# Equivalent Modeling of Wind Farm Based on Fuzzy Clustering

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*Abstract*—**Modeling of wind farm has become the focus research in the field of wind energy. In this paper, the wind generators are clustered through fuzzy clustering, every cluster contains certain number of generators. Each cluster can represent an equivalent generator; correspondingly the total power of wind farm will be the total power of all equivalent generators. This can avoid the gross error caused by taking the whole wind farm as one generator. The proposed method can largely reduce the computational complexity, which might lead to successful modeling of wind farm. The example simulation shows that the approach discussed in this paper can effectively indicate the power fluctuation of wind farm, and it is very helpful for wind power integration.**

*Keywords-wind farm; equivalent modeling; power fluctuation; doubly-fed induction generator; fuzzy clustering*

## I. INTRODUCTION

With constant progress of renewable resources project construction, the ratio of new energies like wind power and photovoltaic in power grid has been on an increase. Due to fluctuation of wind power, when capacity of integrated wind power reaches certain standard, wind farm access to power grid might bring about adverse effect to the power balance. Therefore, modeling research of wind farm is very important and indispensable, it can be applied to analyze and evaluate the effect of the wind power.

Current wind farm modeling is generally based on mathematical models of generator. Transient simulation models of squirrel-cage induction generator (SCIG), doubly-fed induction generator (DFIG) and permanent-magnet synchronous generator (PMSG) are usually used for analyzing generators, and the modeling approaches of wind rotor, transducer and control and protection system are also very important [1]. The double-fed induction generator can be described as a group of algebraic equations, and the equations can be solved with iterative algorithm [2], this modeling method is widely used. Some papers take the wind farm as an equivalent generator [3]. Difficulty in these approaches is the calculation of equivalent wind speed. If the wind speed in wind farm varies largely, with these approaches there would be many errors. As the size of wind farm enlarges, the practice of modeling for each generator in wind farm would obviously enhance computational complexity in modeling, so equivalent modeling of wind farm with low complexity has become the focus of research. As a whole, the wind farm can be simplified into one or more equivalent generators to reduce difficulty of modeling.

In this paper, wind generators in the wind farm are clustered: those with similar dynamic characteristics are put into one cluster and each cluster is represented as one equivalent generator. The number of clusters can be determined according to real demands, and example simulation shows that the approach discussed in the paper can effectively indicate dynamic characteristics of power in wind farm.

## II. MATHEMATICAL MODELS

At present, wind turbine generator mainly includes wind rotor, drive train, generator as well as control and protection system.

### A. Model of Wind Rotor

The power captured by wind rotor is $P_w$, and it can be expressed with (1) [4], $A$ is swept area of wind rotor, $\rho$ is air density, $v$ is wind speed, $C_p(\beta, \lambda)$ is rotor power coefficient, which is high-order non-linear function of tip speed ratio $\lambda$ and pitch angle $\beta$, theory study usually used (2) and (3) [5], $c_1 \sim c_9$ are fitting parameters.

$$P_w = \frac{1}{2} C_p(\beta, \lambda) A \rho v^3 \qquad (1)$$

$$C_p(\beta, \lambda) = c_1 (\frac{c_2}{\lambda_i} - c_3 \beta - c_4 \beta^{c_5} - c_6) e^{-\frac{c_7}{\lambda_i}} \qquad (2)$$

$$\frac{1}{\lambda_i} = \frac{1}{\lambda + c_8 \beta} - \frac{c_9}{\beta^3 + 1} \qquad (3)$$

The wind turbine mechanical torque is (4), $\omega_m$ is mechanical angular velocity.

$$T_m = \frac{P_w}{\omega_m} \qquad (4)$$

### B. Model of Drive Train

Wind turbine drive train system is mainly composed of wind rotor, gear box, shafting system and generator rotor. In research, (5) ~ (7) are usually used to describe the drive train [6], $H_m$ and $H_g$ are inertia time constant of wind turbine and generator rotor respectively; $T_m$ and $T_e$ are mechanical torque of wind turbine and electromagnetic torque of generator; $K_s$ is stiffness coefficient of shafting system; $D_s$ is damping coefficient between the wind rotor and generator, $\theta_s$ is torsional angle of shafting; $\omega_m$ and $\omega_g$ are angular velocity of wind turbine and generator rotor respectively; and $\omega_s$ is synchronous speed of generator.

