The Whole Trajectory Simulation of Cruise Missile Based on Inertial Navigation Error Model

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***Abstract*-For the problem of cruise missile's performance test in complex electromagnetic environment, the whole trajectory simulation of cruise missile is researched in this paper. First, we analyze the role played by inertial navigation system(INS) in the process of cruise missile attack. Then, we build error model of inertial navigation system. At last, we design the whole trajectory simulation platform of cruise missile, which integrates trajectory programming module, inertial navigation calculation module, trajectory simulation module, simulation control module, terminal guidance module, and so on. Our research in this paper is an important technology accumulation, which is highly significant for performance test of cruise missile in complex electromagnetic environment.**

Keyword-Strapdown Inertial Navigation System; Cruise missile; The whole trajectory simulation

I. INTRODUCTION

In order to fully test the performance of cruise missile in the battlefield electromagnetic environment, and to provide an important basis for equipments application, military training, existing equipments improvement and new equipments manufacture, etc, testing cruise missile’s combat performance in complex electromagnetic environment has great significance.

In fact, cruise missile's performance test in complex electromagnetic environment is a fully new field, a series of special projects research are necessary. This paper first analyzes the role played by SINS in the process of cruise missile attack, and then carries out the whole trajectory simulation of cruise missile based on the principle and error models of SINS, to provide support and make technology accumulation for missile's performance test in complex electromagnetic environment.

II. THE ROLE PLAYED BY INS IN THE PROCESS OF CRUISE MISSILE ATTACK

A. Attack process of cruise missile

Cruise missile is different from ballistic missile, it needs to program trajectory before launch, the entire flight will always be in guided flight status, power device and guidance system are always active in flight, so missile can fly along scheduled trajectory and attack target accurately. The whole attack process is divided into the cruise segment and terminal guidance. Cruise segment is from launch to autocontrol endpoint, then missile searches the target and attack it using terminal guidance system. The whole cruise segment mainly uses high-precision INS and topographic matching, and corrects inertial navigation information by satellite navigation to improve the navigation accuracy. Terminal guidance mainly uses infrared imaging matching and missile-borne manned control loop.

B. The role played by INS

INS uses object's inertial characteristics to guide. It has many advantages, such as free from outside interference, all-weather, high reliability, low development costs, short development cycle, mature technology. Composite guidance scheme that based on high-precision INS and using satellite navigation, terrain matching, scene matching to correct is widely used by many powerful military countries[1].

INS is the basis of cruise missile guidance. It is usually composed of a gyro-stabilized platform, inertial instruments (such as accelerometers), program devices, computers, integrators, and multipliers. The basic principle of its work: using accelerometers to measure the acceleration of aircraft, using integrators to fetch aircraft's speed and location information, so the spatial location of the aircraft in a moment could be determined, then comparing it with scheduled theoretical flight trajectory, if they have deviation, the error signal will generate to control aircraft fly along the theoretical trajectory.

The real trajectory of cruise missile can be understood as: trajectory programming and inertial error. Therefore, the whole trajectory simulation of cruise missile needs to build error model of INS first.

III. ERROR MODEL OF INS

Because of components error, initial condition error, interference error etc, the navigation orientation error of INS is increasing with time.

A. Basic error model

SINS is taken as an example to analyze the navigation error models of INS, which contain attitude angle error model, speed error model and location error model[2].

1) Attitude angle error model of INS

When taking geographic coordinate system as navigation coordinate system of INS, attitude angle error vector could be determined by formula (1).

$$\Phi = [\phi_n, \phi_u, \phi_e]^T \tag{1}$$

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ϕ_n is north attitude angle error, ϕ_u is sky attitude angle error, ϕ_e is east attitude angle error.

Attitude error equation of INS is defined by following formula (2).

$$\dot{\Phi} = \left(W_{ie}^e + W_{en}^e\right)\times\Phi + \delta W_{ie}^e + \delta W_{en}^e + \mathbf{C}_b^e \varepsilon^b \quad (2)$$

$\dot{\Phi}$ is the rate of attitude error equation change.

$$\dot{\Phi} = \left[\dot{\phi}_n, \dot{\phi}_u, \dot{\phi}_e\right]^T \quad (3)$$

W_{ie}^e is defined as follows:

$$W_{ie}^e = \left[\omega_{ie}\cos\varphi, \omega_{ie}\sin\varphi, 0\right]^T \quad (4)$$

ω_{ie} is rotational angular velocity of the earth, φ is the latitude of missile's location now.

W_{en}^e is defined as follows:

$$W_{en}^e = \left[\frac{v_e}{R_e}, \frac{v_e}{R_e}\tan\varphi, -\frac{v_n}{R_n}\right] \quad (5)$$

v_n is north speed, and v_e is east speed in navigation coordinate system of INS. R_e is radius of curvature in prime vertical. R_n is radius of curvature in meridian.

