Methods in Solving the Wave Equations for A Loudspeaker

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Abstract—**This paper introduces several methods in solving the wave equations for a loudspeaker. The separation of variables method is very common for solving constant coefficient linear partial differential problems. The finite difference method has been the main method of numerical computation in early work. The direct method is to formulate the boundary integral equation and can be solved using a direct time integration procedure. There are also some transform methods such as Fourier, Laplaces and wavelet transforms etc. to solve the wave equations. The retarded potential technique to solve the wave problems numerically was first used in the early 1960s. Since then, many authors have worked on the method and its applications.**

Keywords- separation of variables, finite difference,integral, transform, retarded potential, loudspeaker.

I. INTRODUCTION

The purpose of this paper is mainly to deal with the numerical methods for solving the wave equation and in the future apply those methods to model the sound radiated by loudspeakers. The main concentration is going to be focused on methods to model transient sound fields, which are the Fourier series method and the retarded potential technique. Furthermore, this work extends the analysis of the retarded potential integral equation into a numerical form which can be easily used to analyse the stability of the problem. A method is developed to help overcome any problems of instability.

In the following sections, we will discuss the relevant literature of different methods of solving the wave equation and its applications.

II. SEPARATION OF VARIABLES

The separation of variables method is very common for solving constant coefficient linear partial differential problems in one dimension or in the spherical or cylindrical domain (Chester [1], Carrier [2]).

Jódar and Almenar [3] extended this method to the time dependent coefficient and proved the existence of the solution and constructed a continuous numerical solution,

although it was only suitable for the one dimensional wave equation.

The method is easy to perform, but hard to apply to a complicated geometry. However, it can give us the exact

solution in the case where a structure has a simple shape (sphere or cylinder), so that we can compare our numerical results with it to check the accuracy of the numerical methods.

III. FINITE DIFFERENCE METHOD

The finite difference method has been the main method of numerical computation in early work. In this method, the time derivative is approximated by a finite difference
form and the equation is solved in time step by step.

The method has been applied to transient and harmonic wave problems. Mitchell and Griffiths [4], Schedin et al.[5] showed some processes in discretising the space and time for the wave equations. An explicit or implicit finite difference scheme could be chosen according to the calculation requirement.

The boundary condition is easily incorporated into the finite difference method, but it is slower to compute and occupies much memory. A small time step and fine grids are usually required to meet the constraints of accuracy, stability and minimum grid dispersion.

Villarreal and Scales [6] used the parallel processing techniques to speed up the calculations and showed the efficiency of the algorithm.

IV. DIRECT INTEGRAL METHOD

The direct method is to formulate the boundary integral equation and derive the ordinary differential equations (usually 2nd-order) for the displacement vector field,
which can be solved using a direct time integration procedure.

Nardini and Brebbia [7] presented the alternative formulation to the parabolic or hyperbolic partial differential equation, which led to a system of ordinary differential equations with constant coefficients which may be easily solved using a direct time integration procedure. They showed that the method had the advantage that it reduced the problem to a boundary-only formulation in a simple manner. However, the accuracy was sacrificed while dramatically reducing the necessary boundary integrations.

Tanaka et al. [8] dealt with the direct integral method for analysing the quasistatic problem in coupled thermoelasticity. The numerical procedure enabled the
authors to treat the coupled thermoelastic problems as an uncoupled one. The accuracy and versatility of the proposed

method were demonstrated in examples to compare with analytical results.

Brebbia and Nardini [9] described how a transient dynamic problem in boundary elements can be reduced to a set of linear time-dependent differential equations in a elastodynamic problems through the formulation of a system mass matrix, which can be expressed as a function of the boundary nodes only. This approach required that the integrations were only carried out on the boundary thus considerably reducing the subsequent computations.

Gaul and Wenzel [10] presented the hybrid boundary element method for the calculation of acoustic fields in compressible fluids in the time domain. The state variables were separated into domain variables and boundary variables, which were approximated by piecewise polynomials. The time step solutions can then be calculated by conventional time integration schemes.

The success of the method is strongly dependent on the choice of the coordinate functions, which are used to approximate the time derivatives of the solution in the time domain [7].

V. TRANSFORM METHODS

A. Fourier and Laplace Transforms

Some authors use the Fourier or Laplace transforms, which replace time derivatives with new variables and hence simplify the differential equations. Then the inverse transformation is performed numerically to obtain the solution in real space. The approach focuses on solving the Helmholtz equation rather than the original time dependent problems.

Duddeck [11] worked on the principle of the generalization of the boundary element method by spatial and temporal Fourier transform. The examples were given for the heat conduction, elasticity, plates, waves, thermoelasticity etc. For the actual calculation a rigorous distributional representation of the boundary element method was developed.

