Two New Calculating Methods for the Irregular Hermite Interpolation Polynomial

Miao Luo

School of Computer Science and Technology
Panzhihua University
Panzhihua, China
e-mail: 4597686@qq.com

Liangliang Ma

School of Computer Science and Technology
Panzhihua University
Panzhihua, China
e-mail: mll198684@126.com

Abstract—**Based on the theory of the regular Hermite interpolation polynomial, two new calculating methods including undetermined coefficient and Lagrange interpolation have been proposed to solve the complex irregular Hermite interpolation polynomial.**

Keywords- Hermite interpolation polynomial; undetermined coefficient; Lagrange interpolation

I. THEORY OF THE HERMITE INTERPOLATION

Set $y = f(x)$ has definition in interval $[a,b]$, and has corresponding values $y_0, y_1, \cdots, y_n$ on the points of $a \le x_0 < x_1 < \cdots < x_n \le b$, if there is a simple function $P(x)$ satisfied the following formula:

$$P(x_i) = y_i \qquad (i = 1, 2, \cdots, n) \qquad (1)$$

where $P(x)$ is the interpolation function of $f(x)$, $x_0, x_1, \cdots, x_n$ are the interpolation nodes, interval $[a,b]$ is the interpolation interval[1].

If $P(x)$ is the polynomial with the number not more than n, namely

$$P(x) = a_0 + a_1 x + \cdots + a_n x_n \qquad (2)$$

where $a_i (i = 0,1,\cdots,n)$ are the real numbers, we can say $P(x)$ is a interpolation polynomial.

Suppose $y_j = f(x_j), m_j = f'(x_j) (j = 0,1,\cdots,n)$ on the nodes $a \le x_0 < x_1 < \cdots < x_n \le b$, if there is an interpolation polynomial $H(x)$ satisfied the conditions (3):

$$H(x_j) = y_j, H'(x_j) = m_j \qquad (j = 0,1,\cdots,n) \qquad (3)$$

then $H(x)$ is the Hermite interpolation polynomial[2].

If y_i and $m_j (i = 0,1,\cdots,n)$ are all known on the nods $a \le x_0 < x_1 < \cdots < x_n \le b$, we can say $H(x)$ is the regular Hermite interpolation polynomial, correspondingly, the methods of seeking $H(x)$ is the regular Heremite interpolation ; If only some of $m_j (i = 0,1,\cdots,n)$ are known on the nodes $a \le x_0 < x_1 < \cdots < x_n \le b$, we can

say $H(x)$ is the irregular Hermite interpolation polynomial, the methods of seeking $H(x)$ is the irregular Heremite interpolation [3-4].

II. TWO PLANS FOR SOLVING THE IRREGULAR HEREMITE INTERPOLATION POLYNOMIAL

If $y_i \ (i = 0,1,\cdots,n)$ and $m_j \ (j = 0,1,\cdots,k; k < n)$ are all known on the points $a \le x_0 < x_1 < \cdots < x_n \le b$, and $m_j \ (j = k+1,\cdots,n)$ are unknown. In this paper, we want to seek an irregular Heremite interpolation polynomial to meet the following equations:

$$\begin{cases} H(x_i) = y_i & (i = 0,1,\cdots,n) \\ H'(x_j) = m_j & (j = 0,1,\cdots,k; k < n) \end{cases} \qquad (4)$$

A. Undetermined coefficient

Bring the known conditions into the irregular Hermite interpolation polynomial, and to determine the required irregular Hermite interpolation polynomial through solving equations to determine its coefficients.

By equations (4), we can know that the equations own $n+k+2$ known conditions. Due to $n+k+2$ known conditions can uniquely determine a polynomial with the number not more than $n+k+1$, namely

$$H_{n+k+1}(x) = H(x) \qquad (5)$$

so we can suppose the form of $H_{n+k+1}(x)$ as follows:

$$H_{n+k+1}(x) = a_0 + a_1 x + \cdots + a_{n+k+1} x^{n+k+1} \qquad (6)$$

Bring the unknown conditions $H(x_i) = y_i \ (i = 0,1,\cdots,n)$ and $H'(x_j) = m_j \ (j = 0,1,\cdots,k; k < n)$ into the formula (6), we can get the following equations:

$$\begin{cases} a_0 + a_1 x_0 + \cdots + a_{n+k+1} x_0^{n+k+1} = y_0 \\ a_0 + a_1 x_1 + \cdots + a_{n+k+1} x_1^{n+k+1} = y_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_0 + a_1 x_n + \cdots + a_{n+k+1} x_n^{n+k+1} = y_n \\ a_1 + 2a_2 x_0 + \cdots + (n+k+1)a_{n+k+1} x_0^{n+k} = m_0 \\ a_1 + 2a_2 x_1 + \cdots + (n+k+1)a_{n+k+1} x_1^{n+k} = m_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_1 + 2a_2 x_k + \cdots + (n+k+1)a_{n+k+1} x_k^{n+k} = m_k \end{cases} \qquad (7)$$

Solving equations (7), we can obtain the coefficients $a_0, a_1, \cdots, a_{n+k+1}$, and determine $H_{n+k+1}(x)$ finally.

B. Lagrange interpolation

Using the known conditions $H(x_i) = y_i \quad (i = 0, 1, \cdots, n)$, we can seek a Lagrange interpolation polynomial $L_n(x)$. Suppose $H_{n+k+1}(x)$ exists, and bring the known conditions $H(x_i) = y_i \quad (i = 0, 1, \cdots, n)$ and $H'(x_j) = m_j \quad (j = 0, 1, \cdots, k; k < n)$ into $H_{n+k+1}(x)$, and we can determine the required irregular Hermite interpolation polynomial.

