Nonlinear Disturbance Observer-enhanced Depth Control for Submarine: A Sum of Squares Technique

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Abstract—**The state feedback control using sum of squares technique (SOStools) combined with the nonlinear observer is applied to the depth control of a submarine. The mathematical model of the submarine is rewritten as the linear-like form, then the uncertainty caused by the variation of the hydrodynamic coefficients is treated as the disturbance, which can be estimated online by a nonlinear observer. The method can provide good robustness against the nonlinearity as well as the uncertainty, which is verified by the simulation.**

Keywords-submarine;depth control;nonliniear distlurbance observer;SOS

I. INTRODUCTION

The submarine control system is a complex nonlinear system with parameter uncertainties and strong coupling. Large numbers of scholars are participating in this study, and put forward some more fruitful thoughts. Guoliang[1] derived a variable structure controller for submarine. Jian[2] discussed a submarine movement control in vertical plane and derived a robust arithmetic based on mix-sensitivity by studied the state function of submarine linear mode. Shu[3] derived a improved variable structure control method for multiple input-multiple output systems of submarine. The nonlinear disturbance observer achieved good effects in actual system, such as in missiles [5], and aircrafts [6].

This paper proposes a new approach which combines a state feedback controller with a nonlinear disturbance observer. The state feedback controller is formulated using the sum of squares technique (SOStools), and the nonlinear disturbance observer is designed to estimate the nonlinear internal system uncertainty and un-modeled dynamics. The simulation results of submarine in the vertical plane show the controller stability and the observer effectiveness.

II. STATE FEEDBACK CONTROLLER USING SOSTOOLS

Consider the system $\dot{x} = f(x) + g(x)u$ written in the linear-like form:

$$\dot{x} = A(x)x + B(x)u \qquad (1)$$

where $A(x)$ and $B(x)$ are polynomial matrices in x.

For the system (1), let $A_j(x)$ denotes the j-th row of $A(x)$, where $J = \{j_1, j_2, ..., j_m\}$ denote the row indices of $B(x)$ whose corresponding row is equal to zero, and define $\tilde{x} = \left(x_{j_1}, x_{j_2}, ..., x_{j_m} \right)$, then $P(\tilde{x})$ is formulated by $\tilde{x}$.

Our objective is to find a state feedback controller $u(x) = K(x)P^{-1}(x)x$ which renders the equilibrium point $x = 0$ stable. Here we present the lemma 1.

Lemma 1 [4] For a symmetric polynomial matrix $P(x)$ which is nonsingular for all x, we have

$$\frac{\partial P}{\partial x_i}(x) = -P(x)\frac{\partial P^{-1}}{\partial x_i}(x)P(x) \qquad (2)$$

Lemma 2 [4] For the system (1), suppose exist an $N \times N$ symmetric polynomial matrix $P(\tilde{x})$, an $N \times N$ polynomial matrix $K(x)$, two constants $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, such that the following sum of squares problem:

$$v_1^T (P(\tilde{x}) - \varepsilon_1 I) v_1 \qquad (3)$$

$$-v_2^T \left(AP + PA^T + K^T B^T + BK - \sum_{j \in J} \frac{\partial P}{\partial x_j}(A_j x) + \varepsilon_2 I \right) v_2 \qquad (4)$$

are sums of squares, where $v_1 \in \Re^N$ $v_2 \in \Re^{M1}$. Then the state feedback stabilization problem is solvable, and a controller that stabilizes the system is given by

$$u(x) = K(x)P^{-1}(\tilde{x})x \qquad (5)$$

Proof: Assume that there exist solutions $P(\tilde{x})$ and $K(x)$ to (3)-(4). Define the function $V(x)$ as follows:

$$V(x) = x^T P^{-1}(\tilde{x})x \qquad (6)$$

We will show $V(x)$ is a Lyapunov function for the closed-loop system. The time derivate of $V(x)$ is given by

$$\frac{dV(x)}{dt} = x^T \left[\sum_{j \in J} \frac{\partial P^{-1}}{\partial x_j}(\tilde{x})(A_j(x)x) + \left[A(x) + B(x)K(x)P^{-1}(\tilde{x}) \right]^T P^{-1}(\tilde{x}) + P^{-1}(\tilde{x})\left[A(x) + B(x)K(x)P^{-1}(\tilde{x}) \right] \right] x \qquad (7)$$

Now, (7) implies that

$$-\sum_{j \in J} \frac{\partial P}{\partial x_j}(\tilde{x})(A_j(x)x) + P(\tilde{x})A(x)^T + K(x)^T B(x)^T + A(x)P(\tilde{x}) + B(x)K(x) + \varepsilon_2 I < 0 \qquad (8)$$

Multiplying the last expression from the left and right

by $P(\tilde{x})$, and using the Lemma1. If (4) holds with $\varepsilon_2 > 0$ for $x \neq 0$, then $\dfrac{dV}{dt}(x)$ is negative definite, and therefore, the zero equilibrium is asymptotically stable. Finally , if $P(\tilde{x})$ is a constant matrix, then $V(x)$ is radially unbounded, and the stability holds globally.

