The Method of Positioning Error Measuring Compensation of the Multiple-Target Motion Simulation System Based on Neural Network

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Abstract—In order to solve the problem of large range of target motion and targets mutual interference in Two-dimensional Cross-Moving Motion Simulation System with Multiple Objects, a multi-objective system is designed, which consists of a number of independent two-dimensional motion simulation systems, and an error measuring compensation program is proposed, which can meet the accuracy requirements. Threedimensional position coordinates of the space measured feature points are got by two theodolites which is based on triangular intersectional measuring method and the experiment is designed by the best measurement area which is obtained through the uncertainty analysis. The position accuracy of the measured feature points can be determined by simulation. The movement reference coordinate system can be set up according to the three-dimensional position data of the measured feature points and the spatial position error can be calculated by coordinate system transformation. RBF neural network is used to establish the error compensation model which is finally validated. Experimental results of actual system indicate that the program can be well applied to the Two-dimensional Motion Simulation System for spatial position error measurement and compensation. The accuracy requirements of the system can be satisfied after compensation.

Keywords-multiple objects; two-dimensional cross-moving motion; triangle intersectional measurement; RBF neural network; spatial position error compensation

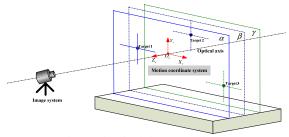
I. Introduction

The target motion simulation system can provide comprehensive test and physical simulation for the subjects through reproducing the trajectory of the target and simulating the motion features of the target. A twodimensional multiple-target motion simulation system is composed of several two-dimensional single target motion systems. And this system can not only provide the motion simulation of multiple targets, but also resolve problems such as the small scope of target motion, independent motion regions of the targets, etc. In view of these difficulties of high positioning precision for the target points in the space and the unification of multiple-target motion reference coordinate system, a set of methods of error measuring compensation are designed in this paper, which are based on the triangular intersectional vision measurement principle and the RBF neural network theory.

The paper, at first, studies the triangular intersectional measurement method, and makes a model of measurement

method with two theodolites. Some factors affecting measurement precision are analyzed and the designed experiment was simulated with MATLAB to determine whether the precision of the three-dimensional coordinates is satisfied with the requirements. Then several feature points of which the ideal positions were known in the motion plane of each target are chose, and distributed reasonably on the whole motion plane of the target. The target motion reference coordinate was set up by acquiring all the threedimensional coordinates of all the feature points and the error of the system was deduced by using the concept of coordinate system transformation. Finally, the RBF neural network was used to build error compensation model and check the model. And then the model was implanted into the control software of the host computer to compensate for the spatial position errors of the system. After the compensation, the multiple-target system has a unified motion reference coordinate system and a high precision of positioning.

II. THE ANALYSIS OF THE SOURCE OF THE SYSTEMATIC SPATIAL ERROR



 $Fig. 1\ Motion\ simulation\ system\ diagram$

The principle of the two-dimensional multiple-target motion simulation system is shown in Figure 1. The system consists of several independent two-dimensional single target motion systems and the targets have their own independent motion planes which parallels to each other. Therefore the problem of the mutual interference with targets' motion when these targets are designed on a same motion plane can be resolved. Moreover, the structure is convenient to increase or decrease the number of motion targets, offering the system a strong extendibility.

The major problem of this structure, which is unavoidable, was that the nonlinear error caused by the transmission or implementation mechanism, results in the disunity of motion coordinate systems of multiple targets and the precision cannot reach the requirements. To ensure that

the multiple motion targets on the image plane of the imaging system has the same two-dimensional motion reference coordinate system, this paper researched a set of error measuring compensation methods that meet the required precision by combining the triangular intersectional vision measurement principle and the RBF neural network theory.