$\boldsymbol{\varepsilon}^b$ is drift error of gyroscope.

$$\varepsilon^b = \left[\varepsilon_x, \varepsilon_y, \varepsilon_z\right]^T \quad (6)$$

ε_x, ε_y, ε_z are drift errors of gyroscope in x-axis, y-axis, z-axis of missile coordinate system separately.

C_b^e is conversion matrix from missile coordinate system to inertial coordinate system.

$$C_b^e = \begin{pmatrix} \cos\gamma\cos\psi + \sin\gamma\sin\psi\sin\theta & \sin\psi\cos\theta & \sin\gamma\cos\psi - \cos\gamma\sin\psi\sin\theta \\ -\cos\gamma\sin\psi + \sin\gamma\cos\psi\sin\vartheta & \cos\psi\cos\theta & -\sin\gamma\sin\psi - \cos\gamma\cos\psi\sin\theta \\ -\sin\gamma\cos\theta & \sin\theta & \cos\gamma\cos\theta \end{pmatrix} \quad (7)$$

ψ, γ, θ are azimuth angle, roll angle, pitch angle of missile coordinate system relative to inertial coordinate system separately.

δW_{ie}^e, δW_{en}^e are total derivatives of W_{ie}^e, W_{en}^e relative to the latitude φ and the speed v_n, v_e of missile now separately.

2) Speed error model of INS

Speed error equation of INS is defined by following formula (8).

$$\Delta\dot{V} = F^e \times \Phi - (2W_{ie}^e + W_{en}^e)\times\Delta V - (2W_{ie}^e + W_{en}^e)\times V^e + \mathbf{C}_b^e \nabla^b \quad (8)$$

ΔV is speed error of INS.

$$\Delta V = \left[\Delta v_n, \Delta v_u, \Delta v_e\right]^T \quad (9)$$

Δv_n is north speed error, Δv_u is sky speed error, Δv_e is east speed error.

$\Delta\dot{V}$ is the rate of speed error change.

$$\Delta\dot{V} = \left[\Delta\dot{v}_n, \Delta\dot{v}_u, \Delta\dot{v}_e\right]^T \quad (10)$$

$\Delta\dot{v}_n$ is the rate of north speed error change. $\Delta\dot{v}_u$ is the rate of sky speed error change. $\Delta\dot{v}_e$ is the rate of east speed error change.

V^e is the speed of carrier.

$$V = \left[v_n, v_u, v_e\right]^T \quad (11)$$

v_n is north speed, v_u is sky speed, v_e is east speed.

F^e is the stress of INS, it could be measured by accelerometer.

$$F^e = \left[f_n, f_u, f_e\right]^T \quad (12)$$

f_n, f_u, f_e are north stress, sky stress and east stress separately.

∇^b is zero bias vector of three-axis accelerometer.

$$\nabla^b = \left[\nabla_x, \nabla_y, \nabla_z\right]^T \quad (13)$$

∇_x, ∇_y, ∇_z are zero bias vectors of three-axis accelerometer separately in missile coordinate system.

3) Location error model of INS[3]

Location equation of INS is defined by following formula (14).

$$\begin{cases} \dot{\lambda} = \dfrac{v_e}{R_e\cos\varphi} \\ \dot{\varphi} = \dfrac{v_n}{R_n} \\ \dot{h} = v_u \end{cases} \quad (14)$$

λ, h are the longitude and the height of missile's location now.

$$\begin{cases} \Delta\dot{\lambda} = \dfrac{\Delta v_e}{R_e\cos\varphi} + \dfrac{v_e\Delta\varphi}{R_e\cos\varphi}\tan\varphi \\ \Delta\dot{\varphi} = \dfrac{\Delta v_n}{R_n} \\ \Delta\dot{h} = \Delta v_u \end{cases} \quad (15)$$

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B. Error model of INS

Error state variable $\mathbf{X}_{ins}$ of SINS error model is defined by following formula (16).

$$\mathbf{X}_{ins}=[\,\Delta v_n,\Delta v_u,\Delta v_e,\phi_n,\phi_u,\phi_e,\Delta\varphi,\Delta\lambda,\Delta h,\nabla_x,\nabla_y,\nabla_z,\varepsilon_x,\varepsilon_y,\varepsilon_z\,]^{\mathrm{T}}_{15\times1} \quad (16)$$

Δv_n, Δv_u, Δv_e are north speed error, sky speed error and east speed error separately. ϕ_n, ϕ_u, ϕ_e are attitude error angles. $\Delta\varphi$, $\Delta\lambda$, Δh are latitude error, longitude error and height error separately. ∇_x, ∇_y, ∇_z are zero bias vectors of three-axis accelerometer. ε_x, ε_y, ε_z are drift errors of three-axis gyroscope.