III. EASE NONLINEAR DISTURBANCE OBSERVERS

In actual application, we should consider the internal system uncertainty and un-modeled dynamics. Consider the system as follows:

$$\dot{x} = A(x)x + B_1 d + B_2(x)u \qquad (9)$$

For the system (9), the uncertainty caused by the nonlinear internal system and un-modeled dynamics is treated as the disturbance. For all x , suppose exist a norm bounded nonsingular matrix B_0 , such that the following expression is equal:

$$B_2(x)B_0 = B_1 \qquad (10)$$

So we can get the disturbance observer from the following lemma.

Lemma 3[5] For the system (6), a nonlinear disturbance observer is designed to estimate the disturbances d , which are slowly time varying. Given by

$$\hat{d} = z + p(x)$$
$$\dot{z} = -l(x)B_1 z - l(x)(B_1 p(x) + A(x)x + B_2(x)u) \qquad (11)$$

where $\hat{d}$ and z are the estimate of the unknown disturbance and internal state of the nonlinear observer, respectively , and $p(x)$ is a nonlinear function to be designed. The nonlinear observer gain $l(x)$ is defined as

$$l(x) = \frac{\partial p(x)}{\partial x} \qquad (12)$$

If $l(x)B_1 > 0$, the estimation error is global exponential stability for all $x \in \Re^n$:

Proof: The estimation error is defined as

$$e_d = d - \hat{d} \qquad (13)$$

Consider the disturbance d is slowly time varying ($\dot{d} \approx 0$), then the time derivative of e_d is given by

$$\dot{e}_d = \dot{d} - \dot{\hat{d}} \approx 0 - \dot{z} - \dot{p}(x) = -\dot{z} - \frac{\partial p(x)}{\partial x} \dot{x} \qquad (14)$$

Submiting (9) and (11), we have

$$\dot{e}_d = l(x)B_1 z + l(x)B_1 p(x) - l(x)B_1 d = -l(x)B_1 e_1 \qquad (15)$$

If $l(x)B_1 > 0$, then the estimation error e_d is global exponential stability for all $x \in \Re^n$.

After the disturbance is estimated by the observer (11), the new closed-loop system controller is given by

$$u^* = u - B_0 \hat{d} \qquad (16)$$

Then the system (9) can be written as

$$\dot{x} = A(x)x + B_1(d - \hat{d}) + B_2(x)u \qquad (17)$$

Then the new controller can eliminate the effect of disturbances, and can improve the control performance of closed-loop system.

IV. DEPTH CONTROLLER OF SUBMARINE

Suppose that only consider the vertical motion of submarine, the other axises motion are neglected. The vertical motion model submarine can be written as follows:

$$\begin{cases} \dot{w} = a_{11}sw + a_{12}sq + a_{13}w|w| + a_{14}q|q| + a_{15}\sin\theta + b_{11}s^2\delta_b + b_{12}s^2\delta_s \\ \dot{q} = a_{21}sw + a_{22}sq + a_{23}w|w| + a_{24}q|q| + a_{25}\sin\theta + b_{21}s^2\delta_b + b_{22}s^2\delta_s \\ \dot{z} = w\cos\theta - s\sin\theta \\ \dot{\theta} = q \end{cases}$$
$$(18)$$

where w is heave velocity; q is the pitch rate; θ is pitch angle; z is submarine depth; δ_b is bow hydroplane angle; δ_s is stern hydroplane angle; the others are kinetic parameters. Let $x = [w \ q \ z \ \theta]^T$, $u = [\delta_b \ \delta_s]^T$, the system (18) can be rewritten as the linear-like form:

$$\dot{x} = \begin{bmatrix} a_{11}s + a_{13}|w| & a_{12}s + a_{14}|q| & 0 & a_{15} \\ a_{21}s + a_{23}|w| & a_{22}s + a_{24}|q| & 0 & a_{25} \\ 1 - \frac{1}{2}\theta^2 & 0 & 0 & s \\ 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} b_{11}s^2 & b_{12}s^2 \\ b_{21}s^2 & b_{22}s^2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u \qquad (19)$$

