# Single Channel Signal Separation of GMSK Signals Based on MLP

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Abstract—In according to the issue of multi-signal jamming in communication reconnaissance, single channel signal separation for multi-GMSK signals has been studied with a method based on MLP. With parameters of Doppler-shift, time-delay, amplitude and coding sequences efficiently estimated, signals could be restructured, and then be separated. Simulations have proved well separation results can be obtained with the method for unequal power signals with certain SNRs.

Keywords-single-channel, GMSK, maximum likelihood principle (MLP), signal separation

# I. INTRODUCTION

Nowadays usually multiple signals might be received in reconnaissance in the communication electromagnetic circumstances. Mutual interferences of signals make it difficult for reconnaissance. Thus study on separation algorithms for communication signals is always an important issue. And signal channel blind signal separation (SCBSS) is a new aspect for studying. In references [1,3,4], blind separation of MPSK signal in condition of single channel has been discussed, and also two QPSK signals blind separation in reference [2]. Otherwise, priori information has not been used, or fully used in these references. And some parameters of real communication systems are usually public, such as modulate pattern, symbol length, symbol rate. Based on this fact, single channel signal separation of multiple GMSK has been studied.

# II. SIGNAL MODEL

In condition of single channel, signal model can be expressed as

$$x(t) = \sum_{i=1}^{K} A_i s_i(f_{di}, \tau_i) + n(t)$$
 (1)

$$s_i(f_{di}, \tau_i) = \exp\{j * [2\pi (f_c + f_d)(t - \tau_i) + \varphi(t - \tau_i)]\}$$
 (2)

here  $f_c$  denotes carrier frequency, K denotes number of signals, and  $A_i$ ,  $f_{di}$ ,  $\tau_i$  denote amplitude, Doppler-shift, time-delay of the  $i^{th}$  signal received. n(t) is a Gaussian noise with power of  $\sigma^2$ .

Waveform of a GMSK-modulated signal could usually be written as

$$s(t) = \exp\{j * [2\pi f_c t + \varphi(t)]\}$$
 (3)

And  $\varphi(t)$  here is carrier phase. It is the signal energy for a symbol period,  $T_b$ .

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$$\begin{split} \varphi(t) &= \pi \sum_{k = -\infty}^{\infty} a_k q(t - kT_b) \\ \text{here } a_k &= \pm 1 \text{, and } q(t) = \frac{1}{2T_b} \int_{-\infty}^{1} g(\tau - \frac{T_b}{2}) d\tau \\ g(t) &= \frac{1}{2} \left\{ erfc \left[ \frac{2\pi B_b}{\sqrt{2\ln 2}} (t - \frac{T_b}{2}) \right] - erfc \left[ \frac{2\pi B_b}{\sqrt{2\ln 2}} (t + \frac{T_b}{2}) \right] \right\} \end{split}$$

g(t) is the response of Gauss filtering to rectangular pulse,  $B_b$  is the 3dB bandwidth. Taking g(t) into q(t)

$$q(t) = \begin{cases} 0 & t < -(L-1)T_b/2 \\ 1/2 & t > (L+1)T_b/2 \\ \frac{1}{2T_b} \int_{-\infty}^{1} g(\tau - \frac{T_b}{2}) d\tau & else \end{cases}$$
 (5)

# III. SIGNAL SEPARATION BASED ON MLE

The core idea of separation algorithms is to separate signals with the basis of signal restructure. Firstly, make fuzzy relation of the received signals, then judge whether there are signals or not, if signals exist, estimate the parameters of the strongest one. Then restructure the signal and separate it. Step by step, while there are no signals, just stop.

### A. Parameters Estimation

# a. Doppler-Shift and Time-Delay

On course of real communication, there might be a certain Doppler-shift and time-delay. And Doppler-shift and time-delay of different signals are often different too. With this basis, we can make two dimensional ambiguity correlations in order to estimate the Doppler-shift and time-delay of GMSK signals.