III. THE ERROR MEASURING METHOD OF THE SYSTEMATIC SPATIAL POSITION

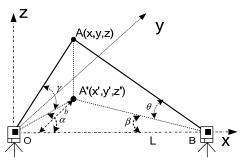


Fig.2 Triangle intersectional measuring principle diagram

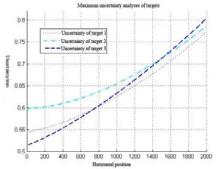


Fig.3 Measuring uncertainty simulation

Two theodolites are used to measure the three-dimensional coordinate of the spatial point with the triangular intersectional method^[1]. The measuring principle is shown in Figure 2.

In the figure, two theodolites whose optical axes of measurement at the same line are put at point O and point B respectively. Supposed that A(x, y, z) is a point in the measured space, and A'(x', y', z') is the projective point of point A on the XOY plane in the coordinate space for measurement, according to the triangular sine theorem, the measuring elements (L, the spacing of the theodolites, and α, β, γ , measured spatial azimuths) and the spatial coordinate of the measured point A(x, y, z) satisfy:

$$\begin{cases} x = OA'\cos\alpha = \frac{L\sin\beta\cos\alpha}{\sin(\alpha+\beta)} \\ y = OA'\sin\alpha = \frac{L\sin\beta\sin\alpha}{\sin(\alpha+\beta)} \\ z = OA'\tan\gamma = \frac{L\sin\beta\tan\gamma}{\sin(\alpha+\beta)} \end{cases}$$
 (1)

In Formula (1), the ranges of variables are: $\alpha = 0 \sim 180^{\circ}$, $\beta = 0 \sim 180^{\circ}$, $\gamma = -90^{\circ} \sim +90^{\circ}$. The Analysis of Factors Affecting Precision [2]

The uncertain degrees of the measured horizontal and vertical angles in the two theodolites A, B are defined as $U_{\alpha}, U_{\beta}, U_{\gamma}, U_{\theta}$ respectively, the measuring uncertain degree of baseline L as U_L , the uncertain degree of the spatial feature point A(x,y,z) as U_A , and the measuring uncertain degrees of coordinate components as U_x, U_y, U_z . As $U_{\alpha}, U_{\gamma}, U_{\beta}, U_{\theta}, U_L$ are mutually unrelated, so U_x, U_y, U_z , and normally they satisfy $U_{\alpha} = U_{\beta} = U_{\gamma}$. It can be acquired by seeking partial derivatives of the three equations in Formula (1) and based on the principle of uncertain degree combination that:

$$U_A = \sqrt{U_x^2 + U_y^2 + U_z^2} = \sqrt{e_1^2 U_\alpha^2 + e_2^2 U_L^2}$$
 (2)

In which:

$$\begin{cases} e_1^2 = \frac{\sin^2 \alpha + \sin^2 \beta \cdot \left[1 + \sin^2 (\alpha + \beta) \cdot (\tan^2 \gamma + \cos^2 \gamma)\right]}{\sin^4 (\alpha + \beta) \cos^2 \gamma} L^2 \\ e_2^2 = \frac{x^2 + y^2 + z^2}{L^2} = \frac{\sin^2 \beta \cdot (1 + \tan^2 \gamma)}{\sin^2 (\alpha + \beta)} \end{cases}$$

Supposed that the precision of the theodolite is 2", the uncertain degree of the measuring angle of the theodolite will be $U_{\alpha} = U_{\beta} = U_{\gamma} = 2'' = 2\pi/(3600 \times 180) rad$; the uncertain U_L of L can be as $U_L = L \cdot U_{\alpha} = 2\pi L/(3600 \times 180)$. As known from Formula (2), if other measuring factors are unchanged, the systematic error is maximal when γ reaches the maximum. Therefore, to judge the feasibility of the measuring method, this system only need to draw the uncertain degrees of position measurement of the measured feature points when they are distributed at the top or the bottom of the target motion scope. With the simulation analysis of MATLAB, the uncertain degrees of the three targets are acquired as shown in Figure 3. Obviously, the measuring uncertain degrees of the three targets are all less than 0.8mm, satisfying the measuring precision requirements of the measured system.

