior probability distribution of system state from Bayesian recursive formula and new measurement result y_k , like this:

$$p(x_k|y_{1:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|y_{1:k-1}) \qquad (3)$$

$$p(x_k|y_{1:k}) = (p(y_k|x_k)p(x_k|y_{1:k}))/p(y_k|y_{1:k-1}) \quad (4)$$

$$p(y_k|y_{1:k-1}) = \int p(y_k|x_k)p(x_k|y_{1:k-1})dx_k \quad (5)$$

B. Particle Filter Algorithm

The PF is a recursive estimation method using Monte Carlo simulation within a Bayesian framework (Farina and Ristic, 2002; Arulampalam et al., 2002). The central idea is to obtain the MMSE minimum mean-square error) of state from a set of random samples (particles) of state space to approximate the required probability density function (PDF). It is often impossible to directly sample from the posterior probability density . Thus，an importance proposal distribution, $q(x_{0:k}|z_{1:k})$, with identical distribution, to the posterior distribution is introduced, where $z_{1:K}$ are all measurements from t=0 to t=k. The bootstrap filter (Gordon et al., 1993) simply takes the prior distribution as the proposal distribution by

$$q(x_k^i|x_{0:k}^i, z_{1:k}) = p(x_k^i|x_{k-1}^i) \approx N(f(x_{k-1}^i), Q_{k-1}) \quad (6)$$

in calculating the importance weights[4]. However，it would cause a larger error if there is little overlap between prior and the likelihood. To obtain more accurate proposal distribution within the particle filter flame, it may be used to update the mean and covariance of the Gaussian approximation to the state distribution given by

$$q(x_k^i|x_{0:k-1}^i, z_{1:k}) = N(\hat{x}_k^i|x_{k-1}^i, P_{k|k}^i), i = 1, \cdots, N, \quad (7)$$

where N is the number of sampling particles[5]. So the PF can be derived (van der Merwe et al., 2000). For completeness one cycle of the PF algorithm can be described in detail as follows：

(1) Initialization. Let $\{x_0^i\}_{i=1}^N$ be a set of particles sampled from the prior $p(x_0)$ at k=1 and set $w_0^i = 1/N$.

(2)Update each particle $x_k^{i^*} \sim p(x_k|x_{k-1}^i)$，i=1, $\cdots$,N. Calculate the likelihood probability $p(z_k|x_k^{i^*})$ of each particle. Update the weight $w_k^{i^*} = p(z_k|x_k^{i^*})$, and make it be normalized, like this $w_k^{i^*} = w_k^{i^*}/\sum_{j=1}^N w_k^{j^*}$。

(3)Let $\{x_k^i, w_k^i\}_1^N$ be Resample, and get the new particles $\{x_k^i, w_k^i\}_1^N$，$w_i^k = 1/N$；

(4) Let k=k+1, and return to the (2).

Wherever Times is specified, Times Roman or Times New Roman may be used. If neither is available on your word processor, please use the font closest in appearance to Times. Avoid using bit-mapped fonts if possible. True-Type 1 or Open Type fonts are preferred. Please embed symbol fonts, as well, for math, etc.

III. KALMAN /PARTICLE FILTER ALGORITHM

The dimensional state vector of dynamic model is divided into two sub-models, $[x_k^\pi \quad x_k^{n\pi}]^T$ is linear, and x_k^l, x_k^n is nonlinear. Then Kalman filter and particle filter are implemented separately.

The error model state vector of SINS can be described like this:

$$x_k = [\delta L \ \delta\lambda \ \delta v_E \ \delta_{vN} \ \phi_E \ \phi_N \ \phi_U \ \nabla_{bx} \ \nabla_{by} \ \omega_{bx} \ \omega_{by} \ \omega_{bz}]^T \quad (8)$$

And then:

$$x_k^l = [\delta L \ \delta\lambda \ \delta v_E \ \delta_{vN} \ \nabla_{bx} \ \nabla_{by} \ \omega_{bx} \ \omega_{by} \ \omega_{bz}]^T$$

$$x_k^n = [\phi_E \ \phi_N \ \phi_U]^T$$

The error model and observation model equation of SINS is:

$$x_{k+1}^l = F_k^l(x_k^n) + A_k^l x_k^l + w_k^l \quad (9)$$

$$x_{k+1}^n = F_k^n(x_k^n) + A_k^n x_k^l + w_k^n \quad (10)$$

$$z_k^l = H_k x_k^l + e_k^l \quad (11)$$

where $F_k^l(x_k^n) = M(I - C_p^n)C_b^p \hat{f}^b$，$F_k^n(x_k^n) = (I - C_p^n)\omega_{in}^n$，$A_k^l$ and A_k^n are linear matrix. They can be calculated from Eq. (9). Where:

