

## Math model and scheduling method for bottleneck station AGA-based approach

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**Abstract**—A math model is built on minimizing the maximum operation time of the equipment for the bottleneck station in which both the switching time and initial state are considered. The objective function is represented as the fitness function in adaptive generic algorithm which is proposed as a solution to this model. Finally, numerical computation is given to show the superiority of this proposed method.

**Keywords**—Scheduling method; adaptive Genetic Algorithms; assembling and testing manufacture

### I. INTRODUCTION

The modeling and optimization of the semiconductor wafer fabrication system has attracted huge attention from researchers and scientists in both academic and practical arenas, which has been proved to be NP-Hard problems. It has characteristics of multi-target, uncertainty. Thus, existing methods had been proposed to solve this problem; however, production scheduling research mainly concentrates in the scheduling mode [1-3], algorithm [4-6] and dispatching strategy [7-9]. Lots of achievements have been obtained. Some parts of the results have related to the genetic algorithm and adaptive genetic algorithm [10-13]. For example, GA is used as an optimizing tool after insert saving algorithm is discussed to get the initial scheme if adequate resources are available [10]. [11-12] proposed that when additional resources are not available, model is firstly built by petri-net technology and then match strategy of batch and equipment is put forward by experience. Finally, GA determines the best strategy.

Construct to the semiconductor wafer production scheduling, few researchers are focused on packaging test. Some existing methods had been proposed to solve this problem. Thus for example, ant colony optimization algorithm is used to semiconductor assembly and test manufacturing [14]. Methods and means of realization about simulation are illustrated [15]. Zhang points that most problems of the semiconductor production scheduling consider multiple resource constraints and scheduling of the parallel machines[16]. He has analyzed semiconductor scheduling problem by methods and ways. Both additional and non-additional resources problem are summarized.

Although the whole semiconductor packaging test production line isn't as complex as wafer manufacturing line, for the test industry market scale is getting bigger and

bigger, fund intensive degree of test procedure is higher and higher, And the semiconductor packaging test production line has character of complex craft, short product life cycle, multi-objective and so on at the same time, which makes the semiconductor packaging test scheduling research also become meaningful in the whole semiconductor manufacturing scheduling.

The reminder of this paper is organized as follows. Section II gives the illustration of the problem to semiconductor package test production line. Subsequently, the math model for the problem is built in section III. The introduction and application to the adaptive genetic algorithm is proposed in section IV. Finally, the conclusion is drawn.

### II. PROBLEM FORMULATION

In actual production, each semiconductor control-site of the packaging test line is composed of specific types of devices like link1, link2, link3...linki. Now, we have to complete some kinds of products which have a certain number of  $X_M$  ( $M=A,B,\dots$ ). Each kind of product could be processed by one or several kinds of equipments; however, one kind of equipment can process only one product in the meantime. The products can be switched between various kinds of equipments according to the schedule time, but have to consider the switching time and initial-state. The initial-state means that it's processing product at first. The switching time means that when one equipment changes from processing one product to another, we have to consider the switching time. Furthermore, the switching time is the same when two kinds of products switch from one equipment to another in the process of producing. Using  $v_{L_i M}$  to express the ability of the  $i$ th equipment  $L_i$  to process some kind of product  $M$ . When some equipments are processing at first, how to finish all the products processed in the limited production ability is our work.

### III. MODELING

From the description of the problem above, we suppose that the  $i$ th equipment  $L_i$  has to process product  $M$  which has a certain number of  $X_{L_i M}$ , where  $X_{L_i M}$  is an integer. In our semiconductor control-site we suppose there are 5 types of device to processing 5 kinds of products. So, we need  $t_A$  to

finish the processing of the product A, where  $t_A$  is shown as follows:

$$t_A = \frac{1}{v_{L_1A}} X_{L_1A} + \frac{1}{v_{L_2A}} X_{L_2A} + \frac{1}{v_{L_3A}} X_{L_3A} + \frac{1}{v_{L_4A}} X_{L_4A} + \frac{1}{v_{L_5A}} X_{L_5A} \quad (1)$$

Provided  $z_{L_iM}$  substitutes the equipment  $L_i$  is processing the product M at first. For example,  $z_{L_1D}$  expresses link1 is processing product M at first. As each equipment processes one kind of product at first, we have

$$\begin{cases} \sum_{M=A,B,\dots,E} z_{L_1M} = 1 \\ \sum_{M=A,B,\dots,E} z_{L_2M} = 1 \\ \sum_{M=A,B,\dots,E} z_{L_3M} = 1 \\ \sum_{M=A,B,\dots,E} z_{L_4M} = 1 \\ \sum_{M=A,B,\dots,E} z_{L_5M} = 1 \end{cases} \quad (2)$$

where  $z_{L_iM}$  is 0 or 1. When we don't consider the initial state, using  $T_{L_iMN}$  to substitute the switching time of the equipment  $L_i$  changes from processing product M to processing the product N, where  $M=A, B, \dots, E$ ;  $N=A, B, \dots, E$  and  $M \neq N$ .