IV. THE CALCULATING METHOD OF THE SYSTEMATIC SPATIAL POSITION ERROR

The motion reference coordinate system can be built by using the three-dimensional coordinates of the measured feature points in the former section. According to the coordinate system transformation principle, the spatial position error of the measured point can be acquired by transforming the three-dimensional coordinates of all measured feature points in the measuring coordinate system into the image plane coordinate system with the medium of motion reference system.

A. The Establishment of the Motion Reference Coordinate System

When the system is in motion, the choice of coordinate system is very important, and the given quantity of the target position is closely related to the two-dimensional motion reference coordinate system defined by the system. Therefore, this system needs to choose an optimal reference plane of the target motion to establish the systematic motion reference coordinate system and transform the three-dimensional position coordinates of the measured feature points in the measurement coordinate system into the above-mentioned coordinate system. To ensure the reliability and precision of the acquired two-dimensional target motion reference plane, this paper takes the following steps:

- (1) Set the minimal number n_{\min} of the three-dimensional coordinate points necessary to fit the plane and the maximum permissible error u_{\max} , to ensure the acquired plane is reliable and requires the precision.
- (2) Acquire the target motion plane by using the regression analysis according to the three-dimensional coordinates of the measured points.
- (3) Acquire the distance from the measured points to the fitting plane, and remove some coordinate points which are far from the plane on the premise of ensuring the remaining number of coordinate points $n > n_{\min}$.
- (4) Refit the target motion plane by using the remaining measured points.
- (5) Acquire the uncertain degree u_{δ} of the plane degree error of the refitting plane to judge whether it is within the maximum permissible error scope of the fitting plane. If $u_{\delta} > u_{\text{max}}$, then repeat step (3), (4) and (5).
- (6) Choose a plane using more measured points and having smaller uncertain degree u_{δ} as the two-dimensional target motion reference plane from several fitting planes. In order to fit a line l, choose some points with smaller distance to the plane. Then define the intersection of the seeker or the optical axis of the other precision instrument and the fitting plane as the origin O_c of the coordinate system, in which the pointing direction of the directional vector of l serves as X-axis, and the pointing direction of the plane normal vector as Z-axis. According to the right hand rule, the systematic two-dimensional motion reference coordinate system $O_c X_c Y_c Z_c$ is established, of which plane XOY is the plane of this two-dimensional motion reference.

B. The Transformation of Coordinate System and the Calculation of the Spatial Position Errors

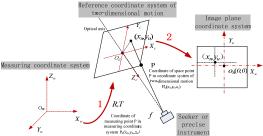


Fig.4 Coordinate transformation and error calculating

This system mainly achieves the control of twodimensional target motion and cannot directly compensate for the error which is vertical to the target motion plane. Therefore, the system needs to be transformed to the twodimensional motion reference plane. In this section, firstly, the measured three-dimensional position coordinate point is transformed into the two-dimensional motion reference coordinate system by rotation and translation. Then the error of the system is acquired in the way of projection, as shown in Figure 4.

(1) Transformation from the Measurement Coordinate System to the Two-dimensional Motion Reference Coordinate System

The coordinate of the spatial point P in the measuring coordinate system is set to be $P_{\scriptscriptstyle w}(x_{\scriptscriptstyle w},y_{\scriptscriptstyle w,}z_{\scriptscriptstyle w})$, and its coordinate in the two-dimensional motion reference coordinate system to be $P_{\scriptscriptstyle c}(x_{\scriptscriptstyle c},y_{\scriptscriptstyle c},z_{\scriptscriptstyle c})$. Then the transformation relationship is as follows:

$$P_{w} = R \cdot P_{c} + T \tag{3}$$

In which: R is the rotation matrix and T is the translation vector.

