The Dynamic Modelling and Simulation of Spatial Multibody Systems with Prismatic Joint Based on Vector Bond Graph

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***Abstract*—For the modelling and simulation of complex spatial multibody systems, the vector bond graph method is proposed. By the kinematic constraint condation, spatial prismatic joint can be modeled by vector bond graph. For the algebraic difficulties brought by differential causality in system automatic modeling and simulation, the effective decoupling method is proposed and the differential causalities in system vector bond graph model can be eliminated. In the case of considering EJS, the unified formulae of system state space equations and constraint forces at joints are derived, which are easily derived on a computer and very suitable for spatial multibody systems. As a result, the unified modelling and simulation for complex spatial multibody systems are realized, its validity is illustrated by a practical example.**

Keywords-Vector bond graph; Modelling and simulation; Spatial multibody system; Prismatic joint; Causality

I. INTRODUCTION

System modeling and simulation are very important to the control and dynamic design of modern mechanical systems. For complex spatial multibody systems, e.g. the spatial multibody systems with prismatic joint , deriving system state space equations and constraint force equations at joints is a very tedious and erroprone task. Although different procedures have been proposed to increase the reliability and efficiency of this process[1,2], most of them can not be used to deal with systems that simultaneously include various physical domains in a unified manner.

Bond graph technique[3] was chosen because it is a computer oriented method which can describe all type of physical systems, thus allowing a single model to represent the dynamic interactions of the spatial multibody system with electrical, hydraulic, pneumatic, and other components. Compared with scalar bond graph[3], vector bond graph is more suitable for modelling spatial multibody systems because of its more concise representation manner[4,5]. But for spatial multibody systems, the kinematic and geometric constraints between bodies result in differential causality loop, and the nonlinear velocity relationship between the mass center and an arbitrary point on a body leads to the nonlinear junction strcture. Current vector bond graph procedures[4] were found to be very difficult algebraically in derivation of system state space equations automatically on a computer.To solve above problems, a more efficient and practical modelling and simulation procedure for spatial multibody systems with prismatic joint based on vector bond graph[4] is proposed here.

II. VECTOR BOND GRAPH MODEL OF SPATIAL MULTIBODY SYSYEM WITH PRISMATIC JOINT

In any spatial multi-body system, the joints impose kinematic constraints on the rigid body elements. A general rigid body moving in space can be modelled by vector bond graph[4]. The diagram of spatial prismatic joint is shown in Figure.1, this constraint limits the relative translation of the two bodies B_α and B_β along two directions, and limits the relative rotation of the two bodies B_α and B_β along three directions. Joint point P and Q are fixed on rigid body B_α and B_β respectively, vector h_α is used to describe the relative motion of the two rigid bodies, $h_\alpha = r_\alpha^P - r_\beta^Q$. Where r_α^P and r_β^Q represent the position vector of joint point P and Q in global coordinates respectively, $r_\alpha^P = [x_\alpha^P \quad y_\alpha^P \quad z_\alpha^P]^T$, $r_\beta^Q = [x_\beta^Q \quad y_\beta^Q \quad z_\beta^Q]^T$. d_β^1 and d_β^2 are two unit vectors fixed on rigid body B_β, which are all orthogonal to axis, and orthogonal to each other. d_α is the unit vector fixed on rigid body B_α along axis, d_α^1 is another unit vector fixed on rigid body B_α and paralell to d_β^1. $d_\alpha^{'}$, $d_\alpha^{'1}$, $d_\beta^{'1}$ and $d_\beta^{'2}$ are the corresponding

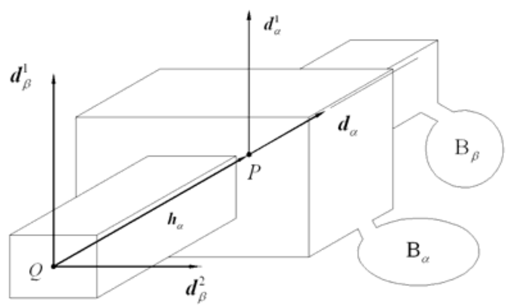

Figure.1 A sketch of prismatic joint

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vectors in body frame. From the Kinematic constraint condition of prismatic joint[1], the position constraint equations can be written as

