Difference Curvature Driven Anisotropic Diffusion for Image Denoising Using Laplacian Kernel

Yiyan WANG[1,2] Zhuoer WANG[3]
1. Department of Physics and Engineering Technology
Sichuan University of Arts and Science
Dazhou, China
2. Laboratory of Image Science and Technology
School of Computer Science and Engineering
Southeast University
Nanjing, China
E-mail: wang_yiyan8207@163.com
3. Library of Sichuan University of Arts and Science
Dazhou, China
E-mail: vivianwang8403@163.com

***Abstract*—Image noise removal forms a significant preliminary step in many machine vision tasks, such as object detection and pattern recognition. The original anisotropic diffusion denoising methods based on partial differential equation often suffer the staircase effect and the loss of edge details when the image contains a high level of noise. Because its controlling function is based on gradient, which is sensitive to noise. To alleviate this drawback, a novel anisotropic diffusion algorithm is proposed. Firstly, we present a new controlling function based on Laplacian kernel, then making use of the local analysis of an image, we propose a difference curvature driven to describe the intensity variations in images. Experimental results on several natural and medical images show that the new method has better performance in the staircase alleviation and details preserving than the other anisotropic diffusions.**

Keywords-image denoising; partial differential equation; difference curvature; Laplacian kernel

I. INTRODUCTION

The degradation of an image is usually unavoidable during its acquisition and transmission[1]. It is necessary to apply an efficient denosing technique to compensate for image corruption[2]. Denoising algorithm performance mainly depends on a suitable representation to describe the original image information. Image noise removal remains a challenge since it introduces artifacts and causes blurring. One of the major concerns in image denoising methods is their edge preservation capability. Among a variety of the develop denoising techniques[3], partial differential equation(PDE) based on models[4-10] have been widely used over the past few decades, due to its great advantage that it can preserve image edges while reducing noise. The basic idea is to deform a curve, a surface or an image using a partial differential equation, and to obtain the desired results as the solution of this equation with the noisy image as initial conditions. Witkin[11] first found that the convolution of a signal with Gaussians as each scale was equivalent to solving a heat diffusion equation with the signal as an initial value. However, this model is referred as isotropic diffusion. The disadvantage of isotropic diffusion is that it is symmetric and orientation insensitive, leading into blurred edges. Perona and Malik(PM)[4] developed an anisotropic diffusion process as a nonlinear image noise removal method, which analogized heat diffusion to adaptively remove the noise of the images. The main idea of anisotropic diffusion is that it encourages intra-region smoothing and discourages inter-region at the edges[5]. The decision on local smoothing is based on a diffusion coefficient, which is a function of the local image gradient. When the gradient is low, the smoothing takes place. Meanwhile, the smoothing is suppressed where the gradient is high and an edge exists. However, it is found that the anisotropic diffusion is sensitive to noise as the whole model is gradient-driven[6], which often suffers the staircase effect and the loss of edge details when the image contains a high level of noise[7]. To overcome this shortcoming, we propose a difference curvature driven anisotropic diffusion to smooth noisy images in this paper. Firstly, we present a new controlling function based on Laplacian kernel, then we employ the difference curvature instead of gradient to control the diffusion coefficient, which plays an important role in filtering. Comparative experimental results on both natural and medical images demonstrate that the proposed method can improve the staircase effect and yields better PSNR and RMSE than the other diffusions.

The outline of this paper is as follows. In Section II, at first, a simple introduction of the anisotropic diffusion is given, then we propose a difference curvature driven anisotropic diffusion scheme based on Laplacian Kernel. Section III shows the extensive experiments results and comparison. Finally, our conclusion is drawn in Section IV.

II. PROPOSED METHOD

A. *Anisotropic diffusion*

The application of partial differential equations(PDEs) in image process[10] has grown greatly in the past years. The original PDE filtering model, proposed by Witkin[11], which found that the convolution of a signal with Gaussians as each scale was equivalent to solving a heat diffusion equation with the signal as an initial value. With regard to an image u_0 , this process can be described with the following partial differential equation:

$$\frac{\partial u}{\partial t} = M\Delta u(x,t) \qquad (1)$$

Where M is the diffusion conductance, and Δ is the Laplaican operator, the original image u_0 is taken as the initial state of the differential equation in (1). This model is referred as linear heat equation that diffuses in all directions and destroys edges. Pernal and Malik(PM) were the first to try such an approach through controlling the speed of the diffusion and proposed a nonlinear adaptive diffusion process[4], termed as anisotropic diffusion. The PM nonlinear diffusion equation is of the form:

$$\frac{\partial u}{\partial t} = div\left[c\left(|\nabla u|\right)\nabla u\right] \qquad (2)$$

Where u is the evolving image derived from the original image u_0 at t time, and "∇" and "div" are the gradient and divergence operators, respectively, i.e.

