Image Fusion Algorithm Based on Wavelet Sparse Represented Compressed Sensing

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***Abstract*—On the basis of the compressed sensing theory, this study proposed an improved wavelet sparse represented compressed sensing based image fusion algorithm. This algorithm firstly got the wavelet sparse domain linear measurement values of the original images by the dual radial sampling mode. Then a simple maximum absolute value fusion rule was adopted on the compressed sensing domain. Finally, the minimum total variation method was used to reconstruct the fused image. The experiment result shows that this algorithm has good fusion effect.**

Keywords-compressed sensing; image fusion, wavelet; minimum total variation

I. Introduction

The image fusion is the process integrating the two or more images or image sequence obtained in the same time or different time to one image. The fused image is more abundance information than any one of the original images. It is fit for the further process in the image processing technology. The image fusion theory is widely applied in the earth observation, airport navigation, safety monitoring, intelligent transportation, medical imaging and diagnosis, Machine Vision, Geographic Information Systems (GIS), Intelligent Manufacturing, industrial processes, military and etc. At present, there are three kinds of image fusion algorithms according to the information abstraction degree. They are pixel level fusion, feature level fusion and decision level fusion. The pixel level fusion algorithm directly process on the original images. So it need larger storage space and it has higher computational complexity. It is an enormous challenge for the fusion system requiring larger sensor scale and higher real-time feature. The advent of the compressed sensing (CS) theory solves this question [1,2].

Compressed sensing [3-5] was proposed by Candes and Donoho in 2004. It is a new information acquisition and impressing theory founded on the basis of signal sparse representation and approximation theory. In this theory, if the signal itself or it in certain transform domain is compressive or sparse, a measurement matrix incoherent with the transform domain projects the signal or transform coefficients in high-dimensional space to low-dimensional space. In the low-dimensional space, less sampling value (much lower than the Nyquist sampling rate) can precisely reconstruct original signal in high probability [6,7] . This new theory breaks the limitation of Nyqusit sampling rate. It highly reduces the signal sampling frequency, signal processing time and the cost of data storage and transmission [8].

Compressed sensing mainly includes sparse representation, measurement matrix and reconstruction algorithm. Where, the sparse representation is the theory basis of the compressed sensing theory. The sparse representation denotes that less coefficients can better describe the main information of the signal. Most actual signals are nonzero. Most coefficients have small values in certain transform base (such as: wavelet basis), while less coefficients which bear most information of the signal have large values. The CS theory shows that the more sparse of the signal, the much accurate of the reconstruction signal. So the suitable transformation base can guarantee the sparsity and independence of coefficient, and guarantee the reconstruction precision of the compressed sensing while reducing the compression measurements. At present, the common transforms are the Fourier Transform, Discrete Cosine Transform, Wavelet Transform etc.

This paper proposed a novel image compressed sensing image fusion algorithm based on wavelet sparse representation. In order to reduce the computational burden, this study firstly constructed the wavelet sparse matrix. On the basis of analyzing the relationship of the reconstruction and fusion quality, the images are fused by the maximum absolute value fusion rule and reconstructed by the minimum total variation method. Finally, the fused image got by this algorithm was compared with the fused image got by other fusion methods. The results show that this algorithm improves the fusion quality.

II. Compressed sensing theory introduction

The compressed sensing theory indicates that if the signals are compressive or sparse in the orthorhombic space, a few observed values of acquisition signal would get by the random projection method, then this signal could be constructed by certain optimization algorithm.

Suppose the signal $y = \Phi s = \Phi \Psi^T x = Ax$ is $N*1$-dimensional column vector. It is sparse in the orthogonal basis or tight frame $\Phi \in R^{N\times N}$. That is, the signal x has only K （$K \ll N$） nonzero coefficients (or much greater than zero coefficients) in the orthogonal basis Ψ .

