

A Research on a Method of Error Separation Based on Wavelet Analysis

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Abstract—According to the error character of the systematic error and random error in calibration data, we put forward using wavelet analysis method to the process of separating error. We give the basic principle and method to dispose random error with Wavelet Analysis. When choosing the threshold, we use GCV(Generalized Cross Validation) principle. Then use IA to search the best value of GCV function and realize the separation simulation of random error in the MATLAB. During the selection operation of IA(Immune Algorithm), the expecting propagation probability is computed in order to guarantee the diversity of the colony and the robustness of algorithm. The results of simulation show that the combination of Wavelet Transform and IA can achieve the aim of separation and have some application prospect in this field.

Keywords- Wavelet Analysis, Immune Algorithm, GCV, error separation, systematic error, random error

I. INTRODUCTION

As one of the maintenance method for weapon system, calibration is an important activity to insure the high accuracy of the weapon. The periodic maintenance is needed during the weapon system using. Along with the increasing utilization time of the weapon, the systematic error will drift away from the norm. So, the calibration is done periodically or before a significant task. Radar is an information source of the weapon system and a part of measuring equipment. The accuracy of detecting target directly affects the shooting accuracy of the weapon. The radar calibration is the key part of weapon system calibration. How to process the calibration data is vital to the calibration work.

During the last decades, wavelets have become a popular tool in signal and image processing. Fundamentally, wavelets are a new type of function which provides an excellent orthonormal basis for functions of one or more variables. They provide a localized basis, and can represent square-integrable functions, but also constant and, more generally, polynomial functions in a locally finite manner. In this paper, we try use the wavelet analysis method to the separation of calibration data error. When choosing the threshold of wavelet, we decide to accept the GCV(Generalized Cross Validation) principle considering the unknown original signal in the practice and make use of IA(Immune Algorithm) to find the best value of GCV.

This paper is broken into the following sections. The introduction section gives a brief summary of the importance of weapon system calibration. In the second section we

formulate the problem. Then, sections are followed by a methodology for the separation of wavelet analysis, and the process for obtaining the optimal value of GCV through IA. In the last, a simulation example and the conclusion are given.

II. THE PRINCIPLE OF CALIBRATION

The aim of calibration is to measure and evaluate the systematic error of weapon, and then confirm the running state and eliminate the systematic error through an error compensation scheme. The basic method is that the calibrated equipment and high-accuracy device measure a target at the same time. While the high-accuracy device reaches 3 times the calibrated equipment, the result from the high-accuracy can be considered as the real value of the target. We can do the error analysis between them. Calibration data contains the real value and the measurement value. Subtracted the measurement value from the real value, the difference contains systematic error and random error. The mathematic model is as follow:

$$y_i = sr_i + rr_i, i = 1, 2 \dots n \quad (1)$$

In the above formula, y_i represents the total error, sr_i is the systematic error and the random error is represented by rr_i .

The first step of error analysis is to separate the random error from the total error and so we can obtain the systematic error. The prior information of original signal of the systematic error is often hard to achieve. How to do a separation of error should be solved firstly.

III. ERROR SEPARATION METHOD

Wavelets have been used broadly in signal and image processing, but the application in error analysis is rare. Guo[1] proposed the application in the error separation of telemetered data and the random error was separated effectively. Wang[2] did a research on the application of the wavelet analysis in the radar accuracy analysis, but didn't give the threshold of the wavelet. The frequencies of the systematic and random error have a distance so that the wavelet analysis method can apply to them. The process of applying wavelet analysis to the error separation is performed in three steps. The first step is to apply a discrete wavelet transform to the total error to produce decomposition coefficients. Next a threshold is applied to the coefficients. Then an inverse discrete wavelet transform is applied to these Modified coefficients.

A. The selection of threshold

Since Donoho and Johnstone proposed the wavelet shrinkage, many thresholds have appeared, such as, VisuShrink Threshold, SureShrink Threshold, HeurSure Threshold and MinMax Threshold. The regulation to evaluate the threshold choosing-strategies is MSE(Mean Square Error). It represents the deviation of the reconstructed signal from the original signal. But most of original signals are unknown to us. Because of this, Weyrich[3] made an application of GCV to the wavelet threshold. GCV is a function of the threshold, only based on the input data.