The unit normal vector of the fitting two-dimensional motion reference plane in the measuring coordinate system $O_w X_w Y_w Z_w$ which can be acquired through the former step is n_z , the unit directional vector of the fitting line l is n_x . Then through the cross product, the unit vector of Y-axis is acquired as:

$$n_{v} = n_{x} \times n_{z} \tag{4}$$

Thus the rotation matrix is constructed:

$$R = [n_x, n_y, n_z] \tag{5}$$

The position vector of the two-dimensional motion reference coordinate system O_w is set to be:

$$O = (o_x, o_y, o_z)^T \tag{6}$$

Then the vector T is translated to satisfy T = O. Through these, the coordinate transformation relationship of the spatial point P from the measuring coordinate system to the two-dimensional motion reference coordinate system is:

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = [n_x, n_y, n_z] \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix}$$
 (7)

(2) The Transformation from the Two-dimensional Motion Reference Coordinate System to the Image Plane Coordinate System

The plane equation of the two-dimensional target motion reference plane is set to be:

$$Ax + By + Cz + D = 0 ag{8}$$

The coordinate of the point projective on the plane, which is the spatial point $P_c(x_c,y_c,z_c)$ in the two-dimensional motion reference coordinate system, was $P_s(x_s,y_s,z_s)$. According to the projection principle from spatial point to the plane, it is acquired that:

$$\begin{cases} \frac{x_c - x_s}{A} = \frac{y_c - y_s}{B} = \frac{z_c - z_s}{C} \\ Ax_t + By_t + Cz_t + D = 0 \end{cases}$$
 (9)

Through Formula (9), it can be acquired that the position of the spatial point $P_c(x_c, y_c, z_c)$ in the two-dimensional motion reference coordinate system and its imaging position (x_u, y_u) in the image plane coordinate system satisfy:

$$\begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} x_s \\ y_s \end{bmatrix} = W \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} + U$$
 (10)

In which:

$$W = \frac{1}{A^2 + B^2 + C^2} \begin{bmatrix} B^2 + C^2 & -AB & -AC \\ -AB & A^2 + C^2 & -BC \end{bmatrix},$$

$$U = -\frac{D}{A^2 + B^2 + C^2} \begin{bmatrix} A \\ B \end{bmatrix}$$

Through Formula (7) and Formula (10), it could be acquired that the relationship between a spatial feature point $P_w(x_w, y_w, z_w)$ in the measuring coordinate system and its coordinate $P_u(x_u, y_u)^T$ in the image plane coordinate system can be expressed as:

$$\begin{bmatrix} x_u \\ y_u \end{bmatrix} = W \cdot R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + W \cdot T + U \tag{11}$$

The given position coordinate of the spatial point P is set to be $P_t(x_t, y_t)$, then the position error $E_t(e_{tx}, e_{ty})$ of point P satisfy:

$$\begin{bmatrix} e_{tx} \\ e_{ty} \end{bmatrix} = \begin{bmatrix} x_t \\ y_t \end{bmatrix} - \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} x_t \\ y_t \end{bmatrix} - W \cdot R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + W \cdot T + U \quad (12)$$

Through Formula (12), the position error of the sampling points in the motion reference plane of each target can be acquired respectively.

V. THE ERROR COMPENSATING METHOD OF THE SYSTEMATIC SPATIAL POSITION

Through measuring the actual system, it was found that the acquired spatial position error is highly nonlinear and

hard to be described by accurate mathematic model, so it is difficult to conduct the accurate error compensation. The neural network has a strong ability of nonlinear mapping. Through learning, the neural network can precisely set the weights and the threshold matrix between the network structure and the neurons. Thus a nonlinear model reflecting the features of the error system can be acquired [3~4]. In this section, the compensation for the spatial position error of the multiple-target simulation system is realized by establishing error compensation model based on the RBF neural network and transforming it into the error compensation module in the controlling software of the host computer through the acquired spatial position error of the targets in the former section.

A. The Determination of the Error Compensation Model of the RBF Neural Network [4,5]

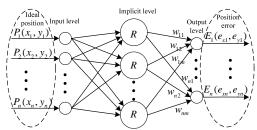


Fig.5 Error compensation model based on RBF neural network

The error compensation model based on the RBF neural network is shown in Figure 5. The input signal of this model is defined to be:

$$P = (P_1, P_2, \dots, P_n) \tag{13}$$

In which: $P_i = (x_i, y_i)$, x_i, y_i are respectively the theoretical position coordinates of the measured feature points numbered i, and the signal source nodes transmit these input signals to the hidden layer.