When we consider the initial-state, the time relates to link1  $T'_{L_iMN}$  can be shown as:

$$T'_{L_iMN} = \left[ z_{L_iM} + \frac{X_{L_iM}}{x_{L_iM} + 1} (1 - z_{L_iM}) \right] \cdot \left[ z_{L_iN} + \frac{X_{L_iN}}{x_{L_iN} + 1} (1 - z_{L_iN}) \right] \cdot T_{L_iMN} \quad (3)$$

So the operation time of the equipment link1 can be shown as

$$T_1 = \sum_{m=1,2,\dots,i} \frac{1}{v_{L_1m}} X_{L_1m} + \sum_{m=1,2,\dots,i, n=1,2,\dots,i, m \neq n} T'_{L_1mn} \quad (4)$$

Accordingly, we can easily get  $T_2, T_3, \dots, T_i$ . In order to process all the products before schedule time, we have to minimize the maximum of all the target-functions. Namely, we have to minimize the maximum of  $T_1, T_2, T_3, \dots, T_i$ . So we choose the optimization object-function as follows:

$$f = \min \{ \max (T_1, T_2, T_3, \dots, T_i) \} \quad (5)$$

The constraint is

$$s.t. \begin{cases} \sum_{m=1,2,\dots,i} X_{L_m1} = X_1 \\ \sum_{m=1,2,\dots,i} X_{L_m2} = X_2 \\ \sum_{m=1,2,\dots,i} X_{L_m3} = X_3 \\ \vdots \\ \sum_{m=1,2,\dots,i} X_{L_mi} = X_i \\ X_{L_11}, X_{L_21}, X_{L_31}, \dots \geq 0 \end{cases} \quad (6)$$

#### IV. APPLICATION OF THE ADAPTIVE GA TO THE OPTIMIZATION MODEL

Genetic Algorithms (GA), a promising alternative to conventional heuristic methods, is a general adaptive optimization search methodology based on a direct analogy to Darwinian natural selection and genetics in biological system. Its concept was developed by Holland and his colleagues in the 1960s and 1970s [17]. As some parameter like mutation rate, cross rate are randomly determined in conventional GA, which result in the loss of the average fitness of the population. Srinivas M has proposed adaptive GA to avoid this defect [18]. Its essence is how to choose the appropriate parameters.

To show the performance of the adaptive GA in the application to the math model, we carried serial calculations based on actual production data. In our model, there are 5 equipments, they have to process 5 kinds of products, including  $X_A=18, X_B=30, X_C=27, X_D=15, X_E=30$  in 72 hours. The processing ability of equipments and switching time of each equipment are shown in table1 and table2 respectively.

Table 1: processing ability of equipments (10000/24h)

	A	B	C	D	E
link1	5	0	0	8	12
link2	7	0	6	0	10
link3	0	8	10	6	0
link4	0	9	8	0	10
link5	6	10	0	10	0

Table2: switch time of equipments (h)

	AB	AC	AD	AE	BC	BD	BE	CD	CE	DE
link1			4	7						6
link2		1		3					2	
link3					8	2		3		
link4					4		3		4	
link5	6		6			8				

The object function can be gotten by

$$\min \{ \max (T_1, T_2, T_3, T_4, T_5) \}$$

$$s.t. \begin{cases} X_{L_1A} + X_{L_2A} + X_{L_5A} = 18 \\ X_{L_3B} + X_{L_4B} + X_{L_5B} = 30 \\ X_{L_2C} + X_{L_3C} + X_{L_4C} = 27 \\ X_{L_1D} + X_{L_3D} + X_{L_5D} = 15 \\ X_{L_1E} + X_{L_2E} + X_{L_4E} = 30 \\ X_{L_1A}, X_{L_2A}, X_{L_3A} \dots \geq 0, \end{cases} \quad (7)$$

where:

$$\begin{cases} T_1 = 24 \cdot \left( \frac{X_{L_1A}}{5} + \frac{X_{L_1D}}{8} + \frac{X_{L_1E}}{12} \right) + (T'_{L_1AD} + T'_{L_1AE} + T'_{L_1DE}) \\ T_2 = 24 \cdot \left( \frac{X_{L_2A}}{7} + \frac{X_{L_2C}}{6} + \frac{X_{L_2E}}{10} \right) + (T'_{L_2AC} + T'_{L_2AE} + T'_{L_2CE}) \\ T_3 = 24 \cdot \left( \frac{X_{L_3B}}{8} + \frac{X_{L_3C}}{10} + \frac{X_{L_3D}}{6} \right) + (T'_{L_3BC} + T'_{L_3BD} + T'_{L_3CD}) \\ T_4 = 24 \cdot \left( \frac{X_{L_4B}}{9} + \frac{X_{L_4C}}{8} + \frac{X_{L_4E}}{10} \right) + (T'_{L_4BC} + T'_{L_4BE} + T'_{L_4CE}) \\ T_5 = 24 \cdot \left( \frac{X_{L_5A}}{6} + \frac{X_{L_5B}}{10} + \frac{X_{L_5D}}{10} \right) + (T'_{L_5AB} + T'_{L_5AD} + T'_{L_5BD}) \end{cases} \quad (8)$$

and  $X_{L_1A}, X_{L_1B}, \dots, X_{L_5E}$  are integers.

