Sensitivity analysis of parameter variation in T network impedance-matching

Jianwen Tan [1, 2], Sijian Deng[1], Fangwei Ye[1], Chunliang Zhao[1], Deping Zeng[1]

1. Department of Biomedical Engineering
Chongqing Medical University
Chongqing, China

2. Key Lab of Special Power Supply
Chongqing Communication College
Chongqing, China

***Abstract*—In the practical application of passive network impedance matching, variations in component tolerance, process, and load impedance may make the performance of impedance-matching network deviate from the designed or simulated result. The current work investigates the effect of these parameter variations on performance of T network impedance matching. The sensitivities of the resulting reflection coefficient to the parameter variations are theoretically studied. The formulas for calculating the resulting reflection coefficient caused by parameter variations are derived from quality factor-based design method. The analysis results can provide reference for design process and an opportunity for a better understanding of the dynamic behavior of the narrowband impedance-matching networks.**

Keywords-Impedance matching; Circuit analysis; Sensitivity analysis; Parameter variation; Component tolerance

I. INTRODUCTION

Impedance matching using a passive network is very important in the design of RF and microwave circuits in order to achieve maximum power transfer, minimum reflection, and adequate harmonic rejection [1–2]. Passive components such as capacitors and inductors play a key role in determining the overall characteristics of impedance-matching networks [3–4]. Thanks to nominal value tolerances, parasitic and manufacturing process variations, the performance of the impedance-matching network is likely to deviate from the results predicted by designs and simulations that consider only ideal component values [5–6]. For example, due to process variation, capacitance can vary up to ±20% for a metal–insulator–metal capacitor [7]. In addition to component variation, variation of load impedance in applications such as the piezoelectric ultrasonic transducers in high-intensity focused ultrasound (HIFU) therapy systems [8] can also cause deviation from perfect matches.

In cases where various parameter variations exist, the sensitivity of the impedance-matching network to parameter variation should be greatly considered. In the aspect of quality control, lower sensitivity to parameter variation means better quality [9]. Hence, investigating and comparing the sensitivity of the commonly used impedance-matching networks to parameter variation is of utmost significance.

The variability quality of L network impedance matching respective to load impedance variation was analyzed by Chung [9]. Chen and Weber [5] presented a process variation-insensitive network with matched passive components. Sun and Fidler discussed component tolerance and parasitic sensitivities of frequency response of Pi network in [10]. However, the most important performance of impedance-matching networks, in many applications, is the resulting reflection coefficient, which represents the effect of impedance matching. Therefore, studying the sensitivity of the reflection coefficient to parameter variation in impedance-matching networks is necessary.

The parameter variation problem can usually be approached through two methods: 1) statistical analysis using electronic design automation software tools capable of Monte Carlo analysis and 2) sensitivity analysis based on sensitivity functions derived from a given circuit configuration [11]. In the present work, parameter variation problems due to component tolerance and process variation in T network are addressed using sensitivity analysis based on Q-based design method. Unlike the analysis in [10], the reflection coefficient is used as the performance parameter of impedance-matching networks.

II. Q-BASED DESIGN METHOD OF T IMPEDANCE-MATCHING NETWORK

The user-friendly Q-based design method is often adopted in designing impedance-matching networks for its simplicity [12]. In order to theoretically analyze the sensitivity and variability of network, the Q-based design process was introduced. The circuit of T network impedance matching is given in Fig. 1. For simplicity, the source impedance and load impedance were considered as pure resistance, denoted by R_1 and R_2, respectively.

Defining $Q_1=X_{L1}/R_1$, $Q_2=X_{L2}/R_2$, $k=R_1/R_2$, the loaded quality factor Q_0 can be defined as $Q_0 = (Q_1+Q_2)/2$. The design process is as follows [1]:

a. An appropriate Q_0 for the network that meets the designable condition is selected:

$$Q_0 \geq Q_{0(\min)} = \frac{1}{2}\sqrt{k-1} \quad \text{for } R_1 \geq R_2 \ (k \geq 1) \text{ and} \tag{1}$$

$$Q_0 \geq Q_{0(\min)} = \frac{1}{2}\sqrt{1/k-1} \quad \text{for } R_1 \leq R_2 \ (k \leq 1)\,; \tag{2}$$

b. Q_1 and Q_2 are calculated using Eqs. 3 and 4 ($k\neq1$):

$$Q_1 = \frac{2Q_0 - \sqrt{4kQ_0^2 - (k-1)^2}}{1-k} \text{ and} \tag{3}$$

$$Q_2 = \frac{2kQ_0 - \sqrt{4kQ_0^2 - (k-1)^2}}{k-1}\,; \tag{4}$$

c. Components L_1, L_2, and C of the network are calculated using Eqs. 5–7:

$$L_1 = \frac{R_1 Q_1}{\omega_0}, \qquad (5)$$

$$L_2 = \frac{R_2 Q_2}{\omega_0} \text{ and} \qquad (6)$$

$$C = \frac{2Q_0}{\omega_0 R_1 (1+Q_1^2)} = \frac{2Q_0}{\omega_0 R_2 (1+Q_2^2)}, \qquad (7)$$

where ω_0 is the matching angular frequency.