$$\underline{\Phi}^{(d_2)}(h_\alpha, d_\beta^1, d_\beta^2) = \begin{bmatrix} d_\beta^{1T} h_\alpha \\ d_\beta^{2T} h_\alpha \end{bmatrix} = \begin{bmatrix} d_\beta^{'1T} A^{\beta T}(r_\beta^Q - r_\alpha^P) \\ d_\beta^{'2T} A^{\beta T}(r_\beta^Q - r_\alpha^P) \end{bmatrix} = 0 \tag{1}$$

$$\underline{\Phi}^{(r_3)}(d_\alpha, d_\alpha^1, d_\beta^1, d_\beta^2) = \begin{bmatrix} d_\beta^{1T} d_\alpha \\ d_\beta^{2T} d_\alpha \\ d_\beta^{2T} d_\alpha^1 \end{bmatrix} = \begin{bmatrix} d_\beta^{'1T} A^{\beta T} A^\alpha d_\alpha^{'} \\ d_\beta^{'2T} A^{\beta T} A^\alpha d_\alpha^{'} \\ d_\beta^{'2T} A^{\beta T} A^\alpha d_\alpha^{'1} \end{bmatrix} = 0 \tag{2}$$

where A^β is the direction cosin matrix of body B_β.

The corresponding velocity(or angular velocity) constraint equations can be written as

$$\dot{\underline{\Phi}}^{(r_3)}(d_\alpha, d_\alpha^1, d_\beta^1, d_\beta^2) = \left[\omega_\alpha - \omega_\beta\right] = 0 \tag{3}$$

$$\dot{\underline{\Phi}}^{(d_2)}(h_\alpha, d_\beta^1, d_\beta^2) = \begin{bmatrix} d_\beta^{1T}\dot{h}_\alpha + h_\alpha^T \dot{d}_\beta^1 \\ d_\beta^{2T}\dot{h}_\alpha + h_\alpha^T \dot{d}_\beta^2 \end{bmatrix}$$

$$= \begin{bmatrix} d_\beta^{'1T} A^{\beta T}(\dot{r}_\alpha^P - \dot{r}_\beta^Q) - h_\alpha^T \tilde{d}_\beta^1 \omega_\beta \\ d_\beta^{'2T} A^{\beta T}(\dot{r}_\alpha^P - \dot{r}_\beta^Q) - h_\alpha^T \tilde{d}_\beta^2 \omega_\beta \end{bmatrix} = 0 \tag{4}$$

where ω_α and ω_β represent the angular velocity vector of the rigid body determined in global coordinates.

$$\tilde{d}_\beta^i = \begin{bmatrix} 0 & -d_{\beta z}^i & d_{\beta y}^i \\ d_{\beta z}^i & 0 & -d_{\beta x}^i \\ -d_{\beta y}^i & d_{\beta x}^i & 0 \end{bmatrix} \quad (i=1, 2)$$

The velocity(or angular velocity) constraint equations shown as Eqs.(3) and Eqs.(4) can be presented by vector bond model shown as Figure.2.

The vector bond graph for the rigid body undergoing spatial motion can be coupled to one another satisfying the kinematic constraints[1-2] at the interfaces to get the over system model. But the kinematic constraints result in differential causality. In the derivation of system state space equations, the current vector bond graph procedures[4] were found to be very difficult algebraically. To eliminate the differential causality, the constraint force vectors at joints can be considered as unknown effort source vectors and added to the corresponding 0-junctions of the system vector bond graph model.

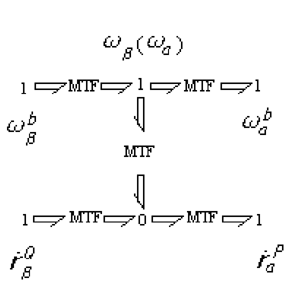

Figure.2 Vector bond graph model of prismatic joint

III. The Unified Formulae of System State Space Equations and Constraint Forces

The basic field and junction structure of system bond graph is imposed is shown in Figure.3[3], where Euler-junction structure[4] (EJS) is added. X_{i_1} represents energy vector variable of independent storage energy field corresponding to independent motion, X_{i_2} represents energy vector variable of independent storage energy field corresponding to dependent motion, Z_{i_1} and Z_{i_2} are the corresponding coenergy vector variables. D_{in} and D_{out} represent input and output vector variables in resistive field, U and V represent input and output vector variables of source field respectively, $U = \left[U_1 \quad U_2\right]^T, V = \left[V_1 \quad V_2\right]^T$. Where U_1 is known source vector, and U_2 represents the constraint force vector of joint. E_{in} and E_{out} are the input and output vector variables in Euler-junction structure （EJS）.