There are four kinds of commonly used basis functions in the hidden layer: multiple quadratic functions, thin spline functions, inverse multiple quadratic functions and Gauss functions. In this paper the most commonly used Gauss function is adopted as the basis function to realize the nonlinear mapping $P \rightarrow R_i(P)$. Gauss function is simple in expression forms, radial symmetry, good in smoothness and analyticity, and convenient in theoretical analysis. Its expression is shown as Formula (14):

$$R_i(P) = \exp\left[-\frac{1}{2}(\frac{\|P - c_i\|^2}{\sigma_i^2})\right], (i = 1, 2, \dots, q)$$
 (14)

In the formula: R_i is the output of the i-st hidden node; c_i is the center vector of the Gauss function of the i-st neuron in the hidden layer, which has the same dimensions with P; σ_i is the normalization constant of the i-st node in the hidden layer, i.e. the i-st perceived variable, which determines the width of the center vector of the basis function; q is the number of the perception units, i.e. the number of the nodes in the hidden layer; $\|P-c_i\|$ shows the distance between P and c_i .

The output of the hidden layer nodes ranges from 0 to

1, and the closer the input sample is to the node center, the greater the output value is. When $P = c_i$, $W_i = 1$.

The input signal of the model is define to be:

$$E = (E_1, E_2, \dots, E_n)$$
 (15)

In which: $E_i = (e_{xi}, e_{yi})$, e_{xi}, e_{yi} are respectively the spatial position errors of the measured feature point numbered i, then the linear mapping relationship of $R_i(P) \rightarrow E_k$ from the hidden layer to the output layer is:

$$E_k = \sum_{i=1}^q w_{ki} R_i(x), (k=1,2,\cdots,n) \tag{16}$$
 In which: w_{ki} is the connection weight between the *i-st* basis

function and the k-st output node, whose purpose is to solve the problem of unidentifiable sample timing by the network in the previous training samples of neural network. According to the rule of the "bigger near and smaller far", the weight factors give different training precision for samples of different timing in the process of training.

The Learning Process of the RBF Neural Network

In this model, c_i, σ_i and w_{ki} are unknown variables, which need to be determined through learning. The learning of the RBF neural network is mainly divided into two stages, i.e. the unsupervised learning stage and the supervised learning stage. At the first stage, c_i and σ_i are determined through input samples, while at the second stage, w_{ki} is acquired by using the least-squares principle after the determination of parameters in the hidden layer. The specific methods are as follows:

(1) The Unsupervised Learning Stage

K-means clustering algorithm is adopted to adjust the center vector, i.e. the center vector c_i of the optimal radial basis function is acquired through sub-families. The steps of the algorithm are as follows:

- 1) Set the initial center vector $c_i(0)$ of each hidden node, the learning rate $\beta(0)$ $(0 < \beta(0) < 1)$ and the threshold ε of the decision to stop calculation.
 - 2) Calculate the node with minimal distance.

$$\begin{cases}
d_i(k) = ||P_k - c_i(k-1)||, 1 \le i \le m \\
d_r(k) = \min d_i(k)
\end{cases}$$
(17)

In the formula, k is the sequent number of the sample; r is the sequent number of the hidden node in the case that the center vector $c_i(k-1)$ is nearest to the input sample distance P_{i} .

3) Adjust the center.

$$\begin{cases} c_i(k) = c_i(k-1), 1 \le i \le m, i \ne r \\ c_r(k) = c_r(k-1) + \beta(k) [X_k - c_r(k-1)] \end{cases}$$
(18)

the In learning rate $\beta(k) = \beta(k-1)/(1+\operatorname{int}(k/q))^{1/2}$; $\operatorname{int}(\cdot)$ indicates the rounding operation of (\cdot) .

4) Determine the quality of clustering.