$$\nabla u = \left(\partial u/\partial x\right) i + \left(\partial u/\partial y\right) j \qquad (3)$$

$$div\left(c|\nabla u|\nabla u\right) = \left(\partial/\partial x\right)\left(c|\nabla u|\partial u/\partial x\right) + \left(\partial/\partial y\right)\left(c|\nabla u|\partial u/\partial y\right) \qquad (4)$$

The function $c\left(|\nabla u|\right)$ controls the speed of the diffusion. It is a nonnegative monotonically decreasing function of the gradient. A desirable characteristic of the conductance function is that the diffusion speed at edges (where $|\nabla u|$ is large) is low and the edges within the image are kept intact, while the diffusion is encouraged within flat regions (where $|\nabla u|$ is small). Using gradient as an edge indicator, it requires that $c\left(|\nabla u|\right)$ goes to zero when $\nabla u \to \infty$ and goes to 1 when $\nabla u \to 0$. The commonly used types of controlling speed function with the above quality include:

$$c\left(|\nabla u|\right) = \frac{1}{1+\left(|\nabla u|/K\right)^2} \qquad (5)$$

$$c\left(|\nabla u|\right) = \exp\left(-\left(|\nabla u|/K\right)^2\right) \qquad (6)$$

Where K is the conductance parameter that influences the diffusion process. In the PM model, diffusion takes place according to the controlling function $c(\cdot)$ to reduce the smoothing effect near edges, however, this controlling function depends on the gradient. It is well known that the PM diffusion often suffers the staircase effect and the loss of edge details especially when noise contained in the image is rather large, because the gradient indicator is sensitive to noise. Moreover, noise will cause infinity of gradient value in theory. As such, it will result in discounted denoising performance of the anisotropic diffusion.

You and Kaveh[8] improved the function $c(\cdot)$ by using the Laplacian image of u (i.e., Δu , where $\Delta u = \partial^2 u/\partial^2 x + \partial^2 u/\partial^2 y$) in the function $c(\cdot)$ instead of the gradient image of u , and proposed the fourth-order PDE model where the controlling speed function $c(\cdot)$ was defined the following form[9] :

$$c\left(\Delta u\right) = \frac{1}{1+\left(|\Delta u|/K\right)^2} \qquad (7)$$

Although the fourth-order PDE can reduce the staircase in the denoised image, they have the disadvantage of blurring edges. Moreover, the mathematical problems is much challenging.

To overcome this shortcoming, we propose a difference curvature method in the following.

B. Difference curvature diffusion

Firstly, we present a new controlling function based on Laplacian kernel as:

$$c_L\left(|\nabla u|\right) = \exp\left(-|\nabla u|/K\right) \qquad (8)$$

It is compared with two controlling functions of PM diffusion from Fig1.

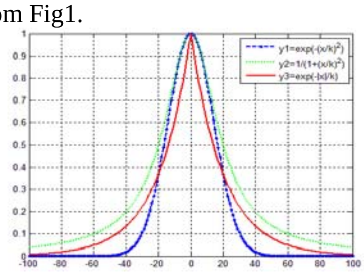


Fig1. Comparison of three controlling functions

As shown in Fig1, compared with the two controlling functions of PM diffusion, the diffusion speed of Laplacian kernel is between them at edge (where $|\nabla u|$ is large), and it is smaller than them within flat regions (where $|\nabla u|$ is small). Meanwhile, unlike gradient and divergence, we define a new edge indicator as:

$$S = \left\| u_{\tau\tau}| - |u_{\zeta\zeta} \right\| \qquad (9)$$

Where $u_{\tau\tau}$ and $u_{\zeta\zeta}$ represent the second derivatives in the direction of the gradient ∇u and in the direction perpendicular to ∇u , respectively.

$$u_{\tau\tau} = \frac{u_x^2 u_{xx} + 2u_x u_y u_{xy} + u_y^2 u_{yy}}{u_x^2 + u_y^2} \qquad (10)$$

$$u_{\zeta\zeta} = \frac{u_y^2 u_{xx} - 2u_x u_y u_{xy} + u_x^2 u_{yy}}{u_x^2 + u_y^2} \qquad (11)$$

In this paper, we call the proposed edge indicator difference curvature. It reflects the intensity variation from the standpoint of difference curvature, and it will be large in the region of edge while small in flat and ramp regions[12], so edges can be distinguished from flat and ramp regions based on the value of difference curvature. Using this new concept to the whole image ,we can obtain the image's corresponding difference curvature space. Applying this difference curvature space and controlling function based on Laplacian kernel, we improve the original anisotropic diffusion as follows:

$$\frac{\partial u}{\partial t} = div\left[c_L(S)\nabla u\right] \qquad (12)$$

Where $c_L(\cdot)$ is the Laplacian kernel function and S is difference curvature, which have been defined in (8) and (9) above.