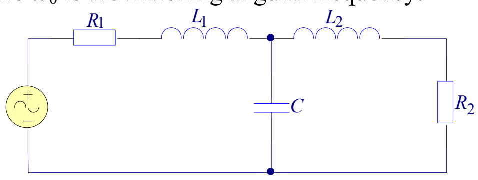

Figure 1. T network impedance matching

III. Analysis of Component Variation Effect on the T Network

Considering the T network in Fig. 1, the resulting reflection coefficient Γ can be obtained by

$$\Gamma = \frac{Z_e - R_1}{Z_e + R_1} = \frac{Z_e - kR_2}{Z_e + kR_2}, \qquad (8)$$

where Z_e is the equivalent impedance of matched load at working frequency, which can be demonstrated as

$$Z_e = j\omega_0 L_1 + \cfrac{1}{j\omega_0 C + \cfrac{1}{j\omega_0 L_2 + R_2}} \qquad (9)$$

For the ideal condition, the equivalent impedance Z_e at working frequency is designed to be equal to R_1 by the T network.

A. Variation of inductor L_1

For a small change of ΔL_1, the inductance of L_1 in T network is given by

$$L_1' = L_1 + \Delta L_1 = L_1(1 + dL_1) \qquad (10)$$

where L_1 is determined by Eq. 5 and $dL_1 = \Delta L_1 / L_1$, the relative change of inductor L_1.

The equivalent impedance of matched load at working frequency is

$$Z_e' = Z_e + j\omega_0 \Delta L_1 = R_1 + j\omega_0 L_1 dL_1 = R_1 + jR_1 Q_1 dL_1 \quad (11)$$

The changed reflection coefficient Γ' can be obtained by

$$\Gamma' = \frac{Z_e' - R_1}{Z_e' + R_1} = \frac{j\omega_0 \Delta L_1}{2R_1 + j\omega_0 \Delta L_1} = \frac{jQ_1 dL_1}{jQ_1 dL_1 + 2} \qquad (12)$$

where $dL_1 = \Delta L_1 / L_1$ is the relative change of inductor L_1.

Using the expression

$$S_x^F = \frac{dF}{dx/x} \qquad (13)$$

as the semi-relative sensitivity of a function $F(x)$ to variations of a variable x [13], the semi-relative sensitivity of reflection coefficient to variation of inductor L_1 in T network can be obtained by

$$S_{L_1}^{\Gamma} = \left.\frac{\Gamma'}{dL_1}\right|_{dL_1 \to 0} = \frac{jQ_1}{2} \qquad (14)$$

Eq. 14 shows that the magnitude of resulting reflection coefficient caused by variation of inductor L_1 in the T network is proportional to the relative change dL_1 and quality factor Q_1.

B. Variation of inductor L_2

Taking into account a small change of ΔL_2 in Fig. 1, the inductance of L_2 is given by

$$L_2' = L_2 + \Delta L_2 = L_2(1 + dL_2) \qquad (15)$$

where L_2 is determined by Eq. 6 and $dL_2 = \Delta L_2 / L_2$ is the relative change of inductor L_2.

Substituting the changed L_2' into Eq. 6, the equivalent impedance of matched load at working frequency is

$$Z_e' = j\omega_0 L_1 + \cfrac{1}{j\omega_0 C + \cfrac{1}{j\omega_0 L_2' + R_2}} \qquad (16)$$

Substituting Eq. 15 and Eqs. 5–7 into Eq. 16, the changed reflection coefficient Γ' caused by variation of L_2 can be obtained by

$$\Gamma = \frac{Z_e' - R_1}{Z_e' + R_1} = -\frac{(k-1)^2 + j(4kQ_0 - p(k+1))}{(k-1)^2 - j(4kQ_0 - p(k+1))} \cdot \frac{j\omega_0 \Delta L_{21}}{j\omega_0 \Delta L_{21} + 2R_2} \qquad (17)$$
$$= -\frac{A + jB}{A - jB} \cdot \frac{jQ_2 dL_2}{2 + jQ_2 dL_2}$$

where p=sqrt $(4kQ_0^2-(k-1)^2)$, $A = (k-1)^2$ and $B = 4kQ_0 - p(k+1)$.

The semi-relative sensitivity of reflection coefficient to variation of inductor L_2 in T network can be derived from

$$S_{L_2}^{\Gamma} = \left.\frac{\Gamma'}{dL_2}\right|_{dL_2 \to 0} = -\frac{A + jB}{A - jB} \cdot \frac{jQ_2}{2} \qquad (18)$$

Eq. 18 illustrates that the magnitude of resulting reflection coefficient caused by variation of inductor L_2 in T network is proportional to the relative change dL_2 and quality factor Q_2.