Step 2) and 3) are repeated over all the samples k until

the following formula is satisfied.

$$J = \sum_{i=1}^{q} \left\| P_k - c_i(k) \right\|^2 \le \varepsilon$$
 (19) (2) Supervised Learning Stage

After c_i is determined, the acquisition of w_{ki} becomes the linear optimization problem and its learning algorithm is:

$$w_{ki}(k+1) = w_{ki}(k) + \eta(\tilde{Y}_k - Y_k)R_i(P)/R^TR$$
 (20)

In the formula, $R = [R_1(P), R_2(P), \dots W_n(P)]^T$, η is the

learning rate, normally whose value is $0 < \eta < 1$, \tilde{Y}_k refers to the actually measured error of the spatial position, i.e. the expected output, and Y_k refers to the value of the output error acquired by using this model. In this way, the compensation for the spatial position error of the target can be realized by using this model.

The Error Contrast of before and after the Compensation

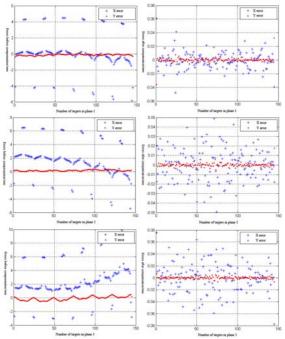


Fig.6 Error compared before and after compensation

Before the model is applied to the compensation for the simulation system of two-dimensional multiple-target motion, the compensation effect of the model needs to be tested to ensure that the compensation model satisfies the requirements of the system. In this section, one target is taken as an example to make the explanation. At first, the model of spatial error compensation of the target is acquired by randomly choosing 120 points from the 145 measured feature points of this target, taking their theoretic coordinate values as the input of the training neural network, and taking the value of the corresponding spatial position error as the output for training neural network. Then the spatial position errors of all the measured feature points after the compensation are acquired by utilizing the trained model. If the simulation system of two-dimensional multiple-target motion is supposed not to have controlling errors, and the measurement of feature points not to have measuring errors, then the compensation result after the compensation by using this method is shown in Figure 6 and the comparison of the errors of before and after the systematic compensation is listed in Table 1, which shows that after modeling for compensation by adopting the RBF neural network and on the premise of not considering the controlling error and the measuring error, the systematic error of the compensated part can be made less than 0.4mm, which is far less than the systematic error before the compensation. Therefore, the compensation effect is very obvious.

According to the above analysis, the model of error compensation satisfies the requirements of the system. Therefore, this model can be implanted into the control software of the host computer to realize compensation for the spatial position error of the simulation system of two-dimensional multiple-target motion through correcting the input quantity of the controller. Through measuring the compensated actual system, it can be acquired that when the range of single axis motion is greater than 2m, the maximum static positioning error of the system is less than 2mm, which satisfies the technical requirement.

TABLE I. ERROR COMPARED BEFORE AND AFTER COMPENSATION

Number of targets	The max error berofe compensation (mm)	The max error after compensati on (mm)	The average error before compensatio n (mm)	The average error after compensatio n (mm)
Target 1X	29.20	0.31	6.77	0.05
Target 1Y	1.95	0.18	0.69	0.01
Target 2X	32.61	0.25	11.35	0.07
Target 2Y	1.99	0.09	0.95	0.01
Target 3X	49.94	0.39	14.76	0.09
Target 3Y	2.79	0.03	1.13	0.01

VI. CONCLUSION

This paper has analyzed the features and the source of the spatial position errors in the two-dimensional multiple-target motion simulation system composed by several independent and mutually parallel two-dimensional single-target motion systems, then deigned a set of error measurement and compensation methods that can satisfy the precision of the system by combining the triangular intersectional vision measurement principle and the RBF neural network theory: the triangular intersectional measurement of points in space is realized by using two theodolites, and the spatial position error of the measured feature points is calculated through the principle of the coordinate system transformation; finally, by using the RBF neural network to establish the error compensation model and implanting the model into the control software of the host computer, compensation for the error of the actual system is realized. According to the result of the measurement in the actual system after compensation, the maximum static positioning error of the system is smaller than 3mm. Therefore, a unified two-dimensional motion coordinate system of multiple targets with high positioning precision for the multiple-target motion simulation system is realized when adopting the structure of which this paper describes.

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