The ambiguity function is as follows

$$r(\omega, \tau) = \operatorname{Re}\left(\frac{\sum_{n} x(t_n) s(t_n - \tau) e^{-j\omega t_n}}{MT_b / T_s}\right)$$
 (6)

 $s_0(t)$  is the unity-magnitude signal of a known symbol sequence modulated by GMSK, such as the sign of midamble and beginning sequence. M is the length of the known symbol sequence.  $T_b$  is the symbol period and  $T_S$  is sampling period.  $t_n$  is the time of sampling. x(t) is the data after sampling of received signals.

For ambiguity functions, as is known to all, while the Doppler-shift and time-delay of one GMSK modulated signal can be fully matched with  $s_0(t)$ , there will be a peak. And because of normalization, the value of peaks just denotes the signal amplitude, and the method for amplitude estimation will be discussed later. So we can judge whether there are GMSK modulated signals or not after the judgment of exist of peaks and the value of peaks. The  $\omega$  and  $\tau$ relative to each peak are actually the estimation of Dopplershift and time-delay.

#### Symbol Sequence Estimation

However there are only two stations, 0 and 1, of the symbol. Thus we can estimate the state value of the next symbol with the known symbol sequences. Step by step, symbol sequence of the whole signal can be estimated.

Here a method for estimating the state value based on MLE has been studied. Considering the correlation length with modulation L equals to 3.

Here an estimation method using the former three known-symbols to deduce the next unknown one has been analyzed.

The detail of this method can be described as follows.

Denote N as the code element length of signals,  $c_{ln}$  as the  $n^{th}$  code value of the  $l^{th}$  signal. Supposing a known code sequence (32bit) is used as the start number, so  $\{c_{l,n}, n=1,2,\cdots 32\}$  is known in advance. Considering n = 33, then calculate  $\alpha$  and  $\beta$  respectively.

Supposing  $c_{ln} = 1$ , and order  $\alpha$  as the correlation result between the sample aggregate of the conference signal of the four code elements  $\{c_{l,n-3},c_{l,n-2},c_{l,n-1},c_{l,n}\}$  and  $x(t_n)$ , here

$$\tau_l + (n-3)T \le t_n \le \tau_l + nT$$

i.e.,

$$\alpha = \left| \sum_{n} x(t_n - \tau_l) s_0^* \{ c_{l,n-3}, c_{l,n-2}, c_{l,n-1}, c_{l,n} \} e^{-j\omega t_n} |_{c_{l,n}=1} \right|$$
 (7)

If  $c_{ln} = 0$ , the correlation result is  $\beta$ 

$$\beta = \left| \sum_{n} x(t_n - \tau_l) s_0^* \{ c_{l,n-3}, c_{l,n-2}, c_{l,n-1}, c_{l,n} \} e^{-j\omega t_n} |_{c_{l,n} = 0} \right|$$
(8)

A judgment need to be made for decision of the value of  $c_{ln}$  through comparison of  $\alpha$  and  $\beta$ .

If 
$$\frac{\alpha}{\alpha + \beta} \ge \frac{\beta}{\alpha + \beta}$$
, i.e.,  $\frac{\alpha}{\beta} \ge 1$ , then we can make  $c_{ln} = 1$ ,

Supposing n = n + 1, that is to say, the same could be done till n = N, and then the whole symbol sequence can be estimated, written as  $\{\hat{c}_{l,n}, n = 1, 2, \dots N\}$ .

# Amplitude Estimation

With the estimated code sequence,  $\hat{c}_{l,n}$ , modulated by GMSK,  $s_{0l}(t)$ , a signal with unit amplitude can be obtained,  $n = 1, 2, \dots N$ . While the observing length of signal is  $N_0$ , noting the observing vector as  $x_k$ ,  $k = 1, 2, \dots N_0$ ,

then the maximum likelihood (ML) function of received signals can be drawn as

$$p(x|A_l) = (2\pi\sigma^2)^{\frac{N_0}{2}} \exp\left\{-\sum_{k=1}^{N_0} \frac{x_k - \sum_{i=1}^K A_i s_{ik}}{2\sigma^2}\right\}$$
(9)

The restriction of MLE is, 
$$\frac{\partial \ln[p(x|A_l)]}{\partial A_l} | A_l = \hat{A}_l = 0$$
 (10)
Simultaneously, code sequences transmitted by each

