The Calculation of the Magnetic Field Produced by an Arbitrary Shaped Current-carrying Wire in Its Plane

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***Abstract*—Instead of performing the integration explicitly to calculate the magnetic field from an arbitrary shaped wire, an improved method was proposed. The magnetic field generated by a straight line segment carrying steady current was calculated based on the Biot-Savart law. The new approach is to break the wire down into a number of straight line segments in its plane. According to the principle of vector superposition and coordinate rotation, the magnetic field from the complete current wire can then be calculated by summing the contribution from each of the separate straight line pieces. The simulation results show that the proposed numerical method has high accuracy and faster computational time and is efficient and convenient to the application in projects.**

Keywords- magnetic field; superposition; segmentation

I. INTRODUCTION

The calculation of the magnetic field is a central problem for degaussing coil systems. An important aspect of designing and adjusting coil loop and degaussing power supply is to determine the magnetic field at any position due to an arbitrary shaped coil. An improved method to achieve this, based on segmentation and coordinate rotation, is described in this paper. Lately, many solutions for the magnetic field from a single rectangular or polygonal loop of wire in a plane can be found in text books and various publications. Integrated method have been used for calculating the magnetic field intensity from a circle and rectangular wire carrying a steady current, for example [1].A series expansion for the magnetic field of a circle current can be found in the survey [2].The formulas were presented for calculating the magnetic fields of line current coils under both common and special conditions [3]. The magnetic field of multiple rectangle loops or polygonal current coil were regarded as the summing contributions from each segment [4-6]. Nevertheless, those methods are unsuitable to calculate the magnetic field of asymmetrical wire in projects.

In this paper we investigate a different method to simplify the calculation. The magnetic field produced by a straight line segment at any position in Cartesian coordinate system was calculated directly from the Biot-Savart law. Rather than performing the integration explicitly to calculate the magnetic field this paper propose an improved method. The useful approach is to break the arbitrary shaped wire down into a number of straight line segments in its plane. According to the principle of vector superposition and coordinate rotation, the magnetic field from the complete current wire can then be calculated by summing the contribution from each of the separate straight line pieces. The rule for segmentation is given as well. The numerical results demonstrate that the proposed method have high precision and faster computational time and is efficient and convenient to the application in projects.

II. THE CALCULATION OF *H* PRODUCED BY A STRAIGHT CURRENT-CARRYING WIRE

The Biot-Savart law is an explicit equation for the magnetic field intensity $\boldsymbol{H}$ in terms of a steady current I and the closed path C that it flows on, such as the path shown in Fig.1 (a).We can obtain the magnetic field equation at an arbitrary field point P (x_p, y_p, z_p) in the Cartesian coordinates, which can be expressed as

$$\boldsymbol{H} = \frac{I}{4\pi}\oint_C \frac{d\boldsymbol{l}\times(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3} \tag{1}$$

The field point P and a short current element $I\boldsymbol{dl}$ are represented by the position vector $\boldsymbol{r}$ and $\boldsymbol{r}'$, respectively, $\boldsymbol{r}-\boldsymbol{r}'$ is the relative position vector.

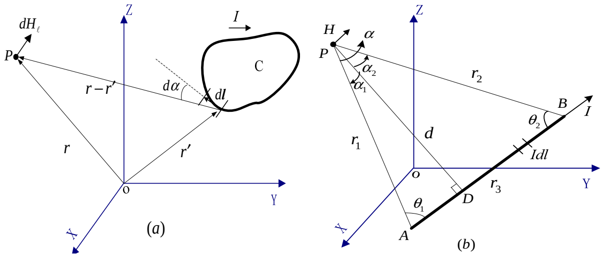

Fig.1 The magnetic field produced by a current-carrying wire at point P

For an regular loop that lies in the x-y plane and carries a steady current I, we can expediently obtain the calculation results by Equation (1), whereas, in the case of arbitrary coli, the resulting integrals are very difficult .In order to develop a simple way to determine the equations for calculation, let us first consider $\boldsymbol{H}$ is generated by separate line segments. Based on the principle of vector superposition and coordinate rotation, the magnetic field from the complete coil can then be determined by summing the field generated by each of the separate line segment.