For independent energy storage field, we have

$$Z_{i_1} = F_{i_1} X_{i_1} \tag{5}$$

$$Z_{i_2} = F_{i_2} X_{i_2} \tag{6}$$

where F_{i_1} and F_{i_2} are the $m_1 \times m_1$ and $m_2 \times m_2$ matrices respectively.

For resistive field, we have

$$D_{out} = RD_{in} \tag{7}$$

where R is $L \times L$ matrix.

For Euler-junction structure (EJS), we have

$$E_{out} = R_E E_{in} \tag{8}$$

where R_E is $L_E \times L_E$ matrix[6].

The corresponding junction structure equations can be written as

$$\dot{X}_{i_1} = J_{i_1 i_1} Z_{i_1} + J_{i_1 i_2} Z_{i_2} + J_{i_1 L} D_{out} + J_{i_1 u_1} U_1 + J_{i_1 u_2} U_2 + J_{i_1 E} E_{out} \tag{9}$$

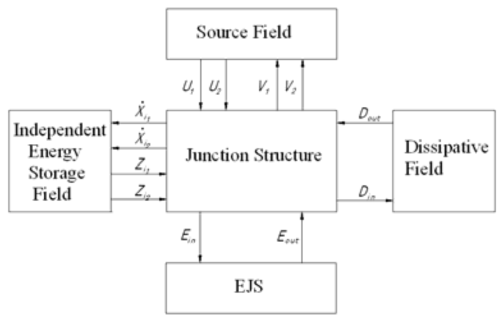

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Figure.3 Basic field and junction structure of system

$$\dot{X}_{i_2} = J_{i_2 i_1} Z_{i_1} + J_{i_2 i_2} Z_{i_2} + J_{i_2 L} D_{out} + J_{i_2 u_1} U_1 + J_{i_2 u_2} U_2 + J_{i_2 E} E_{out} \tag{10}$$

$$D_{in} = J_{Li_1} Z_{i_1} + J_{Li_2} Z_{i_2} + J_{LL} D_{out} + J_{Lu_1} U_1 + J_{Lu_2} U_2 + J_{LE} E_{out} \tag{11}$$

$$E_{in} = J_{Ei_1} Z_{i_1} + J_{Ei_2} Z_{i_2} + J_{EL} D_{out} + J_{Eu_1} U_1 + J_{Eu_2} U_2 + J_{EE} E_{out} \tag{12}$$

From the flow summation of 0-junctions corresponding to m_2 constraint force vectors in system vector bond graph model, we have

$$0 = J_{Ci_1} Z_{i_1} + J_{Ci_2} Z_{i_2} + J_{CL} D_{out} + J_{Cu_1} U_1 + J_{CE} E_{out} \tag{13}$$

By the algebraic manipulation from (5)~(13), the system state space equations can be written as

If $\det(T_{LE}) \neq 0$

$$\left.\begin{aligned} &\dot{X}_{i_1} = T_{i_1 i_1} X_{i_1} + T_{i_1 i_2} X_{i_2} + T_{i_1 u_1} U_1 + T_{i_1 u_2} U_2 && (a) \\ &\dot{X}_{i_2} = T_{i_2 i_1} X_{i_1} + T_{i_2 i_2} X_{i_2} + T_{i_2 u_1} U_1 + T_{i_2 u_2} U_2 && (b) \\ &U_2 = -T_{LE}^{-1}(T_{u_2 i_1} X_{i_1} + T_{u_2 i_2} X_{i_2} + T_{u_2 u_1} U_1) && (c) \end{aligned}\right\} \tag{14}$$