Such parameters like crossover probability and mutation probability together with fitness function are determined by

$$p_c = \begin{cases} k_1 & F_c \leq \bar{F} \\ k_2(F_{\max} - F_c)/(F_{\max} - \bar{F}) & F_c > \bar{F} \end{cases} \quad (9)$$

$$p_m = \begin{cases} k_3 & F_m \leq \bar{F} \\ k_4(F_{\max} - F_m)/(F_{\max} - \bar{F}) & F_m > \bar{F} \end{cases} \quad (10)$$

where  $k_1, k_2, k_3, k_4$  are constant and less than 1.  $\bar{F}$  is the average fitness,  $F_c$  is the bigger fitness of the two cross individuals and  $F_m$  is the fitness of the variation individual.

$k_1, k_2, k_3, k_4$  play a role in the convergence of GA, however, we can't get a qualitative conclusion for this,  $k_3, k_4$  is a little bigger and  $k_1, k_2$  can be adjusted according to the case.

The reciprocal of the objective function is used as the fitness function, which is shown as

$$F = \frac{1}{\min \{ \max (T_1, T_2, T_3, T_4, T_5) \}} \quad (11)$$

In this paper, the size of the population is 60. After 13 times iterative operation, the algorithm can converge to globally optimal solution. The maximum of the fitness function is 1/70.35, which means that the minimum time of

finishing the work is 70.35 hours. The initial solution and appropriate optimal solution can be shown as

$$\begin{pmatrix} z_{L_1A}, z_{L_1D}, z_{L_1E}, z_{L_2A}, z_{L_2C}, z_{L_2E}, z_{L_3B}, z_{L_3C}, z_{L_3D}, \\ z_{L_4B}, z_{L_4C}, z_{L_4E}, z_{L_5A}, z_{L_5B}, z_{L_5D} \end{pmatrix} \\ = (0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0) \quad (12)$$

$$\begin{pmatrix} X_{L_1A}, X_{L_2A}, X_{L_3A}, X_{L_4A}, X_{L_5A}, X_{L_1B}, X_{L_2B}, X_{L_3B}, X_{L_4B}, X_{L_5B}, \\ X_{L_1D}, X_{L_2D}, X_{L_3D}, X_{L_4D}, X_{L_5D}, X_{L_1E}, X_{L_2E}, X_{L_3E}, X_{L_4E}, X_{L_5E} \end{pmatrix} \\ = (6, 8, 4, 13, 7, 9, 13, 10, 11, 4, 2, 14, 10, 6, 10) \quad (13)$$

To prove the superiority of this algorithm in our model, we apply the GA in our model too; the minimum time of completion is 70.4, which means that we need 70.4 hours to process the products. The initial solution and appropriate optimal solution can be shown as

$$\begin{pmatrix} z_{L_1A}, z_{L_1D}, z_{L_1E}, z_{L_2A}, z_{L_2C}, z_{L_2E}, z_{L_3B}, z_{L_3C}, z_{L_3D}, \\ z_{L_4B}, z_{L_4C}, z_{L_4E}, z_{L_5A}, z_{L_5B}, z_{L_5D} \end{pmatrix} \quad (14)$$

$$= (0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0) \\ \begin{pmatrix} X_{L_1A}, X_{L_2A}, X_{L_3A}, X_{L_4A}, X_{L_5A}, X_{L_1B}, X_{L_2B}, X_{L_3B}, X_{L_4B}, X_{L_5B}, \\ X_{L_1D}, X_{L_2D}, X_{L_3D}, X_{L_4D}, X_{L_5D}, X_{L_1E}, X_{L_2E}, X_{L_3E}, X_{L_4E}, X_{L_5E} \end{pmatrix} \\ = (8, 7, 3, 15, 6, 9, 9, 7, 11, 2, 2, 11, 12, 8, 10) \quad (15)$$

## V. CONCLUSION

We carried a research on the modeling, scheduling, and performance evaluation for bottleneck station. Firstly, the scheduling model based on minimizing the maximum equipment running time is built, and then the superiority is shown by applying the adaptive GA to an example compared with the conventional GA, which plays a guide role in production of semiconductor manufacturing scheduling.

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