Compared to the original anisotropic diffusion's flaw of inaccurate estimation of the edge, especially when the image contains a high level of noise, the difference curvature diffusion for images is more efficient than the gradient diffusion, because the difference curvature is more apt than the first order derivative of gradient to extract the intensity oscillations, which is common in an image. Comparative experimental results on both natural and medical images discussed later will demonstrate that the proposed method can improve the staircase effect and yields better PSNR and RMSE than the PM diffusion and fourth-order PDE model.

III. EXPERIMENTAL RESULTS AND COMPARISON

In this section, the performance of the schemes were both tested using natural images(such as "barbara" and "house") and medical images (such as "brain" and "MRI"), see Fig2. In the simulation, four images will be corrupted by Gaussian white noise with zero mean value at different variances, and the restoration performance were quantitatively measured by Root-Mean-Square Error(RMSE) and Peak Signal-to-Noise Ratio(PSNR), which were defined as follows:

$$RMSE = \sqrt{\frac{1}{M \times N}\sum_{i,j}\left(x_{i,j} - y_{i,j}\right)^2} \qquad (13)$$

$$PSNR = 10\log 10\left(\frac{(M \times N)\max(x_{i,j})^2}{\sum_{i,j}(x_{i,j} - y_{i,j})^2}\right) \qquad (14)$$

Where M and N are the total number of pixels in the horizontal and the vertical dimensions of the image; $x_{i,j}$ and $y_{i,j}$ denote the original and distorted image pixels, respectively.

(a) (b) (c) (d)

Fig2. Test Images. (a) barbara(natural), (b)house (natural), (c)brain (medical), (d)MRI (medical)

For the measure of convergency, the "normalized step difference energy"(NSDE)[6] was calculated for each iteration:

$$NSDE = \frac{\left|u_n - u_{n-1}\right|^2}{\left|u_n\right|^2} \qquad (15)$$

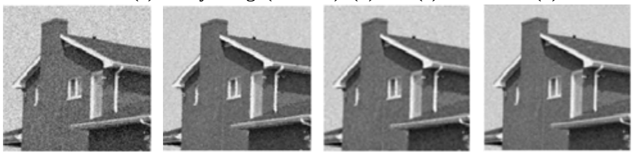
Where u_n and u_{n-1} denote the image vector at the n s and $n-1$ s iteration, respectively. The other parameters of our experiments are as follows: the initial value of iteration is 150, K is from 20 to 40 in different noise conditions and the time step is 0.02.

(a) (b) (c) (d)

Fig3. Denoising results of barbara image with different methods. (a) Noisy image(σ=20), (b)PM, (c)Four-PDE, (d)PA

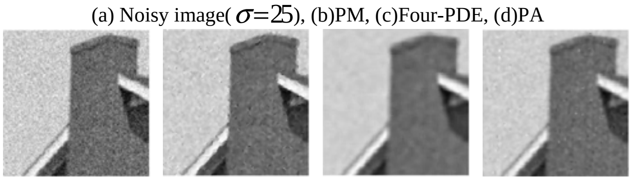
(a) (b) (c) (d)

Fig4. Denoising results of a small part of barbara image with different methods. (a) Noisy image(σ=20), (b)PM, (c)Four-PDE, (d)PA



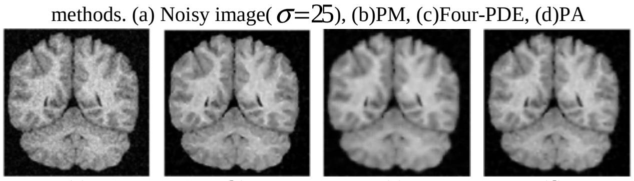
(a) (b) (c) (d)

Fig5. Denoising results of house image with different methods. (a) Noisy image(σ=25), (b)PM, (c)Four-PDE, (d)PA



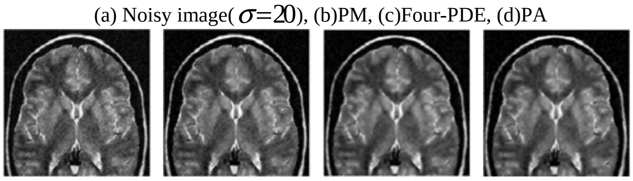
(a) (b) (c) (d)

Fig6. Denoising results of a small part of house image with different methods. (a) Noisy image(σ=25), (b)PM, (c)Four-PDE, (d)PA



(a) (b) (c) (d)

Fig7. Denoising results of brain image with different methods. (a) Noisy image(σ=20), (b)PM, (c)Four-PDE, (d)PA



(a) (b) (c) (d)