where

$$A_1 = [I_2 - J_{EE} R_E - J_{EL} R (I_1 - J_{LL} R)^{-1} J_{LE} R_E]^{-1}$$

$$A_2 = J_{Ei_1} F_{i_1} + J_{EL} R (I_1 - J_{LL} R)^{-1} J_{Li_1} F_{i_1}$$

$$A_3 = J_{Ei_2} F_{i_2} + J_{EL} R (I_1 - J_{LL} R)^{-1} J_{Li_2} F_{i_2}$$

$$A_4 = J_{Eu_1} + J_{EL} R (I_1 - J_{LL} R)^{-1} J_{Lu_1}$$

$$A_5 = J_{Eu_2} + J_{EL} R (I_1 - J_{LL} R)^{-1} J_{Lu_2}$$

$$B_1 = (I_1 - J_{LL} R)^{-1} (J_{Li_1} F_{i_1} + J_{LE} R_E A_1 A_2)$$

$$B_2 = (I_1 - J_{LL} R)^{-1} (J_{Li_2} F_{i_2} + J_{LE} R_E A_1 A_3)$$

$$B_3 = (I_1 - J_{LL} R)^{-1} (J_{Lu_1} F_{i_1} + J_{LE} R_E A_1 A_2)$$

$$B_4 = (I_1 - J_{LL} R)^{-1} (J_{Lu_2} F_{i_1} + J_{LE} R_E A_1 A_5)$$

$$T_{i_1 i_1} = J_{i_1 i_1} F_{i_1} + J_{i_1 L} R B_1 + J_{i_1 E} R_E A_1 A_2$$

$$T_{i_1 i_2} = J_{i_1 i_2} F_{i_2} + J_{i_1 L} R B_2 + J_{i_1 E} R_E A_1 A_3$$

$$T_{i_1 u_1} = J_{i_1 u_1} + J_{i_1 L} R B_3 + J_{i_1 E} R_E A_1 A_4$$

$$T_{i_1 u_2} = J_{i_1 u_2} + J_{i_1 L} R B_4 + J_{i_1 E} R_E A_1 A_5$$

$$T_{i_2 i_1} = J_{i_2 i_1} F_{i_1} + J_{i_2 L} R B_1 + J_{i_2 E} R_E A_1 A_2$$

$$T_{i_2 i_2} = J_{i_2 i_2} F_{i_2} + J_{i_2 L} R B_2 + J_{i_2 E} R_E A_1 A_3$$

$$T_{i_2 u_1} = J_{i_2 u_1} + J_{i_2 L} R B_3 + J_{i_2 E} R_E A_1 A_4$$

$$T_{i_2 u_2} = J_{i_2 u_2} + J_{i_2 L} R B_4 + J_{i_2 E} R_E A_1 A_5$$

$$T_{LE} = J_{CL} R B_4 + J_{CE} R_E A_1 A_5$$

$$T_{u_2 i_1} = J_{Ci_1} F_{i_1} + J_{CL} R B_1 + J_{CE} R_E A_1 A_2$$

$$T_{u_2 i_2} = J_{Ci_2} F_{i_2} + J_{CL} R B_2 + J_{CE} R_E A_1 A_3$$

$$T_{u_2 u_1} = J_{Cu_1} + J_{CL} R B_3 + J_{CE} R_E A_1 A_4$$

If $J_{CL} = 0$, $J_{CE} = 0$

$$\left.\begin{aligned} &\dot{X}_{i_1} = T_{i_1 i_1} X_{i_1} + T_{i_1 i_2} X_{i_2} + T_{i_1 u_1} U_1 + T_{i_1 u_2} U_2 && (a) \\ &\dot{X}_{i_2} = T_{i_2 i_1} X_{i_1} + T_{i_2 i_2} X_{i_2} + T_{i_2 u_1} U_1 + T_{i_2 u_2} U_2 && (b) \\ &U_2 = -H_4^{-1}(H_1 X_{i_1} + H_2 X_{i_2} + H_3 U_1 + J_{Cu_1} \dot{U}_1) && (c) \end{aligned}\right\} \tag{15}$$

where

$$H_1 = \dot{J}_{Ci_1} F_{i_1} + J_{Ci_1} F_{i_1} T_{i_1 i_2} + J_{Ci_2} F_{i_2} T_{i_2 i_1},$$

$$H_2 = \dot{J}_{Ci_2} F_{i_2} + J_{Ci_1} F_{i_1} T_{i_1 i_2} + J_{Ci_2} F_{i_2} T_{i_2 i_2}$$

$$H_3 = \dot{J}_{Cu_1} + J_{Ci_1} F_{i_1} T_{i_1 u_1} + J_{Ci_2} F_{i_2} T_{i_2 u_1},$$

$$H_4 = J_{Ci_1} F_{i_1} T_{i_1 u_2} + J_{Ci_2} F_{i_2} T_{i_2 u_2}$$

For the system state space equations shown as (14) or (15), giving the initial value of state variable vector X_{i_1}, X_{i_2}, the constraint force vector U_2 can be obtained from (14c) or (15c). Thus (14a) and (14b) or (15a) and (15b) is a set of first order differential equations, many numerical solving algorithm that are available can be used. The corrected adaptive step size Runge-Kutta method based on MATLAB program[7] is emplored here.