Simultaneously, code sequences transmitted by each signals are thought to be independent and identically distributed (i.i.d.), i.e.,

$$E\left\{\sum_{m}\sum_{n\neq m}s_{im}s_{in}\right\} = 0, \quad i = 1, 2, \dots, K$$
 (11)

Thus,

$$\hat{A}_{l} = \frac{1}{N_{0}} \sum_{k=1}^{N_{0}} x_{k} s_{l0,k} - \sum_{i=1, i \neq l}^{K} \left( \frac{A_{i}}{N_{0}} \sum_{k=1}^{N_{0}} s_{ik} s_{l0,k} \right)$$
(12)

Considering the signals are independent, so while  $N_0$  is relatively large, the second part of the right of equation (12) is very small due to the first one. That is to say, (12) can be approximately written as follows,

$$\hat{A}_{l} = \frac{1}{N_0} \sum_{k=1}^{N_0} x_k s_{l0,k}$$
 (13)

#### B. Flow Chart of Separation

It is a key problem for how to judge whether there are signals or not through the two dimensional ambiguity figures. And naturally it is a question in designing the threshold.

Considering

$$\xi = \frac{\max_{\omega, \tau} \{ r(\omega, \tau) \}}{\sqrt{E\{x(t)x^*(t)\}}}$$
 (14)

If there is no signal, the

$$x(t) = n(t)$$

Put the above equation into (14)

$$\xi_0 = \frac{\max_{\omega, \tau} \left\{ r(\omega, \tau) \right\}}{\sqrt{E[n(t)n^*(t)]}}$$
(15)

here  $E\{n(t)n^*(t)\}=E_n$ ,  $\max_{\omega,\tau}\{r(\omega,\tau)\}\approx 0$ .

Contrarily, while signals have been received,

$$E\{n(t)n^{*}(t)\} = \sum_{i=1}^{K} A_{i}^{2} + E_{n}$$
 (16)

$$\max_{\omega,\tau} \{ r(\omega,\tau) \} \approx \max_{i} \{ A_{i} \}$$
 (17)

Taking equation (1) into (14),

$$\xi_{s} = \frac{\max_{\omega, \tau} \left\{ r(\omega, \tau) \right\}}{\sqrt{E\left\{ x(t) x^{*}(t) \right\}}} \approx \frac{\max_{i} \left\{ A_{i} \right\}}{\sum_{i=1}^{K} A_{i}^{2} + E_{n}}$$
(18)

Thus the judgment threshold,  $\xi_T$ , can be chosen as

$$\xi_0 \le \xi_T < \frac{\min_i \{A_i\}}{\min_i \{A_i^2\} + E_n}$$
 (19)

Consider  $\xi_T = 0.1$ , and all signals can be separated successfully, then only the weakest one and noises would be left. The equation below can be obtained.

$$\frac{\min_{i} \left\{ A_{i}^{2} \right\}}{E_{n}} > \frac{0.01}{0.99} \tag{20}$$

From equation (20), while the ratio of the weakest signal and noise in power is no less than -20dB, signals can be well separated.

Above all, the separation flow chart is shown in fig.1.

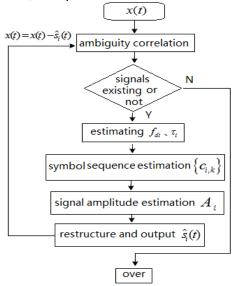
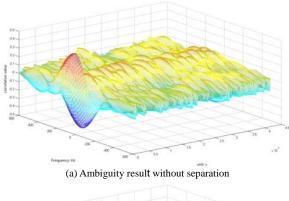
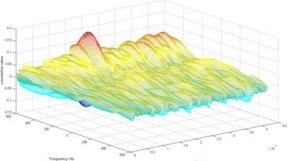


FIG.1 FLOW CHART OF SEPARATION

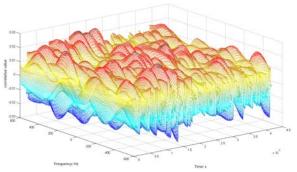
# IV. ALGORITHM SIMULATIONS

Simulation 1: Gaussian and white noise, power ratio of two signals is  $E_1/E_2=10$  dB,  $E_s/E_n=-10$  dB. Here, we mean that  $E_s=E_1+E_2$ , and the amplitude of the two signals is respectively  $A_1=20$  and  $A_2=6.3246$ . The Doppler frequency shift is respectively  $f_{d1}=0$  Hz and  $f_{d2}=400$  Hz. Taking signal  $s_1$  as the referenced signal, the time delay difference between them is  $\Delta \tau = \tau_2 - \tau_1 = 16T_b$ . The threshold  $\xi_T=0.1$ , each code element has been 80 points sampled.