Maarten Janse[4] proved that the minimum of the GCV is an asymptotically optimal threshold. In other words, it is $N \rightarrow \infty, \arg \min GCV(\delta) = \arg \min MSE(\delta)$

The risk estimate function of GCV in the wavelet domain can be represented as

$$GCV(\delta) = \frac{N \|W - W_\delta\|^2}{N_0^2} \quad (2)$$

where W is the wavelet coefficients of the total error, W_δ represents the shrinkage wavelet coefficients, N_0 is the number of wavelet coefficients transacted to zero, N is the number of the total wavelet coefficients.

B. How to design the optimal level of wavelet

The random error in calibration data exhibits a high frequency form while systematic error has a low frequency one. A frequency interval exists between them. Yi[5] proposed a method to determine the optimal step of spline fitting. We are enlightened by them and give the optimal level of the wavelet decomposition. The formula is given by

$$G(L) = \left(\sum_{k=1}^n (y_k - \hat{y}_{kL}) \right)^2 \quad (3)$$

The formula has a meaning that it shows an approximate degree of wavelet fitting close to the low frequency of the total error. \hat{y}_{kL} represents the approximate of systematic error when the level of wavelet decomposition is L . That is to say, we can obtain \hat{y}_{kL} by the following method: make an L -level decomposition of the total error, then deal with the detail wavelet coefficients into zeros and do a reverse wavelet transform. When some frequency bands exist, $G(L)$ has the following trend:

$$\begin{cases} G(L) < G(L+1), 0 < L < L_1 \\ G(L_1) \approx G(L_1+1), L = L_1 \\ G(L) < G(L+1), L_1 < L < L_2 \\ G(L_2) \approx G(L_2+1), L = L_2 \\ \dots \end{cases} \quad (4)$$

The trend indicates that as the level increasing, the wavelet approximation coefficients are closer to the

systematic error and the wavelet detail coefficients are closer to the random error. Between level 1 and level L_1 , the wavelet approximation coefficients stepwise approach to the systematic error. And they haven't approach to the other parts. They approach to the random error after level L_1 . So we can decide the optimal level by computing and observing $G(L)$.

IV. THE APPLICATION OF IA

A. The base of IA

The biological immune system, which is composed of cells, molecules and organs with immune function, is an adaptive, distributed and parallel intelligent system that has the capability to control a complex system. There are two types of immunity, innate immunity and adaptive immunity. The innate immunity is natural resistance of the host to foreign antigens, while the adaptive immunity uses a specific immune response to antigens. Immune system has the ability of producing multiple antibody, self-adaptive organization and immune memory.

The IA(immune algorithm) which is a global optimal algorithm, is an adaptive system, inspired by theoretical immunology and observed immune functions, principles and models, which are applied to problem solving. In order to keep the adaptability and diversity of the antibodies, the immune algorithm stimulates the antibodies with higher affinity and lower density, and restrains the antibodies with lower affinity or higher density. Comparing the immune algorithm with the evolutionary algorithm, the diversity of the antibodies in the immune algorithm avoids the shortcomings of the premature phenomenon and slow converging speed, and improves the global search ability.

Generally, the immune algorithm is described as follows.

Step 1. Analyze the problem. There need to be an analysis to the problem and the character of the solution. Then design a suitable form of solutions.

Step 2. Initialize the antibody colony.

Step 3. Affinity calculation of the antibody.

Step 4. Form the farther-generation colony.

Step 5. Judge whether the end condition is met. If it is met, terminate. Otherwise go to the next one.

Step 6. Produce the new colony.

Step 7. Go to step 3.

B. The optimal threshold of GCV based on IA

Observing the GCV function, we conclude that choosing the best threshold of GCV can be equal to a complicated function optimal problem. Considering the simpleness and high efficiency of IA, IA is propose to solve choosing the optimal value. The antibodies are the needed threshold and the wavelet coefficients are treated as antigens. Choosing the optimal threshold is equal to searching the best antigen which has maximum affinity. The operations of IA are as follow:

1) Coding antibodies and initializing colony

The thresholds are some positive real number. In order to offer convenience to the immune operation, we accept the binary coding form. A random method is used to initialize the colony.

2) Affinity function

The affinity between the antibody and antigen represents the matching degree of them and the extent of solution to the problem. We make the GCV function as the affinity function.