$$x = \Psi s$$

Adopt an observation matrix $\Phi \in R^{M \times N}$ ($M \ll N$) irrelevant with orthogonal basis Ψ to measure the sparse coefficient s of the signal to get the observation vector:

$$y = \Phi s = \Phi \Psi^T x = Ax$$

Where, A is called the measurement matrix ($A_{M \times N} = \Phi_{M \times N} \Psi^T_{N \times N}$). The equation shows that the compressed sensing reduces the signal x of N -dimension to observation signal of M -dimension. Obviously, the dimension is reduced after the sampling. Recovering the signal x from M measured values is an ill-posed problem. Chen, Donoho and Saunders mentioned that the x approximation was got by the l_1 normal optimal recovery. That is:

$$\min \left\| \Psi^T x \right\|_l \quad \text{s.t.} \quad y = \Phi \Psi^T x$$

$s = \Psi^T x$, so the optimization problem in equation (3) can be translated into:

$$\min \left\| s \right\|_l \quad \text{s.t.} \quad y = \Phi s$$

Solve sparse coefficient estimate $\hat{s}$, then do inverse transformation, the reconstructed signal $\hat{x}$ could be got. The common methods are Basic Pursuit, Matching Pursuit and Orthogonal Matching Pursuit etc.

III. THE COMPRESSED SENSING IMAGE FUSION ALGORITHM BASED ON THE WAVELET SPARSE REPRESENTATION

A. Fast Wavelet Algorithm

Inspired by the pyramid image decomposition and reconstruction algorithm proposed by Burt and Adelson, Mallat improved the Mallat's fast wavelet algorithm [9,10]. Suppose H (low-pass) and G (high-pass) are all one-dimensional mirror filtering operator. The subscripts r and c of them correspond to the rows and columns of the image. In accordance with the two-dimensional Mallat algorithm, the Mallat decomposition function on the j-1 scale is shown as follows:

$$\begin{cases} C_j = H_c H_r C_{j-1} \\ D_j^1 = G_c H_r C_{j-1} \\ D_j^2 = H_c G_r C_{j-1} \\ D_j^3 = G_c G_r C_{j-1} \end{cases}$$

Where, C_j , D_j^1 , D_j^2 and D_j^3 correspond to the low-frequency of the image C_{j-1} , high-frequency in the vertical direction, high-frequency in the horizontal direction and the high-frequency in the diagonal direction. The corresponding Mallat reconstruction algorithm is shown as follow:

$$C_{j-1} = H_r^* H_c^* C_j + H_r^* G_c^* D_j^1 + G_r^* H_c^* D_j^2 + G_r^* G_c^* D_j^3$$

Where H^* and G^* are the conjugate transposed matrix of H and G.

The N layer wavelet decomposition was done on the two-dimensional image. There are 3N+1 different sub-bands, including 3N high-frequency sub-band and one low-frequency sub-band.

B. Fusion Steps

In the wavelet compressed sensing algorithm (W-CS), firstly do the wavelet transform on the image whose size is $N \times N$. Secondly, construct the measurement matrix Φ (At present, the measurement matrix are random Gaussian matrix and Bernoulli distribution ± 1matrix that obey （0，$1/N$ ） distribution). Thirdly, get the measurement coefficient matrix of size $M \times N$ （ $M \ll N$ ） by measuring the coefficients after the wavelet transform by the measurement matrix Φ . When restoring image, recover the original image by the OMP algorithm according to the Φ and $M \times N$ measurement coefficients.

Compare with the W-CS algorithm, the algorithm in this paper have the next adjustments and improvements:

Traditional image fusion method is to fuse the original images directly. With the development of the compressed sensing, the researchers discovered that directly fuse the compressed sensing measurement value and then reconstruct the images can greatly improve the computational efficiency [11,12]. On this basis, this study adopted the maximum absolute value method as the fusion rule. The fusion steps are shown as follows:

- **Step1** Input the original images A and B . Their size is $N \times N$.
- **Step2** Construct the measurement matrix Φ .
- **Step3** Do the wavelet transform on the original images to get the coefficients. Then measure the coefficients by the measurement matrix Φ . The result was the measurement coefficient matrixs Y_a and Y_b whose size was $M \times N$ （$M \ll N$）.

$$\begin{cases} Y_a = \left(x_1, x_2, ..., x_M \right)^T = \Phi A \\ Y_b = \left(y_1, y_2, ..., y_M \right)^T = \Phi B \end{cases}$$