(c) Ambiguity result after  $s_1$ ,  $s_2$  being separated

Fig. 2 2-D ambiguity result on course of separation

Fig.2 demonstrates the normalized 2-D ambiguity results on course of separation. (a) is the ambiguity result between separation, and from the picture, there is a quite big peak higher than the threshold because the power of  $s_1$  is far larger than  $s_2$ . So signal  $s_1$  can be separated firstly. (b) gives the ambiguity result after  $s_1$  been separated. And there is still a peak with quite large height. The power of  $s_2$  is relatively low, only about 0.1453, which is a little larger than the threshold. So parameters estimation can be continued to separate  $s_2$ . In fig.1 (c),  $s_1$  and  $s_2$  have been both separated, because the power of signals left is very small, there are so many peaks in the figure with no order. The values of peaks are all very small, just about 0.025, and are lower than the threshold. Then it is thought no signals existed, the separation process is over.

Fig.3 and Fig.4 show the waveforms of each signal source and the separated signal waveforms. Through the figures, a conclusion can be drawn that the result is relatively good.

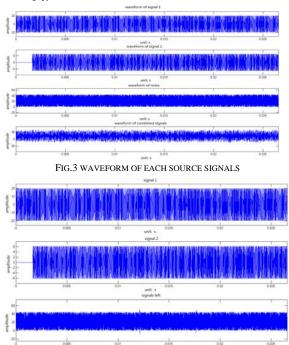


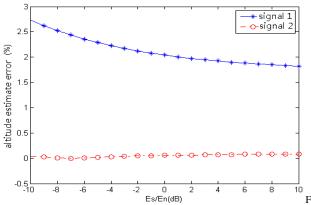
FIG.4 WAVEFORM OF EACH SEPARATED SIGNAL

Simulation 2:  $E_1 / E_2 = 5$  dB, variable  $E_s / E_n$ 

In FIG.5, estimation error of signal amplitude with variable  $E_s/E_n$  is shown with 100 times of Monte Carlo experiments. From the figure, conclusions can be given that while the SNR is increasing, amplitude estimation error of the strong signal  $s_1$  would be gradually smaller and approach to a stable value. And amplitude estimation error of  $s_2$  is nearly to be invariable. In comparison, amplitude estimation error of the strong signal  $s_1$  is far larger than that of  $s_2$ . Because while estimating  $s_1$ , influence of  $s_2$  is actually in existence. And when estimating  $s_2$ ,  $s_1$  has already been separated, only quite a little part of energy left. So only the noise would influence the estimation error.

Simulation 3:  $E_s / E_n = 10 \, dB$ , variable  $E_1 / E_2$ 

In FIG.6, estimation error of signal amplitude with variable  $E_1/E_2$  is shown with 100 times of Monte Carlo experiments. From the figure, conclusions can be given that while  $E_1/E_2$  is increasing, influence of  $s_2$  to  $s_1$  become smaller and smaller. But influence of noise to  $s_2$  has been relatively strengthened. Thus, amplitude estimation error of  $s_1$  becomes smaller, and estimation error of  $s_2$  becomes larger gradually.



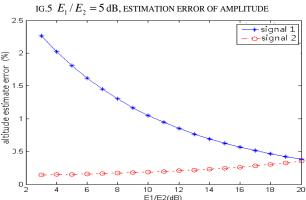


Fig. 6  $E_s / E_n = 10$  dB, estimation error of amplitude

# V. CONCLUSIONS

Signal separation issues for single channel in condition of multiple GMSK signals have been studied in this thesis. And a separation method based on ML criterion has been discussed. Basic principle of this method is through efficient estimation parameters to reconstruct signals, then making efficient separation reality. With simulations, due to unequal power signals, good separation results can be obtained by the algorithm with certain SNR.

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