$$M = \begin{bmatrix} O_{2\times3} \\ 1\ 0\ 0 \\ 0\ 1\ 0 \\ O_{5\times3} \end{bmatrix}，\quad H_k = [I_{4\times4} \ O_{4\times8}]，\quad \omega_k \in N(0, Q_k^l)，$$

$$\omega_n \in N(0, Q_k^n)，\quad e_k^l \in N(0, R_k)$$

After state decomposition, based on the Bayesian, we know that:

$$p(x_k|z_{1:k}) = p(x_k^n, x_k^l|z_{1:k})$$
$$= \underbrace{p(x_k^n|x_k^l, z_{1:k})}_{PF} \underbrace{p(x_k^l|z_{1:k})}_{KF} \quad (13)$$

From (2.1), we find that:

$$z_k^n = x_{k+1}^l + A_k^l x_k^l = F_k^l(x_k^n) + \omega_k^l \quad (14)$$

where $p(x_k^l|z_{1:k})$ is estimated by Kalman filter, and $p(x_k^n|x_k^l, z_{1:k})$ is estimated by Particle filter.

The Kalman/ Particle filter described in detail as follows：

(1) Initialization. We suppose that $x_{1|0}^l, P_{1|0}^l, P_{x_0^n}(x^n)$ are known. To the linear model Eq(9) and Eq(10), initialize $x_{1|0}^l, P_{1|0}^l$. To the non-linear model Eq(11)and(14), the particles $x_0^{n,(i)} \sim p_{x_0^n}(x_0^n)$ come into being random, and the number of the particles is N. The weight is $q_0^i = 1/N$ .Then $F_0^l(x_0^n) = M(I - C_p^n)C_b^p \hat{f}^b$, and k=1。

(2) Update the measurement of Kalman filter:

$$x_k^l = x_{k|k-1}^l + F_{k-1}^l K_K(z_k^l - H_k x_{k|k-1}^l - H_k F_{k-1}^l)(15)$$

$$P_K = (I - K_K H_k)P_{k|k-1} \quad (16)$$

$$K_K = p_{k|k-1} H_k^T \left(H_k P_{k|k-1} H_k^T + R_k \right)^{-1} \qquad (17)$$

(3) Update the time of Particle filter.

Bringing $\left\{ x_{k-1}^{n,(i)}, q_{k-1}^i \right\}_{i=1}^N$ and x_k^l into Eq. （11）, we know that $x_k^{n^*,(i)} \sim p\left(x_k^n | x_{k-1}^{n(i)} \right)$.

(4) Update the measurement of Particle filter:

From Eq. (14), we get z_k^n. Then calculate $q_k^{i^*} = p\left(z_k^n | x_k^{n^*,(i)} \right)$, and update the weight $q_k^{i^*} = q_{k-1}^i q_k^{i^*}$. At last, normalize $q_k^{i^*} = q_k^{i^*} / \sum_{i=1}^N q_k^{i^*}$.

(5) Particle filter resampling

Calculate the number of efficient samples N_{eff}. If $N_{eff} < N_T$, use the method of II to resample the particles, and get $\left\{ x_k^{n,(i)}, q_k^i \right\}$.

(6) Update the time of Kalman filter

Calculate $x_k^n = \sum_{i=1}^n q_k^i x_k^{n,(i)}$ by $\left\{ x_k^{n,(i)}, q_k^i \right\}$. Bringing it to $F_k^l(x_k^n) = M\left(I - C_p^n \right) C_b^p \hat{f}^b$, we can find F_k^l, and

$x_{k+1|k}^l = A_k^l x_k^l + F_k^l$, $P_{k+1|k}^l = A_k^l P_k^l (A_k^l)^T + Q_k^l$.

(7) Let k=k+1, and return to (2).

IV. SIMULATIONS

The dimensional state vector of the system is $x_k = \left[\delta L\; \delta \lambda\; \delta v_{_{v_E}}\; \delta_{_{v_N}}\; \phi_E\; \phi_N\; \phi_U\; \nabla_{bx}\; \nabla_{by}\; \omega_{bx}\; \omega_{by}\; \omega_{bz} \right]^T$. System noise and measurement noise are all Gaussian white noise. The latitude of initial position is 32°, and the longitude is 118°. $\dot{\nabla}_b = 0$. The constant drift of the accelerometer is 3×10^{-3}g. The white noise is 3×10^{-3}g. $\dot{\omega}_b = 0$. The constant drift of the gyroscope is 0.3°/h, and the white noise is 0.1°/h. The initial azimuth misalignment angle is 10°, and the initial horizontal misalignment angle is 0.1°. The mean square error of position measurement noise is 3m. The mean square error of velocity measurement noise is 0.03m /s. Under dynamic environment, the carrier is in uniform velocity rectilinear motion, and its speed is 20m/s, azimuth angle is 45°, the time of simulation is 800m. We implement PF and K/PF separately to simulate. Fig1 to Fig7 display the error curve of PF and K/PF.

