A new joint algorithm based on EMD-ICA to reduce the cross-term in Wigner-Vill distribution

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***Abstract*—Wigner-Vill distribution (WVD) will inevitably have cross-terms when it used for time-frequency representation in a multi-component signal. In order to suppress the cross-terms in Wigner-Vill distribution, this paper proposes a joint algorithm based on EMD-ICA. This algorithm resolves a multi-component signal into several IMF components used by EMD at first, and then each IMF component is used FastICA algorithm for processing, and next seeks the Wigner-Vill distribution of each component, finally, add up the results. This method effectively inhibited the emergence of cross-terms in Wigner-Vill distribution, and keeps the properties of time-frequency concentration higher.**

Keywords—Wigner-Vill distribution; cross-term; EMD; ICA

I. INTRODUCTION (HEADING 1)

In signal processing, the frequency spectrum and power spectrum of signal are two of the most important physical quantity. But frequency spectrum or power spectrum can not express the specific time of frequency component appeared and its change trend. Correlation function and power spectrum of non-stable signals are time-varying functions, what we want the most is the situation of such frequency spectrum or power spectrum changed over time, so we can use the joint function of time and frequency to describe the condition of frequency spectrum or energy density with time. The energy presentation of non-stable signals is called time-frequency distribution.

Wigner-Vill distribution [1] is the first of the time-frequency distribution, and is the most important one. But it has to appear cross-term when it is used for seeking the time-frequency distribution of quadric form in multicomponent signal. Then more and more methods of time-frequency distribution sprung up, like Pseudo-Wigner-Vill distribution, narrow band ambiguity function and so on [2]. Although they have a little effect in reducing the cross-term in Wigner-Vill distribution, it reduces the time-frequency concentration. this paper proposes a joint algorithm based on EMD-ICA. This algorithm resolves a multi-component signal into several IMF components used by EMD at first, and then each IMF component is used FastICA algorithm for processing, and next seeks the Wigner-Vill distribution of each component, finally, add up the results. This method effectively inhibited the emergence of cross-terms in Wigner-Vill distribution, and keeps the properties of time-frequency concentration higher.

II. WIGNER-VILL DISTRIBUTION INTRODUCED

Time impulse function $\phi(u-t,\tau)=\sigma(u-t)$ (τ is not limited and take the instantaneous value in time-domain) is used for window function, correlation function in part is given by

$$\begin{aligned} R_z(t,\tau) &= k_z(t,\tau) \\ &= \int_{-\infty}^{\infty}\sigma(u-t)z(u+\frac{\tau}{2})z*(u-\frac{\tau}{2})du \\ &= z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2}) \end{aligned} \quad (1)$$

Instantaneous correlation function $k_z(T,\tau)$ is Fourier transformed about hysteretic τ to get given as

$$W_z(t,\omega)=\int_{-\infty}^{\infty}z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})e^{-j\omega\tau}d\tau=F_{\tau\to\omega}[k_z(t,\tau)] \quad (2)$$

Here, $F_{\tau\to\omega}$ expresses Fourier transformed about hysteretic τ .

WVD is also defined as this through $Z(\omega)$ frequency spectrum of signal.

$$W_z(\omega,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}Z(\omega+\frac{v}{2})Z^*(\omega-\frac{v}{2})e^{jvt}dv \quad (3)$$

$z(t)$ in formula (1) and $Z(\omega)$ in formula (2) have appeared twice, so WVD is one of bilinear time-frequency distribution (BTFD), and cross-term will be appeared unavoidable.

Figure 1 shows the WVD of signal consist of four components of Gaussian components; it contains four autocorrelation and six cross-terms (two of them overlap).

III. A NEW JOINT ALGORITHM BASED ON EMD-ICA IN WVD

A. EMD(Empirical Mode Decomposition)

EMD is a digital signal processing method put forward by N.E.Huang [3] comes from NASA. In essence, it seeks Eigen vibration model through characteristic time scale, and then, the time series data is separated by Eigen vibration model. Any signal can be expressed as numbers of components IMF (intrinsic mode function) and residual component RES through EMD [4]. The general procedure is as follows:

A) Find all the maximum and minimum value of signal $x(t)$, mingle them for the upper and lower envelopes through cubic spline function, and calculate the average m_1, then calculate the difference h_1 between m_1 and source signal, it can be written as

$$h_1 = x(t) - m_1 \tag{4}$$

B) Repeating the above process until h_1 satisfy the condition of IMF, that h_1 is for the new signal, and calculate the average, then calculate the difference h_{11} between m_{11} and h_1 it can be written as

$$h_{11} = h_1 - m_{11} \tag{5}$$

C) The process is proceeding until it gets the first of IMF component c_1, then r_1 has been determine which is the difference between c_1 and source signal, it can be written as

$$r_1 = x(t) - c_1 \tag{6}$$

D) Repeat process of Step1 under r_1 as the source signal, then $r_2 = r_1 - c_1, ..., r_n = r_{n-1} - c_{n-1}$, until r_n can not break down or meaningless. EMD can be written as

$$x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t) \tag{7}$$

B. Independent Component Analysis

ICA (Independent Component Analysis) [5], which is introduced into the field of process industry as a data analysis method, its characteristics are to decompose the target signal into independent components without knowing any other priori knowledge besides the statistical independence between source signals. FastICA algorithm is the fast algorithm one in ICA [6, 7]; it is based on the maximization principle of non-Gaussian, and seeks non-Gaussian maximum value through fixed point iteration theory. In this algorithm, plenty of sampling points of observed variable is processed by Newton iteration algorithm, maximize negentropy as the objective function; an independent component is separated from observation signal every time. The general procedure is as follows:

A) Signal X is removed mean and processed whitening, and then the standardizing x is obtained.

B) The number of source signal m which are chose is confirmed;

C) All the ω are initialized by matrix $W = (\omega_1 \omega_2, ..., \omega_m)^T$, so that they have unit norm, and is
made symmetric and orthogonal by the following formula

$$W \leftarrow (WW^T)^{-\frac{1}{2}} W \tag{8}$$

D) ω_i is updated for every $i = 1, 2, ..., m$

E) The above steps continue to update until W which has been already symmetric and orthogonal is convergent.

F) Every source signals are extracted from composite signal, and it can be written as

$$y = Wx. \tag{9}$$

C. The new joint algorithm based on EMD-ICA

The new joint algorithm based on EMD-ICA is put forward in this paper. The cross-term in WVD is effectively eliminated through this joint algorithm. The general procedure of the new joint algorithm is as follows:

A) The analysed signal decomposes for limited intrinsic mode functions and a residual component by EMD, it can be written as

$$X(t) = \sum_{i=1}^{n} imf_i + r_n \tag{10}$$

B) The related coefficient is calculated between the analysed signal and every imf_i. Because of the error of the interpolation and boundary effect in the calculation method of local mean numerical through the process of EMD, the numbers of component which are decomposed by EMD are more than components of source signal. So the imf pseudo components which have the low related coefficient are removed. The remainder imf components are used by FastICA algorithm which is the fast algorithm one in ICA algorithm, and then the new imf' components have been obtained. From the FastICA algorithm, the independence can maximize between each imf', and the frequency position is clearer.

C) Every imf' component is sought for its Wigner-Vill distribution, and adding the results, this is the Wigner-Vill distribution of original signal.

D. Simulation Experiment

In order to validate the effectiveness of the method proposed in this paper, mixed three sine signals for the original signal, the three sine signals are

$s_1 = 5\sin(200\pi t)$ $s_2 = 2\sin(100\pi t)$ $s_3 = \sin(50\pi t)$, sampling frequency is 1 kHz.

Figure 2 is time-domain chart of the mixed signal composed by the above three sine signals. The above three sine signals are respectively to seek the Wigner-Vill distribution, and add the results, the time-frequency representation as shown in Figure 3. Figure 4 is the Wigner-Vill distribution of the mixed signal, there is a lot of cross-terms when it is compared with Figure3, it serious influence the time-frequency distribution effect, and make the results of analysis cannot be explained.

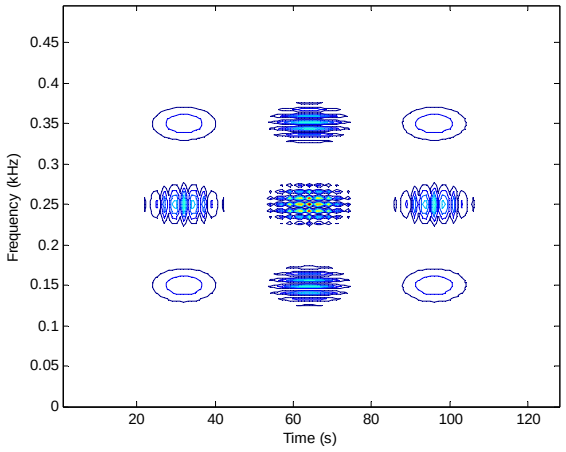

Figure 1. The WVD of four Gaussian components signal.

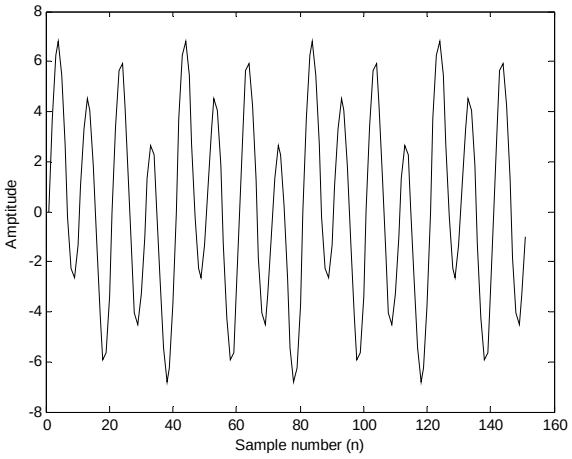

Figure 2. The time domain chart of mixed-signal.