$$2H_m \frac{d\omega_m}{dt} = T_m - K_s \theta_s - D_s(\omega_m - \omega_g) \qquad (5)$$

$$2H_g \frac{\mathrm{d}\omega_g}{\mathrm{d}t} = -T_e + K_s\theta_s + D_s(\omega_m - \omega_g) \qquad (6)$$

$$\frac{\mathrm{d}\theta_s}{\mathrm{d}t} = \omega_s(\omega_m - \omega_g) \qquad (7)$$

### C. Model of DIFG

Most of wind turbine generators are synchronous induction generators, among which doubly-fed induction generator is the most popular mode. The rotor side usually equaling as a controlled voltage source expressed by (8) [7]. Its state equations in dq axis synchronous rotating reference frame can be expressed with (9) and (10). The subscripts s and r respectively indicate the stators and the rotors in the generator; subscripts d and q indicate d and q axial components; $s$ is slip ratio of the generator; $u$ and $i$ are voltage and current winding. $T_0'$ is transient time constant of generator; $X_s$, $X_r$ and $X_m$ are stator reactance, rotor resistance and magnetizing resistance respectively.

$$\bar{E}' = e_d' + je_q' \qquad (8)$$

$$\frac{\mathrm{d}e_d'}{\mathrm{d}t} = -\frac{1}{T_0'}(e_d' - (X_s - X_s')i_{qs}) + s\omega_s e_q' - \omega_s \frac{X_m}{X_r}u_{qr} \qquad (9)$$

$$\frac{\mathrm{d}e_q'}{\mathrm{d}t} = -\frac{1}{T_0'}(e_q' + (X_s - X_s')i_{ds}) - s\omega_s e_d' + \omega_s \frac{X_m}{X_r}u_{dr} \qquad (10)$$

Generator electromagnetic torque $T_e$ is expressed by (11).

$$T_e = \frac{X_m}{\omega_s}(i_{ds}i_{qr} - i_{qs}i_{dr}) \qquad (11)$$

### D. Control and Protection System

Control and protection system of wind turbine generators are composed of pitch control and power control, etc. The protection system would cut out the generator, or lock and cut out corresponding converter when there is breakdown or abnormality in monitor data [8]. The connection would be recovered when breakdown is removed or monitor data becomes normal. Control system in electrical machine is usually simplified with differential equation of first order, while the protection system is usually ignored in practical research due to its limited effect.

### III. Fuzzy Clustering

Some generators in wind farm share similar dynamic characteristics; as a result they can be put into the same cluster. Similar dynamic characteristics means there remains uncertainty, that is, they are not identical. Certain ambiguity is in existence. Therefore, the approach of fuzzy clustering is used in this paper to cluster generators.

As there are many state variables in doubly-fed induction generator, the computational complexity would be rather high if all variables are to be analyzed. Affected by fluctuation of wind speed during operation, the rotor speed of generator would change. In order to ensure operation with constant frequency, the control system would regulate frequency of rotor excitation current according to slip ratio $s$. So the slip ratio can indicate fluctuation of wind speed and it is also important index of control system. In addition,

because the generator voltage $U_T$ and active power $P_a$ are closely related with dynamic characteristics of generator, they can be also used as index in clustering. The three characteristic quantities are easy to obtain with actual measurement or indirect calculation.

Fuzzy C-means (FCM) clustering is applied to divide the generators into $c$ fuzzy classes. $u_{ij}$ is the value of membership degree that $x_j$, the $j$th sample, belongs to the class $i$. Samples in the paper are characteristic quantities of single wind turbine generator; the $i$ class is the $i$th cluster [9]. Value of $u_{ij}$ should be within the range of [0, 1] and satisfy:

$$\sum_{i=1}^{c} u_{ij} = 1, \; j = 1, 2, \cdots n \qquad (12)$$

Target function of FCM algorithm can be expressed by (13). Here, $m > 1$ is a fuzzy coefficient; $d_{ij}$ means distance from sample point $x_j$ to cluster center $c_i$ [10], and usually it can be expressed with the Euclidean distance.

$$\min J = \sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^m d_{ij}^2 \qquad (13)$$

To reach minimum value in (13), an essential condition is that (14) and (15) are satisfied [11].

$$c_i = \frac{\sum_{j=1}^{n} u_{ij}^m x_j}{\sum_{j=1}^{n} u_{ij}^m}, \quad i = 1, 2, \cdots c \qquad (14)$$

$$u_{ij} = \left[ \sum_{k=1}^{c} \left( \frac{d_{ij}}{d_{kj}} \right)^{\frac{2}{m-1}} \right]^{-1} \qquad (15)$$