From a magnitude perspective, we follow the development of field equations [7] and obtain：

$$\begin{aligned}|\boldsymbol{H}| &= \frac{I}{4\pi}\oint_C \frac{|d\boldsymbol{l}\times(\boldsymbol{r}-\boldsymbol{r}')|}{|\boldsymbol{r}-\boldsymbol{r}'|^3} = \frac{I}{4\pi}\oint_C \frac{|\boldsymbol{r}-\boldsymbol{r}'|^2\,d\alpha}{|\boldsymbol{r}-\boldsymbol{r}'|^3} = \frac{I}{4\pi}\oint_C \frac{d\alpha}{|\boldsymbol{r}-\boldsymbol{r}'|} \\ &= \frac{I}{4\pi}\int_{-\alpha_1}^{\alpha_2} \frac{\cos\alpha\, d\alpha}{d} = \frac{I}{4\pi d}(\sin\alpha_1+\sin\alpha_2)\end{aligned} \tag{2}$$

Fig.1 (b) shows a straight current line $\overrightarrow{AB}$ with length r_3, directed along an arbitrary orientation. r_1 and r_2 are the relative position vectors from field point P to terminals A and B respectively. Here, the angle α is defined as the smaller angle between $\overrightarrow{AP}$ and $\overrightarrow{BP}$, $\overrightarrow{DP}$ is defined as the distance vector from P to $\overrightarrow{AB}$, with length d. Using the law of cosine, we find that Equation (2) can be expressed as

$$\begin{aligned}|\boldsymbol{H}| &= \frac{\boldsymbol{I}}{4\pi d}\left(\frac{r_3^2+r_1^2-r_2^2}{2r_3r_1}+\frac{r_3^2+r_2^2-r_1^2}{2r_3r_2}\right) \\ &= \frac{\boldsymbol{I}}{4\pi}\frac{(r_1+r_2)(r_3^2-r_1^2-r_2^2+2r_1r_2)}{r_1r_2\sqrt{2r_1^2r_2^2+2r_1^2r_3^2+2r_2^2r_3^2-r_1^4-r_2^4-r_3^4}}\end{aligned} \tag{3}$$

Since each current element $\boldsymbol{Idl}$ is defined as a vector with the same direction as $\overrightarrow{AB}$, the direction of magnetic field generated by each element at point P is the same as well. According to the cross product, we obtain the direction of $\boldsymbol{H}$, which can be written as

$$\boldsymbol{e}_H=\frac{\overrightarrow{AB}\times\overrightarrow{AP}}{\left|\overrightarrow{AB}\times\overrightarrow{AP}\right|}=\begin{vmatrix}\boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ x_B-x_A & y_B-y_A & z_B-z_A \\ x_P-x_A & y_P-y_A & z_P-z_A\end{vmatrix}=\frac{v_x}{\sqrt{v_x^2+v_y^2+v_z^2}}\boldsymbol{e}_x+\frac{v_y}{\sqrt{v_x^2+v_y^2+v_z^2}}\boldsymbol{e}_y+\frac{v_z}{\sqrt{v_x^2+v_y^2+v_z^2}}\boldsymbol{e}_z \tag{4}$$

Here, $v_x=(y_B-y_A)*(z_P-z_A)-(y_P-y_A)*(z_B-z_A)$,

$v_y=(x_P-x_A)*(z_B-z_A)-(x_B-x_A)*(z_P-z_A)$, $v_z=(x_B-x_A)*(y_P-y_A)-(x_P-x_A)*(y_B-y_A)$.

Then, we can easily obtain three components of $\boldsymbol{H}$ in the Cartesian coordinates.

$$\begin{aligned}\boldsymbol{H} &= |\boldsymbol{H}|\bullet\boldsymbol{e}_H = \boldsymbol{H}_x\boldsymbol{e}_x+\boldsymbol{H}_y\boldsymbol{e}_y+\boldsymbol{H}_z\boldsymbol{e}_z \\ &= \frac{|\boldsymbol{H}|v_x}{\sqrt{v_x^2+v_y^2+v_z^2}}\boldsymbol{e}_x+\frac{|\boldsymbol{H}|v_y}{\sqrt{v_x^2+v_y^2+v_z^2}}\boldsymbol{e}_y+\frac{|\boldsymbol{H}|v_z}{\sqrt{v_x^2+v_y^2+v_z^2}}\boldsymbol{e}_z\end{aligned} \tag{5}$$

