The Application of Queuing Theory—

Reasonable Arrangements of Ophthalmic Beds

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***Abstract*—The utilization of hospital bed is critical to hospitals. A reasonable evaluation of hospital bed utilization helps hospital administrators to strengthen hospital management in allocating sickbeds to improve efficiency in the use of hospital beds. Therefore, through establishing model to analyze bed utilization and adjust bed arrangement, we can improve the hospital benefit, improve ward management, tap potential and strengthen the service capacity.**

***Keywords*- theory of queuing up, ophthalmic beds, simulation of computer, linear programming.**

I. INTRODUCTION

To queue up in a hospital is very familiar for everybody. For example, going to the clinic, pricing to cashier, fetching drug to drugstore, having an injection to injection room and waiting for being hospitalized need to queue up and wait for accepting a service.

II. "FCFS" MODEL

The clinic of ophthalmology is opened in a hospital everyday. There are 79 sickbeds in patient department altogether. Operation of ophthalmology is divided into cataract, retinal disease, glaucoma and trauma. Operation of cataract is simple, and isn't emergency. At present cataract is operated in Monday or Wednesday. Readiness time of operation of cataract only need one day or two days. The patient of operating two eyes is more than the patient of operating one eye. The patient of being operated two eyes is about 60%. If being operated two eyes, he is operated on Monday first, on Wednesday again. Trauma usually belongs to emergency. If sickbed is empty, patient of trauma is immediately arranged to be hospitalized, and is arranged to operate in the following day. The other ophthalmology disease is complicated. The patient may accept operation in two days or three days after being hospitalized, but observational time is long after operation. Operation of these diseases isn't commonly arranged on Monday or Wednesday. These diseases may not be considerd emergency. Operation condition is plenty in this hospital. As the question of arranging doctor of operation is considerd, operation of cataract isn't arranged the same day with the other operation (except emergency). All non- emergency patients are arranged to be hospitalized according to "FCFS" （First come, First serve）, but patient queue of waiting for being hospitalized is longer and longer.

After he is hospitalized, a patient needs preparative time before operation. A patient of cataract needs preparative time 2 days, and the other patient needs preparative time 3 days. Suppose that t is waiting time, and k is preparative time, losing time (a_{1j}) is

$$a_{1j} = \begin{cases} t-k & t \ge k \\ 0 & t < k \end{cases}$$

The degree of non-satisfaction (λ_{1j})[1] is

$$\lambda_{1j} = \left[\frac{a_{1j} - \min(a_{1j})}{\max(a_{1j}) - \min(a_{1j})} \right]^2$$

Using matlab, then we may get Table 1.

Table 1 losing time, the number of person and the degree of non-satisfaction

losing time(a_{1j})	0	1	2	3	4	5
the number of person(n_{1j})	218	23	16	21	8	8
the degree of non-satisfaction (λ_{1j})	0	$\frac{1}{25}$	$\frac{4}{25}$	$\frac{9}{25}$	$\frac{16}{25}$	1

The average degree of non-satisfaction (w_1) is

$$w_1 = \frac{\sum \lambda_{1j} \times n_{1j}}{\sum n_{1j}}$$

Before he is hospitalized, a patient needs queuing time(a_{2j}). Then the degree of non-satisfaction (λ_{2j}) is

$$\lambda_{2j} = \left[\frac{a_{2j} - \min(a_{2j})}{\max(a_{2j}) - \min(a_{2j})} \right]^2$$

Using matlab, then we may get Table 2.

Table 2 queuing time, the number of person and the degree of non-satisfaction

queuing time(a_{2j})	≤ 10	11	12	13	14	15	≥ 16
the number of person(n_{2j})	7	32	109	99	40	6	1
the degree of non-satisfaction (λ_{2j})	0	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{9}{36}$	$\frac{16}{36}$	$\frac{25}{36}$	1

The average degree of non-satisfaction(w_2) is

$$w_2 = \frac{\sum \lambda_{2j} \times n_{2j}}{\sum n_{2j}}$$

Suppose that weight of w_1 is α, then weight of w_2 is $1-\alpha$, the total degree of non-satisfaction (w) is

$$w = \alpha w_1 + (1-\alpha) w_2 .$$

When α=0.7, we may get w=0.17.

When sickbed is arranged with adopting "FCFS" model, there is a problem that accumulative total length of group is longer and longer, and the total degree of non-satisfaction (w) isn't reasonable. We set model of priority.[2]

According to current situation of being hospitalized, suppose that d_2=the time of leaving hospital – first time of operation(d_0).

Using spss, d_2 of five kind patients obey Poisson distribution. Then we may get Table 3.

Table 3 poisson distribution value (λ) of five kind patients

	cataract of one eye	cataract of two eyes	glaucoma	retinal disease	trauma
λ	2.9	4.96	8.08	10.17	6.04

The time of leaving hospital(d_1) is polyfited ($d_1 = d_0 + d_2$). We may solve how many patients leave hospital. For example in Table 4:

Table 4 the number of patients from Sep 10 to Sep 23

time	Sep 10	Sep 11	Sep 12	Sep 13	Sep 14	Sep 15	Sep 16
the number of patients	2	2	1	17	3	4	5
time	Sep 17	Sep 18	Sep 19	Sep 20	Sep 21	Sep 22	Sep 23
the number of patients	5	3	9	9	4	5	2

III. PRIORITY MODEL

In order that a patient is operated in the shortest time after being hospitalized, and waiting patients isn't more and more, we adopt priority model to arrange patients as follows in Table 5 .[3]

Table 5 priority model to arrange patients in proper order

time	arranging in proper order
Saturday or Sunday	cataract of two eyes, cataract of one eye, non- cataract
Monday or Tuesday	cataract of one eye, non- cataract
Wednesday, Thursday or Friday	non- cataract, cataract of two eyes, cataract of one eye

According to the time of drafting to leave hospital and queuing time, and considering priority model, we may arrange patients to be hospitalized.