Geaves et al. [12] described the Laplace and Fourier transforms in dealing with the transient structural wave propagation in loudspeakers. The approach was verified by comparing the laser measured result with the simulated result.

The transform method involves solving a large number of separate problems in the frequency domain to obtain a reasonable degree of accuracy. This thesis intends to show that working in the time domain will be more efficient.

B. Wavelet Transform

The wavelet transform method is more efficient compared with other direct iterative methods and it has some advantages such as a lighter computational burden. It is an expansion of compactly supported functions.

Hong and Kennett [13] introduced a wavelet-based method for acoustic and elastic wave equations for the simulation of wave propagation. They demonstrated the way in which the scheme can be used when the physical parameters were varying with depth or distance. They

illustrated elastic wave propagation in a homogeneous medium with sinusoidal topography at the free surface and in a stochastic heterogeneous medium with a fluid-filled crack.

Although the wavelet transform method has some advantages, its engineering application is still quite new.

C. Wave Envelope

The wave envelope approach can be regarded as a modification of the infinite element concept. It is closely related to the time-harmonic problem. The discretisation is almost independent of the wavelength.

Astley et al. [14], [15] and Li [16] used infinite wave envelope elements combined with conventional finite elements to model the transient wave equation in unbounded regions. The method was based on a Fourier transform of the wave envelope approach applied to steady wave problems and was valid over a full range of excitation frequencies. The transient formulation was derived by taking an inverse Fourier transform of the discrete time harmonic problem to a mapped wave envelope formulation in the frequency domain, which can be solved using any suitable implicit or explicit scheme. Cylindrical or spherical polar symmetry were applied to the model solutions to show the close agreement with the analytic results.

The wave envelope method permits the use of very large elements which are much larger than a wavelength. It requires the high order integration of the element matrices.

VI. RETARDED POTENTIAL TECHNIQUE

The retarded potential integral equation (RPIE) to solve the wave problems numerically was first used in the early 1960s. Since then, many authors have worked on the method and its applications. The collocation and Galerkin methods are two common approaches used to discretise the resulting integral equation.

A. Collocation Method

Mitzner [17] and Farn and Huang [18] discussed the scattered field for a given incident wave for a hard boundary. They provided Courant-Friedrichs-Levy(CFL) conditions and obtained the simplified iterative equation. The validity and practicality of the retarded potential technique have been verified by applying it to the problem of scattering from a sphere. Comparison was made against results of known high accuracy obtained by a combination of separation of variables and the numerical Fourier transformation. The agreement was extremely good.

Davies [19] presented a stability analysis of a mixed finite element approximation of the time dependent electric field integral equation. The analysis showed that the solution had fast growing unstable modes which were due to a combination of the finite element basis functions and the integral quadrature rule. These modes cannot be removed by a standard time averaging method, but can be eliminated by averaging in space appropriately. Averaging the computed current in time and space at each time step resulted in an algorithm that was stable on a coarse mesh and did not degrade the solution.

The collocation method is easy to perform and simple to understand, but the stability is its problem.

B. Galerkin Method

Ha-Duong [20], [21], [22], Filipe et al. [23] formed the retarded potential equation into a matrix operator equation. The properties for the integral operator such as the existence and uniqueness were deduced from associated harmonic problems with complex frequencies. The solution could be obtained by the Galerkin approximation. The piecewise constant functions in space as in time were used to discretise the equation.

Ding et al. [24] obtained a variational formulation of the space-time integral equation from an energy identity, leading to a Galerkin-type numerical procedure. By choosing an appropriate basis for the discretised functional space, this scheme was of the marching-in-time type with matrix coefficients. The piecewise-constant elements used were sufficient to give better numerical results than the collocation methods where polynomial elements of degree two were used. This technique was applied to a sphere, a box and dihedron targets, and the solution remained stable for a long time interval.

The Galerkin method is complicated to implement, but it has better stability than the collocation method, therefore it is utilised by many authors.

VII. SUMMARY

There are a number of ways to solve the wave equation, and each method has its advantages and disadvantages.

For the simple structure such as a sphere or cylinder, the separation of variables method is inevitably a robust way to obtain an analytical solution. Of the numerical schemes, the finite difference method is one of the easiest to understand. However, its stability and how to apply it to unbounded domains are issues that need to be considered. Similar problems arise with the FEM. The direct integral method can reduce the necessary boundary integrations, but it is not very easy to choose the appropriate basis functions. Transform methods, such as the Fourier or Laplace transforms, can be applied to the time component of the wave equation. If a Fourier transform method is used, the time-dependent wave equation is replaced by a sequence of Helmholtz equations, each of which can be solved using the boundary integral method. An alternative is to use a direct boundary integral type representation of the solution to the wave equation. The most commonly used technique is the retarded potential integral equation (RPIE).

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