Through $H(x_i) = y_i \quad (i = 0, 1, \cdots, n)$, we can seek the Lagrange interpolation polynomial $L_n(x)$:

$$L_n(x) = \sum_{i=0}^{n} y_i l_i(x) \qquad (8)$$

where

$$l_i(x) = \frac{(x - x_0) \cdots (x - x_{i-1})(x - x_{i+1}) \cdots (x - x_n)}{(x_i - x_0) \cdots (x_i - x_{i-1})(x_i - x_{i+1}) \cdots (x_i - x_n)}$$
$$(i = 0, 1, \cdots, n) \qquad (9)$$

Due to $l_i(x)(i = 0, 1, \cdots, n)$ satisfy the following conditions on the nodes $x_0 < x_1 < \cdots < x_n$

$$l_i(x_j) = \begin{cases} 1, & i = j, \\ 0, & i \neq j \end{cases} \qquad (i, j = 0, 1, \cdots, n) \quad (10)$$

namely $l_0(x), l_1(x), \cdots, l_n(x)$ are the interpolation basis functions on the nodes $x_0, x_1, \cdots, x_n$. By formula (8), we can obtain the expression (11) and (12):

$$L_n(x_j) = \sum_{i=0}^{n} y_i l_i(x_j) = y_j \quad (j = 0, 1, \cdots, n) \qquad (11)$$

$$L'_n(x_j) = \sum_{i=0}^{n} y_i l'_i(x_j) = m_j \quad (j = 0, 1, \cdots, n) \qquad (12)$$

Suppose $\quad H_{n+k+1}(x) = L_n(x) + Q_{n+k+1}(x) \qquad (13)$

Set the highest coefficients of $Q_{n+k+1}(x)$ as 1, and bring (x_0, y_0) into (13), through $L_n(x_0) = y_0$, we can obtain

$$Q_{n+k+1}(x_0) = 0 \qquad (14)$$

namely x_0 is one of the zero point of $Q_{n+k+1}(x)$. Then bring $(x_1, y_1) \ (x_0, y_0)$ into (13), through $L_n(x_1) = y_1$, we can obtain

$$Q_{n+k+1}(x_1) = 0 \qquad (15)$$

namely x_1 is one of the zero point of $Q_{n+k+1}(x)$. Similarly,

bring $(x_2, y_2), (x_3, y_3), \cdots, (x_n, y_n)$ into (13), through

$$L_n(x_j) = \sum_{i=0}^{n} y_i l_i(x_j) = y_j \quad (j = 2, 3, \cdots, n) \text{, we can}$$

obtain

$$Q_{n+k+1}(x_j) = 0 \quad (j = 2, 3, \cdots, n) \qquad (16)$$

namely $x_j \ (j = 2, 3, \cdots, n)$ are the zero points of $Q_{n+k+1}(x)$. Because the number of $Q_{n+k+1}(x)$ is $n + k + 1$, so we can suppose

$$Q_{n+k+1}(x) = (x - x_0)(x - x_1) \cdots (x - x_n) A_d(x) \quad (17)$$

where $A_d(x)$ is a polynomial with the number of d.

Through $H_{n+k+1}(x) = L_n(x) + Q_{n+k+1}(x)$, we can obtain

$$H'_{n+k+1}(x) = L'_n(x) + Q'_{n+k+1}(x) \qquad (18)$$

bring $H'(x_j) = m_j \quad (j = 0, 1, \cdots, k; k < n)$ into (17),

through $L'_n(x_j) = \sum_{i=0}^{n} y_i l'_i(x_j) = m_j \quad (j = 0, 1, \cdots, k; k < n)$,

we can obtain

$$Q'_{n+k+1}(x_j) = 0 \ (j = 0, 1, \cdots, k; k < n) \qquad (19)$$

namely $x_j (j = 0, 1, \cdots, k; k < n)$ are the double roots of $Q_{n+k+1}(x)$, so

$$Q_{n+k+1}(x) = (x - x_0)^2 (x - x_1)^2 \cdots (x - x_k)^2 (x - x_{k+1}) \cdots$$
$$(x - x_n) B(x) \qquad (20)$$

Owning to the number of $Q_{n+k+1}(x)$ is $n + k + 1$, and the number of the polynomial

$$(x - x_0)^2 (x - x_1)^2 \cdots (x - x_k)^2 (x - x_{k+1}) \cdots (x - x_n) \quad (21)$$

is $n + k + 2$, so the number of $B(x)$ is -1, namely

$$B(x) = x^{-1}$$

To sum up, $H_{n+k+1}(x)$ can be uniquely determined by $L_n(x)$ and $Q_{n+k+1}(x)$.

REFERENCES

[1] Yang Shijun, and Wang Xinghua, "Expressions of Hermite interpolation polynomials as divided differences with applications," Appl. Math. J. Chinese Univ. Ser. A, vol. 21, pp. 70-78, January 2006.

[2] Wang Ziyu, Shi Yingguang, Tian Jishan and Liang XueZhang, "Some derelopment and question of polynomials interpolation (1)," Journal of Baoji College of Arts and Sciences (Natural Science), vol. 2, Febuary 1963, pp. 1-5.

[3] Deng Yongchang, "Existence and uniqueness theorem of Hermite interp0Iati0n polynomial," JournaL of Gansu University of Technology, vol.6, pp.1-5, Febuary 1995.

[4] Li Qignyang, Wang Nengyi, and Yi Dayi, Numerical Analysis. Wuhan: Huazhong University of science and technology press, 2006.