- Step4 Get the measured value $Y_c = \left(z_1, z_2, ..., z_M \right)^T$ by the maximum absolute value fusion rule.

$$Y_c = \left(z_1, z_2, ..., z_M \right)^T$$

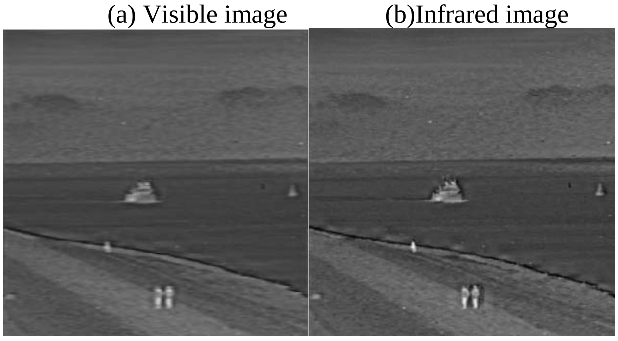
$$z_i = \begin{cases} x_i & \left| x_i \right| \ge \left| y_i \right| \\ y_i & \left| x_i \right| < \left| y_i \right| \end{cases} \qquad i = 1, 2, ..., M$$

- Step5 Construct the fused image C by the minimum total variation method.

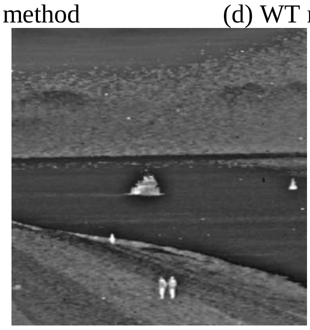
$$Min - TV(x) \quad s.t. \quad \Phi x = y$$

IV. Simulation Results and Analysis

(a) Visible image (b)Infrared image



(c) DCT method (d) WT method



(e) Method in this paper

Fig.1 Visible image, infrared image, fused images using different methods

In order to verify the effectiveness of the proposed algorithm, this study adopted a infrared image and a visible light image with the size of 256 X 256 to do simulation test. Then the study compared this algorithm with the DCT and WT algorithm. In the experiment, the WT method chose wavelet function “sym8” to do 4- layer decomposition on the image. The result was the transform coefficients matrix with size of 256 X 256.

This algorithm chose wavelet function’’9-7’’ to do monolayer decomposition whose direction number was 8. The result was that the low-frequency coefficient was 128×128 and the high-frequency coefficient was 256×256. In order to ensure the coefficients points have the same number as before after measuring, the value of M adopted 128, 128 and 64. Then a simple maximum absolute value fusion rule was adopted on the compressed sensing domain. Finally, the minimum total variation method was used to reconstruct the fused image.

Figure 1 shows that the image reconstructed by the DCT algorithm has obvious distortion. The image reconstructed by the WT algorithm has slight distortion. And the image reconstructed by the algorithm in this paper approximates the original image.

Assessing from the objective standpoint, entropy (EN), mutual information (MI) and average gradient (AvG) are used as the object evaluation factors of the image fusion effect. The entropy is one important factor of the image information rich degree. The mutual information is used to calculate how much information is transferred to the fusion result. The average gradient is used to describe the clarity of the image. The larger the three values, the better the fusion results. The table 1 shows that the entropy, mutual information and average gradient are all higher than other three methods. So the method in this paper is better than other three methods.

Tab.1 Objective evaluation of fusion results

Method	EN	MI	AvG
DCT	6.5638	2.2842	4.7128
WT	6.625	2.4221	5.5377
This Method	7.0328	2.6540	5.8239

V. Conclusion

This paper introduced the theory and method of the compressed sensing algorithm. Then proposed a Contourlet transform based compressed sensing algorithm by the better sparse representation ability of the Contourlet transform on the image. Finally compared this algorithm with the DCT and wavelet transform compressed sensing algorithm. The experiment results show that the reconstructed image quality has a significant improvement. Especially, this algorithm has better effect on the images with rich curve.

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