III. The Deviation of $\boldsymbol{H}_C$ Generated by an Arbitrary Shaped Coil Carrying Steady Current

The generated magnetic field at any point can be calculated directly from the Equation (5) when the coordinate variables of the straight current line are given. For an arbitrary current curve C, a more useful approach is to break the complete current curve into a number of arc segments $\overset{\frown}{M_0M_1},\overset{\frown}{M_1M_2},\cdots,\overset{\frown}{M_{N-1}M_N}$, N is the number of segments. Thus, the arc segment $\overset{\frown}{M_{i-1}M_i}$ ($i=1,2,\cdots,N$) can be thought of as straight segment $\overline{M_{i-1}M_i}$ if N is selected properly. The magnetic field from the complete coil can then be determined by summing the contributions from each of the separate line segments.

$$\boldsymbol{H}_c=\sum_{i=1}^{n}\boldsymbol{H}^i=\sum_{i=1}^{n}\left|\boldsymbol{H}_x^i\right|\boldsymbol{e}_x+\sum_{i=1}^{n}\left|\boldsymbol{H}_y^i\right|\boldsymbol{e}_y+\sum_{i=1}^{n}\left|\boldsymbol{H}_z^i\right|\boldsymbol{e}_z \tag{6}$$

Fig.2 (a) shows a steady current I flows along an arbitrary path DE in the Cartesian coordinate system. When the termination points $D(x_D,y_D,z_D)$, $E(x_E,y_E,z_E)$ and any other point $F(x_F,y_F,z_F)$ on the curve are given, the local coordinate system can be determined accordingly. As shown in Fig.2 (b), a termination point D of curve is located at the origin in the new coordinates, the X' axes is defined with the same direction of the vector $\overline{DE}$, the Z' axes is perpendicular to the plane of the loop , the Y' axes is specified by the right-hand rule. In the local coordinate system $X'Y'Z'$, the curve is broken down into straight line segments, and then we can obtain the coordinate variables of breakpoint M_i by coordinate rotation.

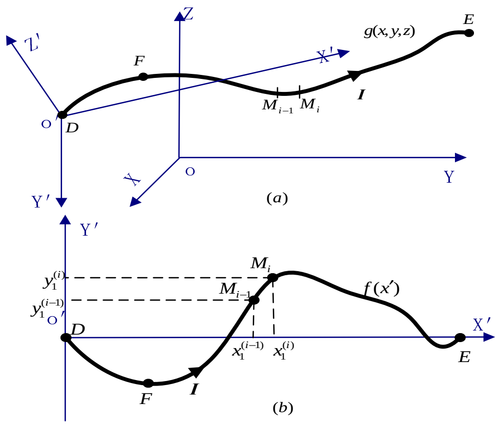

Fig.2 The subdivision and coordinate transformation of an arbitrary shaped wire carrying a current I

To evaluate coordinate variables, $\boldsymbol{\alpha}=(\boldsymbol{e}_x,\boldsymbol{e}_y,\boldsymbol{e}_z)^{\mathrm{T}}$ is specified as the base vector in Cartesian coordinates, $\boldsymbol{\beta}=(\boldsymbol{e}_{x'},\boldsymbol{e}_{y'},\boldsymbol{e}_{z'})^{\mathrm{T}}$ is the base vector in the local coordinate system $X'Y'Z'$, we obtain

$$\begin{cases}\boldsymbol{e}_{x'}=\dfrac{\overline{DE}}{\left|\overline{DE}\right|}=t_{11}\boldsymbol{e}_x+t_{12}\boldsymbol{e}_y+t_{13}\boldsymbol{e}_z; \\ \boldsymbol{e}_{z'}=\dfrac{\overline{DE}\times\overline{DF}}{\left|\overline{DE}\times\overline{DF}\right|}=t_{31}\boldsymbol{e}_x+t_{31}\boldsymbol{e}_y+t_{33}\boldsymbol{e}_z; \\ \boldsymbol{e}_{y'}=\boldsymbol{e}_{z'}\times\boldsymbol{e}_{x'}=t_{21}\boldsymbol{e}_x+t_{22}\boldsymbol{e}_y+t_{23}\boldsymbol{e}_z\end{cases} \tag{7}$$

$$\boldsymbol{\beta}=\begin{bmatrix}t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33}\end{bmatrix}\bullet\boldsymbol{\alpha}=T'\bullet\boldsymbol{\alpha} \tag{8}$$