According to priority model arranging to be hospitalized, operate and leave hospital, using appraiseing quota: $w = \alpha w_1 + (1-\alpha) w_2$, we may get Table 6 and Table 7.

Table 6 losing time, the degree of non-satisfaction and the number of person

losing time(a_{1j})	0	1	2	3	4	5
The degree of non-satisfaction (λ_{1j})	0	0.04	0.16	0.36	0.64	1
the number of person(n_{1j})	160	3	7	6	1	2

Table 7 queuing time, the number of person and the degree of non-satisfaction

queuing time(a_{2j})	≤ 11	12	13	14	15	≥ 16
The degree of non-satisfaction (λ_{2j})	0.03	0.11	0.25	0.44	0.69	1

the number of person(n_{2j})	9	26	38	13	16	28

Thus, we may get w=0.098. Because 0.098<0.17, the priority model is more reasonable than FCFA model[4].

IV. OPERATION ISN'T ARRANGED ON SATURDAY OR SUNDAY

If operation is arranged on Saturday or Sunday, operation should be non- cataract. If operation isn't arranged on Saturday or Sunday, the time of leaving hospital and being hospitalized of non- cataract is only influenced. Adopting priority model, we draft the time of leaving hospital and the number of patients of leaving hospital everyday by excel.

If operation isn't arranged on Saturday or Sunday, the number of patients of leaving hospital is in Table 8.

Table 8 the number of patients from Sep 11 to Sep 22

time	Sep 11	Sep 12	Sep 13	Sep 14	Sep 15	Sep 16
the number of patients	2	3	17	3	2	1
time	Sep 17	Sep 18	Sep 19	Sep 20	Sep 21	Sep 22
the number of patients	8	4	16	10	6	5

If operation isn't arranged on Saturday or Sunday , we research whether the time of all operation is adjusted. Firstly, trauma patients aren't considered. Secondly, We may compute the time of patients being hospitalized as follows.

If operation isn't arranged on Saturday or Sunday, the time of patients being hospitalized is in Table 9.

Table 9 operation isn't arranged on Saturday or Sunday, the time of patients

	cataract of one eye	cataract of two eyes	glauco ma	retinal disease	trau ma
time (day)	5	7	11	13	9

But, If operation is arranged on Saturday or Sunday, the time of patients being hospitalized is Table 10.

Table 10 operation is arranged on Saturday or Sunday, the time of patients

	cataract of one eye	cataract of two eyes	glauco ma	retinal disease	trau ma
time (day)	4	6	9	12	8

V. THE ARRANGEMENT OF HOSPITAL SICKBED

In the general case, according to the times of every kind patients being hospitalized, We may set optimizational model of the arrangement of hospital sickbed.

According to data statistics, using spss, the number of every kind outpatients obey Poisson's distribution[4]. Then we may get mean value a_i (i=1,2,3,4,5) of the number of every kind outpatients in Table 11.

Table 11 mean value a_i (i=1,2,3,4,5) of the number of every kind outpatients

	cataract of one eye	cataract of two eyes	glauco ma	retinal disease	trauma
a_i	1.8947	2.4182	1.5366	2.8814	1.6

According to data statistics, using spss, the time of every kind patients being hospitalized obey normal distribution[5]. Then we may get mean value b_i (i=1,2,3,4,5) and standard deviation s_i (i=1,2,3,4,5) of the number of every kind patients being hospitalized in Table 12 .

Table 12 mean value and standard deviation of the number of every kind patients

	cataract of one eye	cataract of two eyes	glaucoma	retinal disease	trauma
b_i	5.24	8.56	10.49	12.54	7.04
s_i	1.45	2.09	1.699	2.45	1.84

We assume that c_i (i=1,2,3,4,5) is sickbed of every kind patients , we may solve average service intensity ρ_i (i=1,2,3,4,5) of every kind patients: $\rho_i = \frac{a_i \times b_i}{c_i}$. When $\rho_i \to 1$, the average staying time is short in system. The arrangement of hospital sickbed is good. We may set optimizational model of the arrangement of hospital sickbed. The target function is $T = \min \sum_{i=1}^{5} (\rho_i - 1)^2$. Constraint condition is $\sum_{i=1}^{5} c_i = 79$. Using lingo, we may get [5]:

T=0.014102,

$c_1 = 8$, $c_2 = 19$, $c_3 = 10$, $c_4 = 35$, $c_5 = 7$,

$\rho_1 = 1.241029$, $\rho_2 = 1.084957$, $\rho_3 = 1.611893$,

$\rho_4 = 1.032364$, $\rho_5 = 1.6109143$.

Because $\rho_i > 1$, average service probability is smaller than average arriving probability. With time extending, the number of waiting patients is more and more in queuing system of hospital. Hospital should make reasonable adjustment[6].

If $\rho_i \le 1$, the hospital may increase sickbed c_i. We may solve:

$c_1 = 10, c_2 = 21, c_3 = 17, c_4 = 37, c_5 = 12$.

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