From Fig1, Fig2 and Fig3, it shows that, in the circumstances of the large misalignment angle, PF and K/PF are similar in the convergence rates of course and attitude error are the same. However, the PF is more accurate than K/PF in the estimation of course angle and attitude angle. The velocity error and position error can be measured directly, and the two methods are the same in estimation accuracy.

The state dimensions of PF is 12, it is 3 for K/PF. The numbers of particle are 10000 and 600 separately. It takes 389.27s to iterative 800 times for PF, and it is 5.07s for K/PF. From the analysis above, the time the PF consumes is 75.8 times more than the K/PF consumes. From these simulation results, it can be seen that, the K/PF are better than PF in the adaptability, the accuracy and the time.
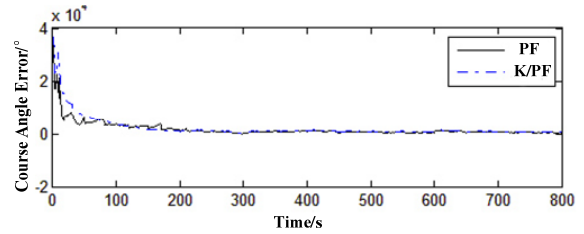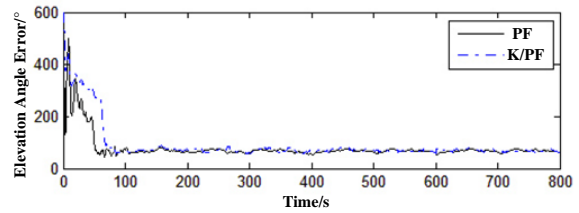

Figure 1. Course angle error
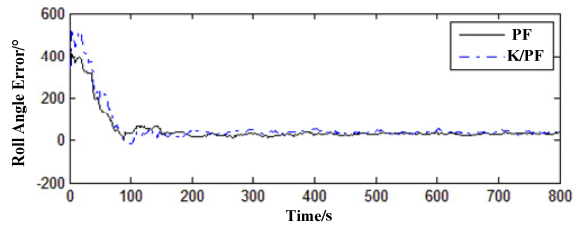

Figure 2. Elevation angle error

Figure 3. Roll angle error
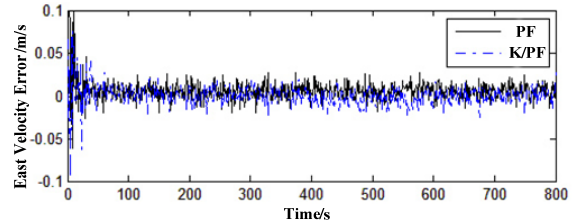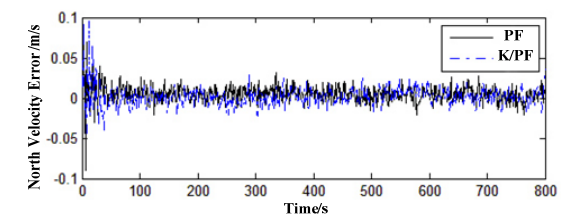

Figure 4. East velocity error
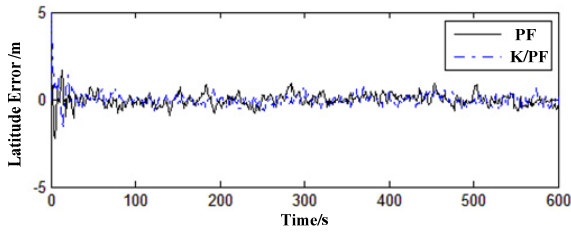

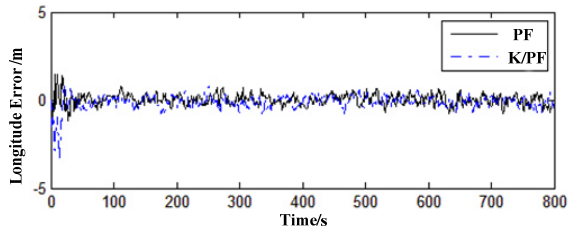Figure 5. North velocity error

Figure 6.   Latitude error

Figure 7.   Longitude error

V.   CONCLUSION

Particle filter is introduced. According to the Rao-Blackwellization theorem and the dynastic error model, a new Kalman/Particle mixed filter used on SINS/GPS integrated navigation system was proposed. The new method divides the system into two sub-models, one is linear, the other one is nonlinear, and then implement Kalman filter and particle filter separately. From the simulation results, it can be seen that, the K/PF are better in the adaptability, the accuracy and the time.

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