Substituting Equation (7) into Equation (8), we have

$t_{11}=\dfrac{x_E-x_D}{\sqrt{(x_E-x_D)^2+(y_E-y_D)^2+(z_E-z_D)^2}}$; $t_{12}=\dfrac{y_E-y_D}{\sqrt{(x_E-x_D)^2+(y_E-y_D)^2+(z_E-z_D)^2}}$

$t_{13}=\dfrac{x_E-x_D}{\sqrt{(x_E-x_D)^2+(y_E-y_D)^2+(z_E-z_D)^2}}$; $t_{21}=t_{13}t_{32}-t_{12}t_{33}$;

$t_{22}=t_{11}t_{33}-t_{13}t_{31}$; $t_{23}=t_{12}t_{31}-t_{11}t_{33}$, moreover, the variables t_{31}, t_{32} and t_{33} can be determined by Equation (4). Using rotation and translation, we obtain the complete coordinates of an arbitrary point $P(x',y',z')$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = T \bullet \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} x_D \\ y_D \\ z_D \end{bmatrix} \tag{9}$$

The arbitrary wire carrying steady current is firstly subdivided in local coordinates and the coordinate variables of breakpoint $M_i(x'^{(i)}, y'^{(i)}, z'^{(i)})$ can be determined. Generally, we obtain the coordinates according to the length of wire, which can be expressed as $x'^{(i)} = \left|\int_{DE} f(x')ds\right| \bullet i/N$, $y'^{(i)} = f(x'^{(i)})$, $z'^{(i)} = 0$, $i = 1, 2, \cdots, N$ .This means the calculation accuracy ascends proportionally to the number of the segments N .To better demonstrate the algorithm of subdivision, we take arc coil as an example. Terminal point $E(x'_E, y'_E, z'_E)$ and the center of curvature $F(x'_F, y'_F, z'_F)$ in the local coordinate system is shown in Fig.3. d is the distance from field point to the chord with length ℓ .

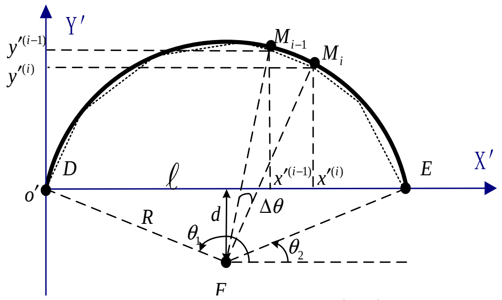

Fig.3 The subdivision of an arc coil in the x' - y' plane

The Equation (9) can be expressed as

$$\begin{bmatrix} x'_E \\ y'_E \\ z'_E \end{bmatrix} = T^{-1} \begin{bmatrix} x_E - x_D \\ y_E - y_D \\ z_E - z_D \end{bmatrix} ; \quad \begin{bmatrix} x'_F \\ y'_F \\ z'_F \end{bmatrix} = T^{-1} \begin{bmatrix} x_F - x_D \\ y_F - y_D \\ z_F - z_D \end{bmatrix} \tag{10}$$

Obviously, we have $z'_E = z'_F = 0$, $R = \sqrt{(x'_F)^2 + (y'_F)^2}$ in the x' - y' plane, the radius angle of the arc is determined by $\theta = \theta_1 - \theta_2 = \pi \mp 2\arctan(-y'_F / x'_F)$, subtraction is selected when $\theta \le \pi$, and $\Delta\theta$ is defined as the radius angle of the differential arc $\widehat{M_{i-1}M_i}$, which can be derived as

$$\Delta\theta = (\theta_1 - \theta_2)/N \tag{11}$$

Thus, the coordinate variables at point $M_i(x'^{(i)}, y'^{(i)}, z'^{(i)})$ can be written as

$$\begin{cases} x'^{(i)} = R\cos(\theta_1 - i\Delta\theta) + x'_F \;;\; z'^{(i)} = 0; \\ y'^{(i)} = R\sin(\theta_1 - i\Delta\theta) + y'_F ; \quad i = 1, 2, 3, \cdots, N \end{cases} \tag{12}$$

Substituting these terms into Equation (9) and Equation (6), then we can obtain $\boldsymbol{H}$ generated by an arbitrary shaped coil carrying steady current.

IV. SIMULATIONS AND PRECISION ANALYSIS

In the simulations, we use integrated method [1] and subdivision method to calculate the magnetic field intensity, represented as H and H_s respectively, generated by arc wires with different radius angles. Other parameters are taken as R=1m, I=20A.The relative errors of magnetic field intensity e_r is defined as $e_r = (H_s - H)/H \times 100\%$. The simulation results are shown in Fig.4.