IV. EXAMPLE SYSTEM

Figure.4 shows a spatial multibody system. The components for this example are three rigid bodies, two revolute joints J_1, J_2 and a prismatic joint J_3. These components are parameterized with the following data: $m_1 = m_2 = 1\,Kg$ are the mass of the rigid body m_1 and m_2, $I_{xx} = I_{yy} = I_{zz} = 4.167e\text{-}4Kgm^2$ are the principal moment of inertia of m_1 and m_2. $F_1 = F_2 = -9.8N$ are the weight of m_1 and m_2. $M = 0.1\ N \cdot m$ is a moment imposed to J_1, $K = 500N/m$ is spring stfffness, a=0.3m, b=0.1m, c=0.15m are the distances shown in Figure.4.

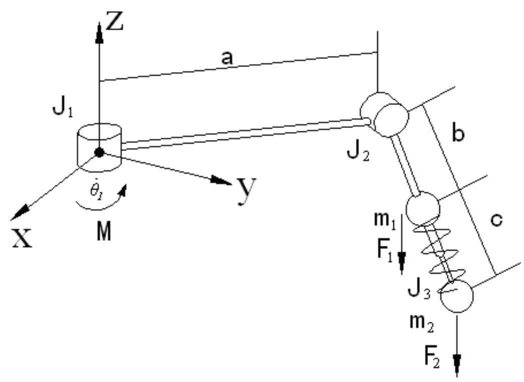

Figure.4 Spatial multibody system

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For revolute joint, three translations and two rotational degrees of freedom are constrainted, leaving only one Rotation degree of freedom free. From this constraint condition, its vector bond graph can be obtained. By assembling the vector bond graph of a single space moving rigid body, the revolute joints, and the prismatc joint, the overal system vector bond graph model can be obtained and shown as Figure.5. The constraint force vectors of joints can be considered as unknown source vectors, such as Se_2, Se_5, Se_7, Se_{10} in Figure.5, and added to the corresponding 0-junctions to eliminate differential causality. As a result, all differential causalities in this system vector bond graph can be eliminated, thus the procedure presented here can be used.

In Figure.5, $J_{1z} = [\mathrm{I_{ZZ}}]$, $J_2^b = diag[\mathrm{I_{XX}} \quad \mathrm{I_{YY}} \quad \mathrm{I_{ZZ}}]$, $J_3^b = diag[\mathrm{I_{XX}} \quad \mathrm{I_{YY}} \quad \mathrm{I_{ZZ}}]$. $\dot{r}_{C_2}$, $\dot{r}_{C_3}$ are the mass center velocity vector of body m_1 and m_2 in global coordinates, ω_2^b, ω_3^b are the angular velocity vector of body m_1 and m_2 in body frame respectively.

Inputting the initial values of state variable vector, the physical parameters of the mechanism, and the coefficent matrices of (5)~(13) into the program associated with the procedure presented here based on MATLAB[7], the system responses and the resultant constraint forces at joints are obtained and shown in Figure.6~Figure.9. These results are good agreement with that obtained by the procedure in [5].

V. CONCLUSIONS

A general procedure was presented for using vector bond graph to model spatial multibody systems with prismatc joint. Compared with standard scalar bond graph model, the procedure presented here is more concise. Because nonlinear junction structure and differential causality exit in shch complex systems, current vector bond graph methods are found to be very difficult algebraically in the derivation of system state space equations. The constraint force vectors of joints can be considered as unknown source vectors, and added to the corresponding 0-junctions to eliminate differential causality. In the case of considering EJS, the unified formulae of system state space equations and constraint forces at joints are derived, which are easily derived on a computer and very suitable for spatial multibody systems. These lead to a more efficient and practical automated procedure for modelling and simulation of complex spatial multibody systems over a multi-enegy domains in a unified manner.

ACKNOWLEDGMENT

This work was financially supported by National Natural Science Foundation of China (Grant No. 51175272).