Suppose the desired state is $x_0 = [0 \ 0 \ z_0 \ 0]^T$, let $e = x - x_0$, the system (18) can be written as the error equation as follows:

$$\dot{e} = A(x)e + B(x)u \qquad (20)$$

In order to verify the effectiveness of the nonlinear disturbance observer, we design a disturbance observer to estimate the uncertainty caused by the pitch angle velocity state equation . The parameters of the disturbance observer can be chosen as follows:

$$l(x) = [0 \ 0.1 \ 0 \ 0] \qquad (21)$$

$$B_1 = [0 \ 1 \ 0 \ 0]^T \qquad (22)$$

$$B_0 = \left[\frac{b_{12}}{u^2(b_{12}b_{21} - b_{11}b_{22})} \ \frac{-b_{11}}{u^2(b_{12}b_{21} - b_{11}b_{22})} \right]^T \qquad (23)$$

V. SIMULATIONS

For the error equation (20), we design a state feedback controller using the result stated in lemma 2. The values of ε_1 and ε_2 are fixed at 0.001.

In this paper, our objective is that the submarine drives from 30m to 60m and moves straight at the certain depth (60m) in the end. The initial state is $x_0 = [0 \ 0 \ 30 \ 0]^T$, the desired state is $x^* = [0 \ 0 \ 60 \ 0]^T$.

$$\delta_b = -0.09w^2 - 0.44wq - 1.35wz + 0.36w\theta -$$
$$67.15q^2 - 0.1q\theta + 5.84qz + 0.01\theta^2 - 0.02\theta z +$$
$$0.23z^2 + 7.9w - 34.05q + 0.69\theta + 1.2z - 5 - \hat{\delta}_b \quad (24)$$

$$\delta_s = -0.11w^2 - 0.90wq - 1.8wz + 1.3w\theta - 35.15q^2 -$$
$$0.04q\theta + 5.7qz + 0.02\theta^2 - 0.04\theta z + 0.53z^2 + 10.1w -$$
$$21.05q + 1.2\theta + 1.5z - 10.3 - \hat{\delta}_s \quad (25)$$

The time history of the depth, the pitch, the bow hydroplane angle, and the stem hydroplane angle are shown from Fig.1 to Fig.4 .From the simulation results, it can be shown that the controller can complete control of the spatial movement of submarine mission, and effectively against the nonlinearity as well as the uncertainty.
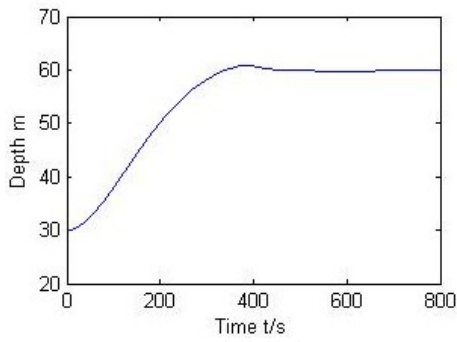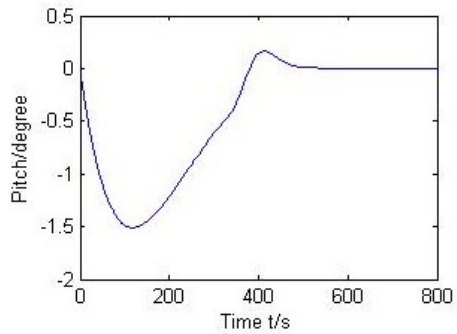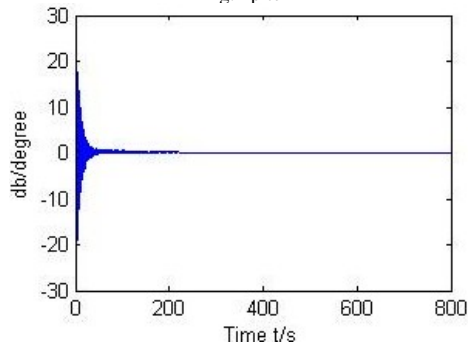

Fig.1 depth

Fig.2 pitch
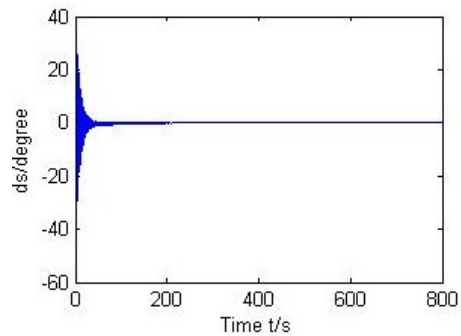

Fig.3 bow hydroplane angle

Fig.4 stem hydroplane angle

VI. SUMMARY

According to the simulation in this paper, the state feedback controller combined with the nonlinear disturbance observer has a good control performance against the nonlinearity and the uncertainty. The state feedback control using the sum of squares technique (SOStools) can complete control of the spatial movement of submarine mission. The controller combined with the nonlinear disturbance observer, which can well eliminate the control error which caused by the model uncertainy and satisfactory dynamic behavior.

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