In order to simplify the problem, we assume that zero bias vector of accelerometer and drift error of gyroscope are both random constants. Error model of INS could be determined by formula (17).

$$\dot{\mathbf{X}}_{ins}(t)=H_{ins}(t)\mathbf{X}_{ins}(t)+\mathbf{G}_{ins}(t)\mathbf{W}_{ins}(t) \quad (17)$$

$\mathbf{G}_{ins}(t)$ is defined by following formula (18).

$$\mathbf{G}_{ins}(t)=\begin{bmatrix}\mathbf{I}_6\\ \mathbf{O}_{9\times6}\end{bmatrix} \quad (18)$$

I_6 is 6×6 unit matrix, $O_{9\times6}$ is 9×6 zero matrix. $\mathbf{W}_{ins}(t)$ in formula (8) is modeling noise. $\mathbf{W}_{ins}(t)=[w_{v_n},w_{v_u},w_{v_e},w_{\phi_n},w_{\phi_u},w_{\phi_e}]^T$. $H_{ins}(t)$ is the state transition matrix of system.

$$H_{ins}=\begin{bmatrix}\mathbf{A}&\mathbf{B}&\mathbf{C}&\mathbf{C_b^e}&\mathbf{O}\\ \mathbf{G}&\mathbf{I}&\mathbf{J}&\mathbf{O}&\mathbf{C_b^e}\\ \mathbf{K}&\mathbf{O}&\mathbf{L}&\mathbf{O}&\mathbf{O}\\ \mathbf{O}&\mathbf{O}&\mathbf{O}&\mathbf{O}&\mathbf{O}\\ \mathbf{O}&\mathbf{O}&\mathbf{O}&\mathbf{O}&\mathbf{O}\end{bmatrix}_{15\times15} \quad (19)$$

$\mathbf{O}$ is 3×3 zero matrix. A、B、C、G、I、J、K、L could be determined by following formulas.

$$\mathbf{A}=\begin{bmatrix}-\frac{v_u}{R_n}&-\frac{v_n}{R_n}&-(2\omega_{ie}+\frac{v_e}{R_e\cos\varphi})\sin\varphi\\ \frac{2v_n}{R_n}&0&(2\omega_{ie}+\frac{v_e}{R_e\cos\varphi})\cos\varphi\\ (2\omega_{ie}+\frac{v_e}{R_e\cos\varphi})\sin\varphi&-(2\omega_{ie}+\frac{v_e}{R_e\cos\varphi})\cos\varphi&\frac{v_n}{R_n}\tan\varphi-\frac{v_u}{R_e}\end{bmatrix} \quad (20)$$

$$\mathbf{B}=\begin{bmatrix}0&-f_e&f_u\\ f_e&0&-f_n\\ -f_u&f_n&0\end{bmatrix} \quad (21)$$

$$\mathbf{C}=\begin{bmatrix}-(2\omega_{ie}\cos\varphi+\frac{v_e}{R_e}\sec^2\varphi)v_e&0&0\\ -2v_e\omega_{ie}\sin\varphi&0&0\\ (2\omega_{ie}\cos\varphi+\frac{v_e}{R_e}\sec^2\varphi)v_n+2v_u\omega_{ie}\sin\varphi&0&0\end{bmatrix} \quad (22)$$

$$\mathbf{G}=\begin{bmatrix}0&0&\frac{1}{R_e}\\ 0&0&\frac{\tan\varphi}{R_e}\\ -\frac{1}{R_n}&0&0\end{bmatrix} \quad (23)$$

$$\mathbf{I}=\begin{bmatrix}0&-\frac{v_n}{R_n}&-(\omega_{ie}+\frac{v_n}{R_n})\sin\varphi\\ \frac{v_n}{R_n}&0&(\omega_{ie}+\frac{v_n}{R_n})\cos\varphi\\ (\omega_{ie}+\frac{v_n}{R_n})\sin\varphi&-(\omega_{ie}+\frac{v_n}{R_n})\cos\varphi&0\end{bmatrix} \quad (24)$$

$$\mathbf{J}=\begin{bmatrix}-\omega_{ie}\sin\varphi&0&0\\ \omega_{ie}\cos\varphi+\frac{v_e}{R_e}\sec^2\varphi&0&0\\ 0&0&0\end{bmatrix} \quad (25)$$

$$\mathbf{K}=\begin{bmatrix}\frac{1}{R_n}&0&0\\ 0&0&\frac{1}{R_e\cos\varphi}\\ 0&1&0\end{bmatrix} \quad (26)$$

$$\mathbf{L}=\begin{bmatrix}0&0&0\\ \frac{v_e\tan\varphi}{R_e\cos\varphi}&0&0\\ 0&0&0\end{bmatrix} \quad (27)$$

IV. THE WHOLE TRAJECTORY SIMULATION PLATFORM OF CRUISE MISSILE

The simulation platform is based on Microsoft Visual Studio 2005 development environment. It contains trajectory programming module, inertial navigation calculation module, trajectory simulation module, simulation control module, terminal guidance module, etc. The composition of simulation platform is shown in Figure 1(a). Figure 1(b) shows the working flow of simulation system. Figure 2 shows the interface of simulation system.