Concrete steps of FCM algorithms are: to set number of clusters $c$ and fuzzy coefficient $m$, and initialize cluster center of each class; to carry out iterative computations repeatedly according to (14) and (15) until the (16) is satisfied. C is a matrix composed of $c_i$ cluster center vector and $\varepsilon$ is given precision.

$$\left\| C^{(k)} - C^{(k+1)} \right\| \le \varepsilon, \; k \ge 1 \qquad (16)$$

### IV. Example Analysis

Data of wind turbine generators of some wind farm at nine o'clock on 1st March, 2012 is provided as cluster data. These generators are clustered with FCM clustering after extraction of characteristic quantities. The wind farm is flat and contains 38 1.5MW doubly-fed induction generators, one of which is under maintenance, so there are 37 generators in actual use, marked as number 1~37. Matlab\ SimPowerSystems is applied to create a simulation environment of wind farm [12]. As to parameter calculation after generator equivalence, reference [8] can be of use. The generator is connected with external power network through point of common coupling (PCC) under the constant power factor. Detailed model and single turbine equivalent model of the wind farm are also built to compare with the approach discussed in this paper. Through fuzzy clustering, generators in the wind farm are divided into four clusters: Cluster 1 {1,2,9,16,17}, Cluster 2 {3,5,6,13,20,22,28}, Cluster 3

{4,7,10,11,12,15,18,21,23,24,25,26,29,35,36}, and Cluster 4 {8,14,19,27,30,31,32,33,34,37}.

Fig. 1 indicates fluctuation of active power and reactive power at point of common coupling in the wind farm. According to the figure, the power fluctuation of wind farm after clustering is consistent with the detailed model, but the single turbine equivalent model is apparently different from the detailed model. Therefore, the multiple-turbine equivalent model after clustering can effectively reduce errors in modeling.
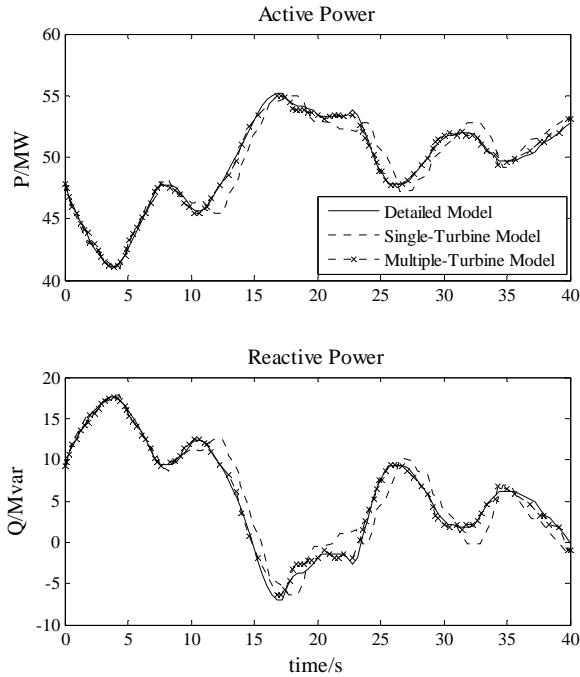


Figure 1. Power fluctuation

To quantify the errors in comparative modeling, (17) and (18) are used to calculate errors of active power and reactive power respectively [3]. In the formula, $P_i$ and $Q_i$ are numerical value of active power and reactive power of the detailed model at moment $i$; $P'_i$ and $Q'_i$ are magnitudes of power of equivalent model at moment $i$. The $E_P$ and $E_Q$ in single-turbine model are 14.32% and 12.67%, and in multiple-turbine model are 2.19% and 1.84%. As a result, multiple-turbine model proposed in this paper is more precise.

$$E_P = \frac{\sum_{i=1}^{n} |P_i - P'_i|}{\sum_{i=1}^{n} P_i} \times 100\% \qquad (17)$$

$$E_Q = \frac{\sum_{i=1}^{n} |Q_i - Q'_i|}{\sum_{i=1}^{n} Q_i} \times 100\% \qquad (18)$$

## V. CONCLUSIONS

Fuzzy clustering is applied in this paper to cluster generators with similar dynamic characteristics. The generators in one cluster can represent one equivalent generator. Through an example simulation, it is proved that multi-turbine equivalent model based on fuzzy clustering has more similarities with detailed model of wind farm, and it is more precise than single-turbine equivalent model. Besides, the number of clusters can be fixed flexibly according to both computational complexity and model precision. In this way, the wind farm can be represented by multiple equivalent generators, so complexity of modeling is reduced.

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