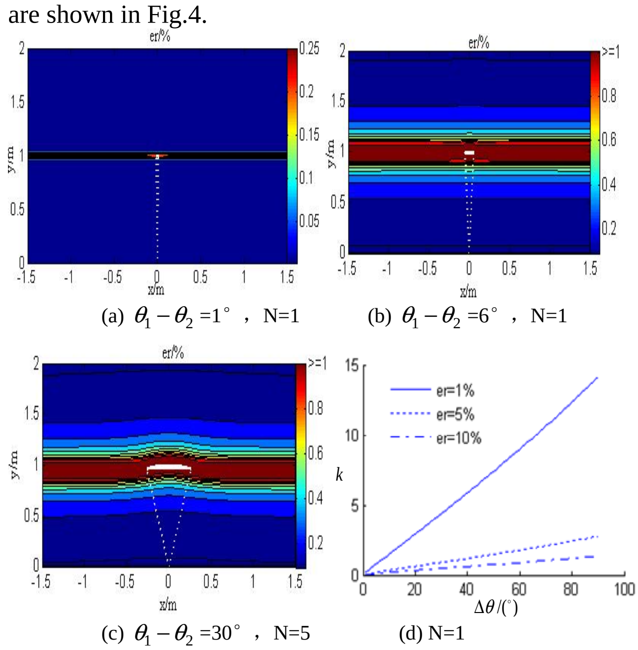

Fig.4 Relative errors of the calculation of magnetic field intensity produced by different arc wires

From the Fig.4 (a) and (b), we find that the relative errors of calculating magnetic field, generated by arc wires with different radius angles, present uniform distribution in a rectangular band when N=1.The Fig.4 (b) and (c) show that the distribution of e_r in the plane varies proportionally to the differential angle $\Delta\theta$. Their tendencies are shown in Fig.4 (d). We defined the ratio of d and ℓ as $k = d/\ell$, then it can be seen from the figures that k increases linearly when e_r is fixed and $\Delta\theta$ increases. The trend means that an arc with larger central angle can be treated as a straight line in the calculation of magnetic field when k is greater. To improve the efficiency of the method, we can express the relationship between $\Delta\theta$ and k as

$$\begin{cases} \Delta\theta = 1.93 - 45.37e_r + (-1.263 + 70.339e_r)k \\ N = [(\theta_1 - \theta_2)/\Delta\theta]_d \end{cases} \tag{13}$$

$[x]_d$ rounds the elements of x to the nearest integers greater than or equal to x.

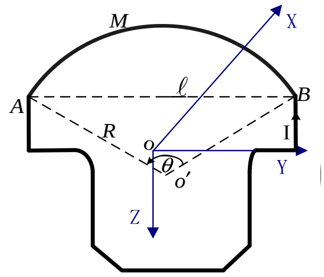

Fig.5 Geometry for a simulated coil

Taking the precision of the arc current as e_r =1% in the simulated coil, shown in the Fig.5, we choose sample points along the central axis and calculate the relative errors of the entire coil. During the simulation, the parameters coincide with that in reference [3].

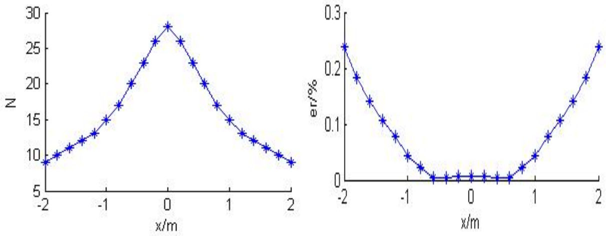

Fig.6 Simulation Result of former-wound coil

The curves in Fig.6 represent the computation results of the number of segments *N* and the relative errors of the complete coil, and indicate that the proposed method is efficient and have high accuracy.

V. CONCLUSIONS

The presented method of calculating the magnetic field from an arbitrary shaped wire is to break the wire down into a number of straight line segments in its plane. Instead of performing the integration of the complete current wire explicitly, the proposed approach is to sum the contribution from each segment based on the principle of coordinate rotation and vector superposition. The rule for segmentation is given to improve the efficiency of the method as well. The simulation results show that the proposed numerical method have high accuracy and faster computational time and is efficient and convenient in projects.

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