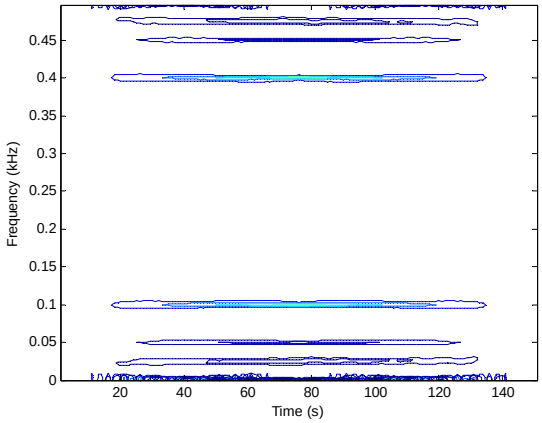

Figure 3. The WVD of the three sine signal.

The mixed signal is decomposed by EMD, and then the IMF components which have the low related coefficient compared with original signal are removed. Each of the remaining three IMF components is sought for its Wigner-Vill distribution, and adding the results, it as is shown in Figure 5, the cross-terms have been reduced.

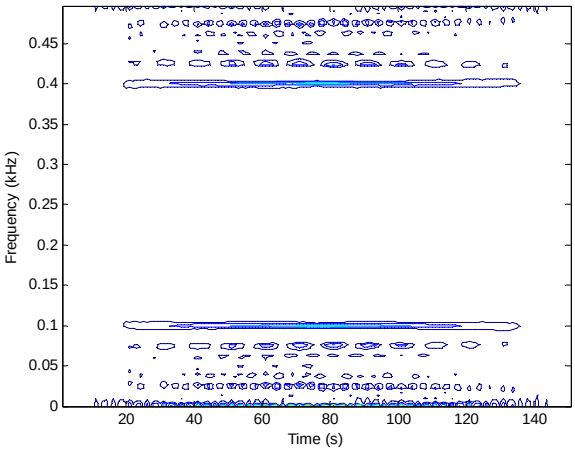

Figure 4. The WVD of the mixed-signal.

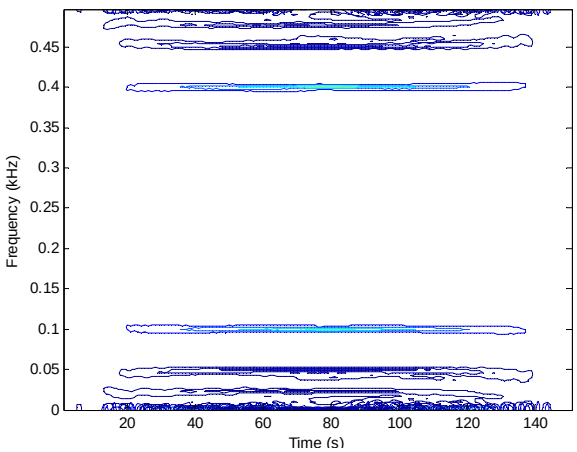

Figure 5. The WVD of the three intrinsic mode functions after separation using EMD.

IV. CONCLUSIONS

Wigner-Vill distribution is a powerful tool to describe the energy, density and intensity of signal in different time or frequency, but the cross-term in WVD is very troubled. From pseudo Wigner-Vill distribution to smoothed pseudo Wigner-Vill distribution, the main purpose of them is decreased the interference of cross-term, but the time-frequency resolution is reduced by them. From the references [8] we know that there is not have the time-frequency distribution which has high time-frequency resolution and doesn't have cross-term. So all of the method which structuring the kernel function to eliminate the cross-term can reduce the time-frequency resolution. In this paper, the cross-term has been reduced in another way. Mixed signal is decomposed into several single-component signals through the joint algorithm based on EMD-ICA, and then the each decomposed signal is computed by WVD and adds the results. At last, the simulation results proved that the joint algorithm based on EMD-ICA has been reduced the cross-term in Wigner-Vill distribution and keep the high time-frequency resolution.

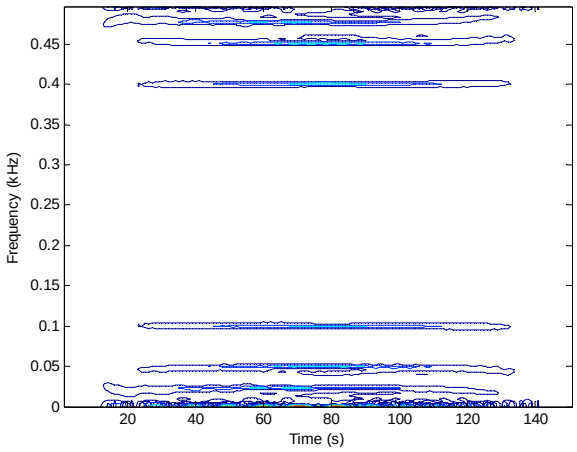

Figure 6. The WVD of the mixed signal with using the joint algorithm based on EMD-ICA.

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