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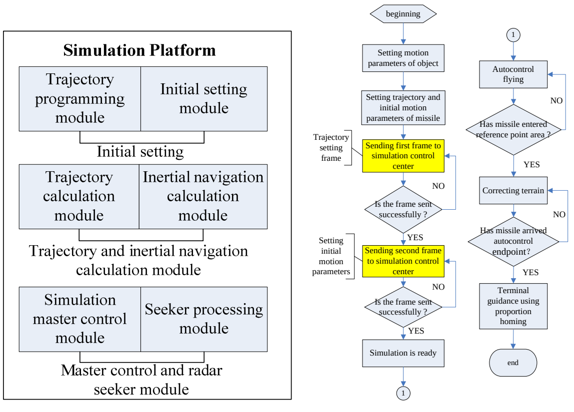


(a) (b)

Figure 1. The composition and working flow of simulation system

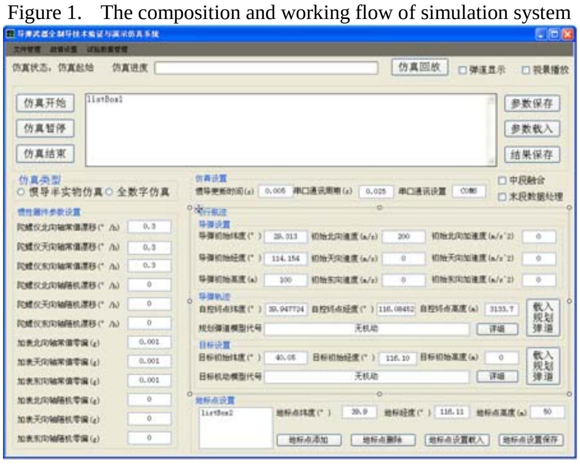


Figure 2. The interface of simulation system

Figure 3 shows trajectory programming and initial setting. Trajectory programming can be composed of 9 trajectories: uniform, climb, acceleration (deceleration), horizontal bank, vertical turn, WR motor, S maneuver, dive, terminal guidance.

Figure 3. Trajectory programming and initial setting

V. SIMULATION TEST

The typical trajectory of indirect attack mode is shown in Figure 4, the initial parameters of missile are as follows: location($40^{\circ}, 116^{\circ}, 100m$), speed($100m/s$, $0m/s$, $0m/s$), acceleration($0m/s^2$, $0m/s^2$, $0m/s^2$), attitude(0° , 0° , 0°). The meaning of every parameter is that: location (longitude, latitude and height), speed (north speed, sky speed, east speed), acceleration (north acceleration, sky acceleration, east acceleration), attitude (heading, roll, pitch).

The total flight time is $144s$, the autocontrol endpoint is (40.3622° , 115.9459° , $277.2542m$). Figure 4 shows the whole trajectory simulation result without considering the inertial error.

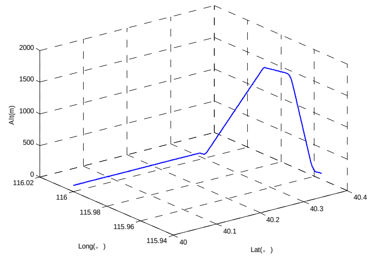
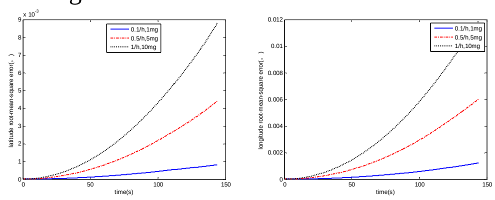


Figure 4. Typical trajectory of indirect attack mode

According to the typical trajectory in figure 4, we make 100 times Monte Carlo simulation, navigation location errors of different inertial navigation accuracy are shown in figure 5. Figure 5 (a) shows the root-mean-square of latitude errors changing with time. And figure 5 (b) shows the root-mean-square of longitude errors.



(a) (b)

Figure 5. Navigation location errors in typical trajectory of indirect attack mode

Because of the influence of gravity acceleration, guidance height is generally provided by altimeters to improve the accuracy.

VI. CONCLUSION

In this paper, we first analyze the role played by INS in the process of cruise missile attack. Then, we build error model of INS according to its principle. Last, we design the whole trajectory simulation platform of cruise missile. Our research not only provides support for missile's performance test in complex electromagnetic environments, but also is an important technology accumulation for further work.

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