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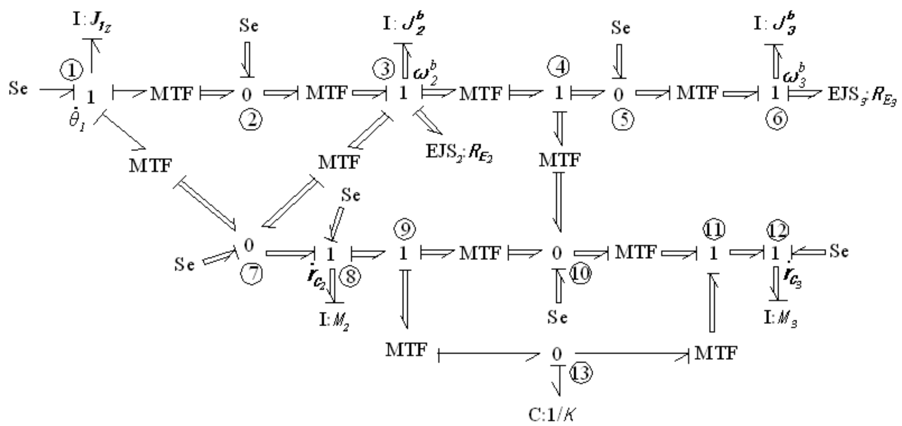

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Figure.5 Vector bond graph model of system

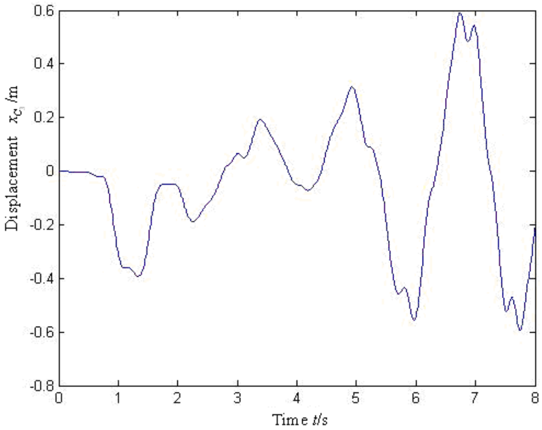

Figure.6 Displacement of m_2 in global X-axis

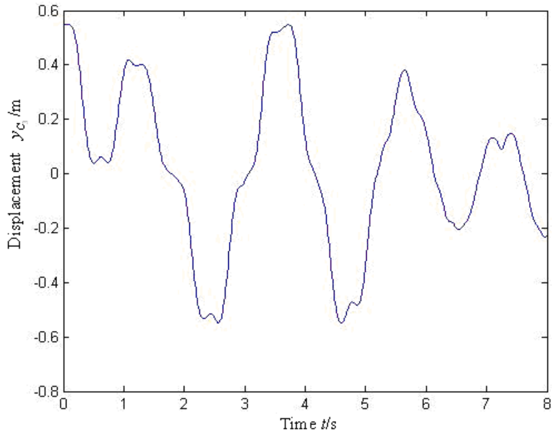

Figure.7 Displacement of m_2 in global Y-axis

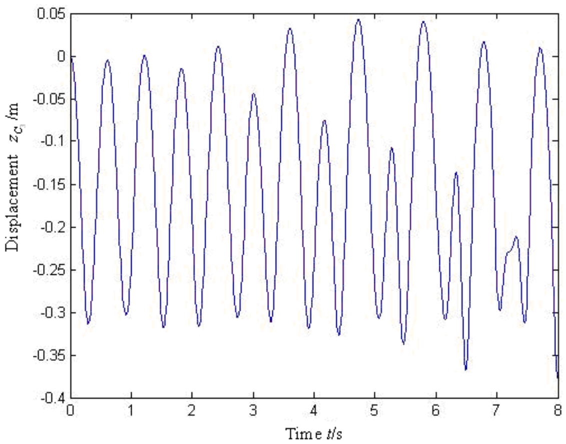

Figure.8 Displacement of m_2 in global Z-axis

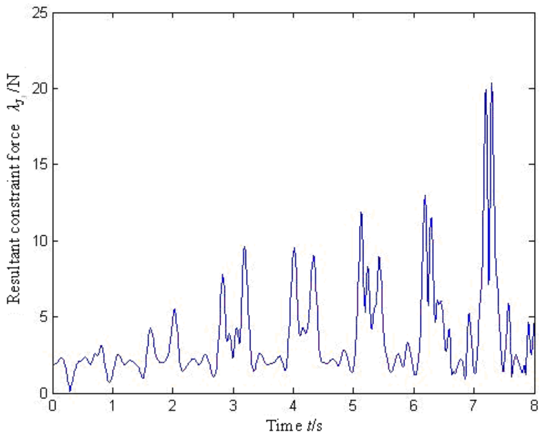

Figure.9 Resultant constraint force at joint J_3

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