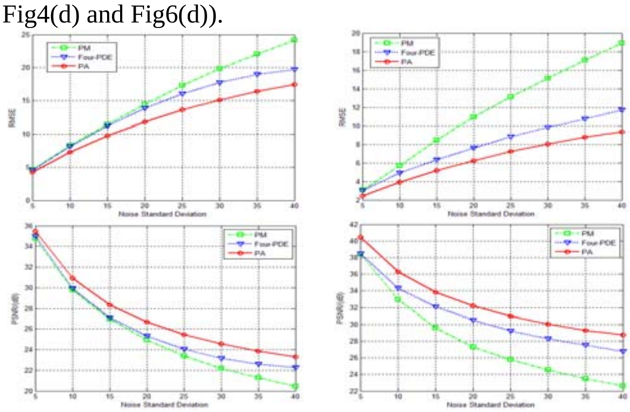
Fig8. Denoising results of MRI image with different methods. (a) Noisy image(σ=25), (b)PM, (c)Four-PDE, (d)PA

TABLE I. RMSE AND PSNR FOR FOUR TEST IMAGES WITH VARIOUS METHODS.（THE BETTER VALUES OF RMSE AND PSNR ARE SIGNED WITH BLACK FONT.）

Method		Noise Image	PM	Four-PDE	PA
barbara	RMSE	20.0011	14.4313	13.8426	**11.8341**
(σ=20)	PSNR	22.1097	24.9447	25.3065	**26.6681**
house	RMSE	25.0332	13.1519	8.8042	**7.2155**
(σ=25)	PSNR	20.1605	25.7510	29.2370	**30.9655**
brain	RMSE	20.0793	12.5978	11.7902	**9.5719**
(σ=20)	PSNR	22.0758	26.1249	26.7003	**28.5108**

MRI	RMSE	24.9864	15.4463	14.8755	**12.3627**
(σ=25)	PSNR	20.1767	24.3543	24.6813	**26.2886**

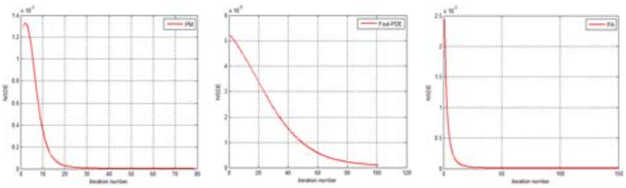
Fig 3~8 show the comparison of different methods, performed on natural image "barbara" , "house" and medical image "brain "and "MRI" at different noise standard deviations, respectively. From these six figs, it can be observed that the PM diffusion has the staircase effect(see Fig4(b) and Fig6(b)), and fourth-order PDE model has the disadvantage of blurring edges(see Fig4(c) and Fig6(c)). On the contrary, our proposed scheme has better performance in the staircase effect alleviation and has fine details preserving than the other two methods(see Fig4(d) and Fig6(d)).



(a) (b)

Fig. 9 Comparison graph for quantitative parameters of various methods for Gaussian white noise with different standard deviation column. (a) results for "barbara" image. (b) results for "house" image. Row 1 shows RMSE versus noise standard deviation. Row 2 shows PSNR versus noise standard deviation.

Table 1 and Fig 9 show the comparison of different methods in terms of RMSE and PSNR. As shown in them, we can see clearly that the results of proposed algorithm (PA) indicate better RMSE and PSNR values compared with PM diffusion and fourth-order PDE model, which demonstrates that the PA is promising in improving the denoising ability of the other two methods. Moreover, when the image contains a higher noise standard deviation, the PA can improve the RMSE value smaller and PSNR value larger than that lower noise standard deviation case (see Fig 9). This also verifies that the PA works more robust than the other two methods when the image is contaminated by large noise.



(a) (b) (c)

Fig10. Normalized step difference energy for barbara image with different methods. (a)PM, (b)Four-PDE, (c)PA

Fig 10 shows the graph of the NSDE for barbara image with different methods. It is apparent from Fig10 that the proposed algorithm achieves the least iteration number when NSDE decreases approach to zero. It means the PA takes much less computation time than that of the PM and Four-PDE methods.

From the comparison of the visual and quantity measures in Fig.3~10, we can conclude that the proposed algorithm not only removes Gaussian noise properly and achieves the best visual result, but also has higher PSNR, less RMSE and iteration number.

IV. CONCLUSION

A novel denoising algorithm based on Laplacian kernel has been proposed in this paper. By making use of the local analysis of an image, we propose a difference curvature, which can describe the intensity variation better than the gradient, especially when the image is contaminated with a high level of noise. We tested our method on both natural and medical images, and experimental results show that the proposed algorithm yield better PSNR and RMSE, has less staircase in the restoration images than the PM anisotropic diffusion and Four-order PDE models do. In addition, it takes fewer iteration number and has low computational cost. Our future efforts will be focused toward further improving the proposed algorithm to color images.

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