C. Variation of capacitor C

For the ideal condition where the passive components are the designed ideal value, the reflection coefficient in front of the capacitor is zero. Such condition indicates that the equivalent load impedance looking into the front of the capacitor C is equal to R_1-$j\omega_0 L_1$ at working frequency.

With regard to the small change of ΔC in Fig. 1, the inductance of C is given by

$$C' = C + \Delta C = C(1 + dC) \qquad (19)$$

where C is determined by Eq. 7 and $dC = \Delta C / C$ is the relative change of inductor C.

The changed equivalent impedance of matched load at working frequency is

$$Ze' = j\omega_0 L_1 + \cfrac{1}{j\omega_0 \Delta C + \cfrac{1}{R_1 - j\omega_0 L_1}} \qquad (20)$$

Substituting Eqs. 5 and 7 into Eq. 20, Eq. 8 expresses that the resulting reflection coefficient due to variation of capacitor leads to

$$\Gamma' = \frac{(Q_1 + j)Q_0 dC}{(-Q_1 + j)(-Q_0 dC + j)} \qquad (21)$$

The semi-relative sensitivity of reflection coefficient to variation of inductor C in T network can be obtained by

$$S_C^{\Gamma} = \left.\frac{\Gamma'}{dC}\right|_{dC\to 0} = \frac{(Q_1 + j)Q_0}{(-Q_1 + j)j} \qquad (22)$$

Eq. 22 shows that the magnitude of resulting reflection coefficient caused by variation of capacitor in T network is proportional to the relative change dC and quality factor Q_0.

IV. ANALYSIS OF LOAD IMPEDANCE VARIATION EFFECT ON THE T NETWORK

To analyze the resulting reflection coefficient variation caused by the change in total load impedance, the load reflection coefficient Γ_l should be used. Based on Fig. 1 the reflection coefficient of the side load is Γ_l. Note that the function of the impedance matching network is to make the resulting reflection coefficient Γ zero.

The change in the load impedance causes the resulting reflection coefficient to deviate from zero. For a small change of Γ_d, the changed load reflection coefficient is given by

$$\Gamma_l' = \Gamma_l + \Gamma_d \quad \Gamma_l' \leq 1 \qquad (23)$$

and the changed load impedance is

$$Z_l' = \frac{1+\Gamma_l'}{1-\Gamma_l'} R_1 = \frac{1+\Gamma_l+\Gamma_d}{1-\Gamma_l-\Gamma_d} R_1 \qquad (24)$$

The small change in load reflection coefficient can make the equivalent impedance vary, which is defined by

$$Z_e' = j\omega_0 L_1 + \cfrac{1}{j\omega_0 C + \cfrac{1}{j\omega_0 L_2 + Z_l'}} \qquad (25)$$

Substituting Eqs. 5–7 and Eq. 24 into Eq. 25, the resulting reflection coefficient caused by load impedance variation can be obtained by Eq. 8 after simplification and factorization [14]:

$$\Gamma' = \frac{(Q_1 + j)(Q_2 + j)\Gamma_d}{(Q_1 - j)(Q_2 - j)\left(1 - \Gamma_l\Gamma_d - \Gamma_l^2\right)} \qquad (26)$$

The semi-relative sensitivity of reflection coefficient to variation of load impedance in the T network can be obtained by

$$S_{\Gamma_d}^{\Gamma} = \left.\frac{\Gamma'}{\Gamma_d/\Gamma_l}\right|_{\Gamma_d\to 0} = \frac{(Q_1 + j)(Q_2 + j)\Gamma_l}{(Q_1 - j)(Q_2 - j)\left(1 - \Gamma_l^2\right)} \qquad (27)$$

Eq. 27 shows that the magnitude of reflection coefficient cause by load variation in the T network is proportional to the relative change in load reflection coefficient and independent of quality factor.

V. DISCUSSIONS AND CONCLUSIONS

a) In practical application, the components deviation and load impedance variation can make the matching effect worse (i.e. make the reflection coefficient deviate from zero).

b) In the T network, the magnitude of resulting reflection coefficient caused by component variation is proportional to the relative change of component and quality factor. Therefore, the flexibility for Q-factor in the T network can optimize the sensitivity to components variation due to tolerance or process variation.

c) The magnitude of the resulting reflection coefficient caused by load impedance variation in T network is independent of quality factor and is proportional to the load reflection coefficient and relative change of load reflection coefficient. Hence, the sensitivity to load impedance variation is higher when the difference of source and load impedance is larger. The Q-factor in the T network can only affect the phase of resulting reflection coefficient.

d) In designing T network impedance matching, the quality factor can not be too high in order to ensure good dynamic stability. However, the harmonic rejection ability is proportional to quality factor. Therefore, it is need to make the tradeoff between dynamic stability and harmonic rejection ability.

This sensitivity analysis provides an opportunity for a better understanding of the dynamic behavior of the impedance-matching networks.

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