Analyse of the Mixed-Integer Nonlinear Programming Method for the Application in the Cellular Network

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***Abstract*—This paper discussed several methods to maximize the sum of the data rate. After the analysis of the graphic algorithm, we found the difficulty of the determination of the variables. To solve the problem, we propose a new idea to operate the variable and proved it to be efficient in the simulation results.**

Keywords-data rate; cellular network; intercell interference; graphic algorithm; power control.

I. INTRODUCTION

In recent years, as the demand of the high quality wireless remote communication is growing fast, more and more attention has been attracted by the technology of telecommunication [1]. We have two methods to increase the speed of the communication and the reliability of the system, one solution is to use the MIMO (multiple-input, multiple-output) communication system which is proved to be an efficient method. Another solution is to use the multiuser diversity strategy at the resource allocation [2]-[3].

II. SYSTEM MODEL

There is a cellular communication system with frequency reuse 1/7. Seven cellular with different frequency ranges compose a cluster. In our case, there are 7 clusters. In each cellular, the user receives the signal from its function node but is also influenced by the other six nodes which work on the same frequency [4]-[6].

For each cellular in one cluster, the situation of the MIMO channels is symmetrical, so we can just analysis a kind of cellular with the frequency. Our goal is to maximize the data rate of the system, and the data rate is determined by the SINR. So our work is to analyze the SINR and maximize the sum data rate by modify the value of SINR. For one user, the formula of SINR can be expressed as:

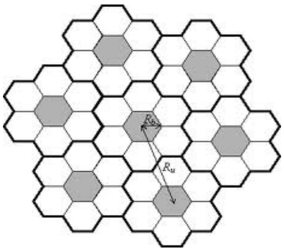

Figure 1 cellular communication system model

$$SINR = \frac{P \| H_{j,j}\omega_j \|^2}{\frac{P}{\| H_{j,j}\omega_j \|} \sum_{k\in\Phi(j)} | w_j^* H_j^* H_{k,j}\omega_k |^2 + Z_0} \tag{1}$$

In this expression, P is the allocation of power; the $H_{j,j}$ stands for the channel matrix between the functional node j and the user in the cellular j. $H_{k,j}$ means the channel matrix between the interference node k and the user in the cellular j. ω_j means the beam forming vector of the node j. ω_j choose a code word in the code book F. According to the formula (1), the data rate of a user j can be expressed as:

$$E_j = \log(1 + \frac{P \| H_{j,j}\omega_j \|^2}{\frac{P}{\| H_{j,j}\omega_j \|} \sum_{k\in\Phi(j)} | w_j^* H_j^* H_{k,j}\omega_k |^2 + Z_0}) \tag{2}$$

For user j, modification of ω_j will increase the date rate E_j, but it may also decrease the other user's data rate $E_{k\neq j}$. Therefore we can't choose the vector of ω simply to maximize the sum of data rate. It's not linear optimization, but the mixed-integer optimization.

III. BASIC METHOD SOLUTION

A. Genie-Aided method

It's a kind of traversal method. The main idea is the code book has M codeword, and then to compare all the results of the E (W). But the computational complexity is about M^J times than the matrix multiplication. This method can get the global optimization [7]. However to get the calculation done, it takes too much computational resource. During the communication, the channels and users are changing all the time, so we can't get the solution timely.

B. Greedy Beam Forming

The goal of this method is to maximize the received power for each user, and ignore the interference power, which can be expressed as the following line:

$$\max_{\omega_j\in F} \| H_{j,j}\omega_j \|^2$$

We calculate the received power, we test M codeword for the vector ωj , and find ωj leading to the greatest $\| H_{j,j}\omega_j \|$.

The computational complexity is M×J, which is much smaller than the Genie-Aided method. Comparing to the MIMO technique, it is just like the Matched filter, which maximize the received power.

C. Minimum Generated Interference Beam Forming

It is like the zero-forcing method, to minimize the interference power we received interference power, and can be expressed as the following line:

$$\min_{\omega_j \in F} \sum_{k\in\Phi(j)} \| H_{j,k}\omega_j \|^2$$

And the computational complexity is $M * J * \Phi(j)$.

D. Maximum Signal-to-Generated-Interference-plus-Noise-RatioBeam Forming

Let $\omega_k = \omega_j$ when we calculate each SINR, so the optimization problem can be transformed to

$$\max_{\omega_j \in F} \frac{P\| H_{j,j}\omega_j \|^2}{\dfrac{P}{\| H_{j,j}\omega_j \|} \displaystyle\sum_{k\in\Phi(j)} | w_j^* H_{j,j}^* H_{k,j}\omega_j |^2 + Z_0}$$

And the computational complexity is reduced to M*J.

IV. BASIC SIMULATION

A. Simulation results

To compare the result and the efficiency of each algorithm, we have used Matlab to simulate the wireless communication system. Because all the cells in one cluster are symmetric, so we can just calculate the cellular with the same serial number.