3) The expecting propagation probability of the individual

The expecting propagation probability of the individual is composed of two parts: affinity and density. Affinity is computed above. The density has a direct relationship with similarity. The similarity function of an antibody reflects how similar it is with other antibody. The similarity function $S_{j,i}$ is calculated by R-contiguous matching regulation.

$S_{j,i}$ is given by:

$$S_{j,i} = \frac{\max(k_{j,i})}{L} \quad (5)$$

where $\max(k_{j,i})$ represent the maximum number of contiguous matching between the j th antibody and the i th one, L is the length of coding.

The density of antibody is a proportion of the similar antibodies. The concentration function of the j th antibody is defined by

$$C_j = \frac{1}{N-1} \sum_{i=1}^{N-1} S_i \quad (6)$$

$$S_i = \begin{cases} 1, & S_{v,s} > T \\ 0, & \text{other} \end{cases} \quad (7)$$

where, T is a preestablished similar threshold, N is the total number of antibodies.

The expecting propagation probability of the j th antibody is defined as follow:

$$P_j = \alpha \frac{A_j}{\sum_{i=1}^N A_i} + (1-\alpha) \frac{C_j}{\sum_{i=1}^N C_i}, \quad (8)$$

where α is a constant. The function assures that the high density antibody can be restrained while the high concentration antibody can be restrained, so the diversity of the colony can be kept.

4) The immune operation

Selection: The selection strategy is roulette-mechanism. The selected probability is calculated by formula (8).

Cross: Choose two random individuals to do the cross operation in single point.

Variation: Variation happens in some position randomly.

V. SIMULATION

In this simulation, we assume that the calibrated radar error contains two parts: the system error made up of a sine trend and a constant, the random error built up by a norm noise. The sine function is

$$3 * \sin(2 * \pi * 0.005 * t), t = 1, 2, \dots, 1000$$

and the constant error is 2. The random error is $N(0, 0.64)$.

In the wavelet decomposition process, perform the discrete wavelet transform of the total error by db6. The best level is decided by formula (3). As is shown in Fig.1, $G(4) \approx G(5)$. So we choose the optimal level as 4. We use IA to choose the threshold. The comparison is HeurSure threshold. GCV threshold has two instances: one is level-threshold separation; the other is global threshold separation. The simulation figures are shown in the following.

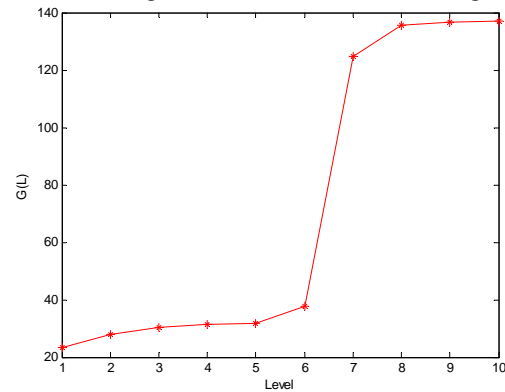


Figure 1. The G(L) of levels

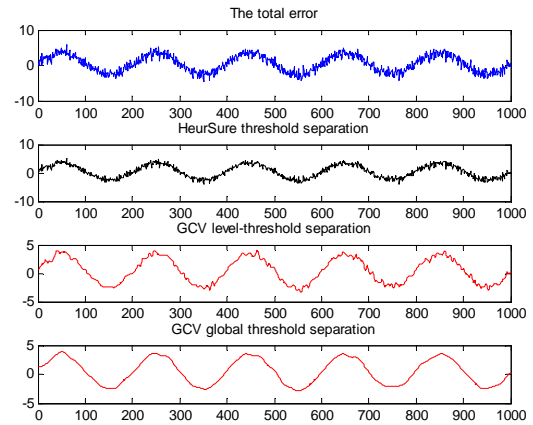


Figure 2. Comparison of different thres

In Fig.2, it is obvious that GCV global threshold method has better separation result than others. The MSE of HeurSure threshold, GCV level-threshold, GCV global threshold are 0.3357, 0.0926, 0.0351. The HeurSure threshold is a conservative method and the result has the most random error.

VI. SUMMARY

We apply the wavelet analysis into the error separation. A selection method for the best level is given. During the wavelet threshold choosing, GCV is introduced to solve the unknown prior information of original signal. IA is a good method to search the optimal value of the GCV risk function. In the end, the simulation has proved that the combination of them has an excellent effect on the separation.

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