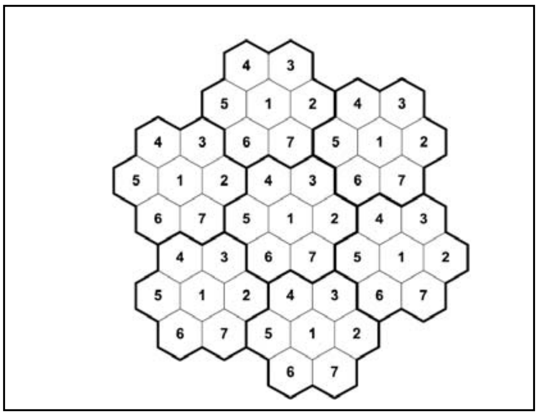

Figure 2 Structure of the cellular network

- We suppose there is a user appearing in the random place of the cell 1 for each cluster.
- We calculate the distance between the user and the node d (j, k), with the method shown in Fig.3.
- We calculate the throughput of the user by using different method. The performance is shown in the Fig.4.
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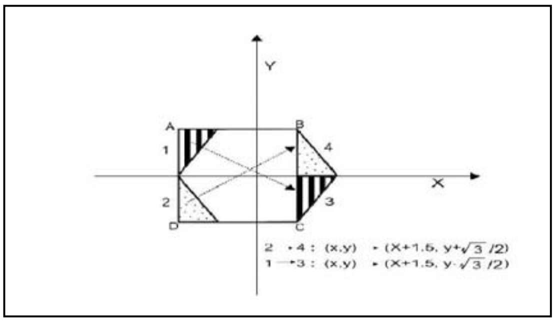


Figure 3 Method to calculate the distance

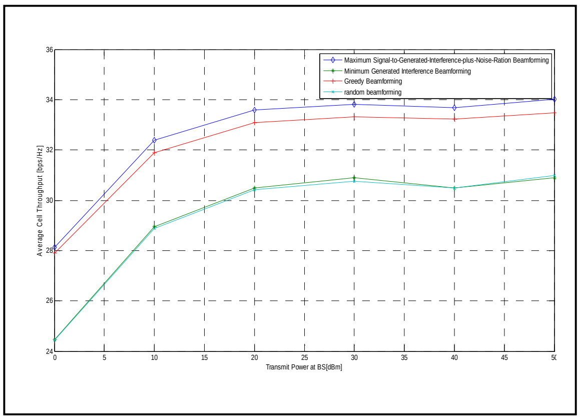


Figure 4 Performance of the 4 method

- The channel H can't be well-known in the real case, so there may be error in the channel estimation.

When we calculate the ω, we use the CSI channel H, but when simulate the data rate, we will use the channel matrix $\tilde{H}$, which presents by $\tilde{H} = \sqrt{1-\sigma^2}H + \sigma H_{Gauss}$, H_{Gauss} is the random matrix in the same distribution as H. The performance is shown in the Fig.5.

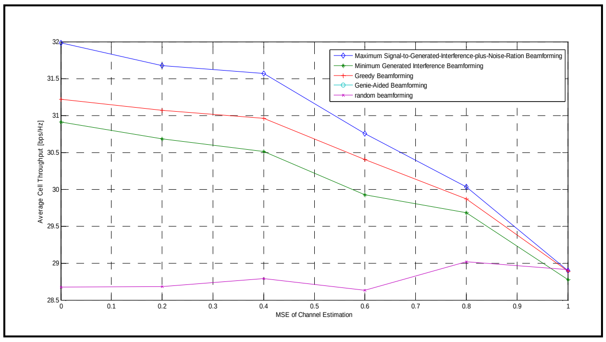


Figure 5 Comparison of the performance of the 4 methods

B. Analysis of the simulation results

As it is shown in Fig.4, the performance of the Maximum Signal-to-Generated-Interference-plus-Noise-Ratio Beam forming is better than that of Greedy Beam forming, Greedy Beam forming is better than Minimum generated interference beam forming. The traverse method (Genie-Aided method) has the best performance, but the running time is too long because the computational complexity is exponential. The random method has the worst performance, because the every algorithm has the contribution to the throughput. As it is shown in Fig.5, when σ is increasing, the throughput decreases, because the degree of association between ω and $\tilde{H}$ are reduced. When it equals to 1, the choosing of ω has no relation with the matrix $\tilde{H}$. It equals we choose the ω randomly; so all the curves go to the same point.

V. THE GRAPHIC ALGORITHM

The principle of the algorithm is interactive method[8].

1. Transform the formula of the sum data rate.

$$E(W) \approx \sum_{j=1}^{J} \log(\|H_{j,j}\omega_j\|^4 / |\Phi(j)|) + \sum_j \sum_{k \neq j} \log(|\omega_j^* H_{j,j}^* H_{k,j}\omega_k \| \omega_k^* H_{k,k}^* H_{j,k}\omega_j|) \quad (4)$$

2. Separate the formula into two parts.

$$E_a^{(1)} = \log(\| H_{j,j}\omega_j \|^4 / |\Phi(j)|) \quad (5)$$

$$E_a^{(2)} = \sum_{k \neq j} \log(| \omega_j^* H_{j,j}^* H_{k,j}\omega_k \| \omega_k^* H_{k,k}^* H_{j,k}\omega_j |) \quad (6)$$

$E_a^{(1)}$ is the factor of the received power, and $E_a^{(2)}$ is the factor of the interference power.

3. Build the correlation coefficient about the algorithm.

$$\hat{v}_{a\to j}^{(t)}(\omega_j) \propto \sum_{\omega_k} \exp(\beta E_{a_j}^{(2)}(\omega_j)) \hat{v}_{k\to a}^{(t)}(\omega_k) \quad (7)$$

$v_{(a\to j)}$ means the message send from the function node to the user in its cellular.

$$\hat{v}_{j\to a}^{(t+1)}(\omega_j) \propto \exp(\beta E_{a_j}^{(1)}(\omega_j)) \prod_{b \in \partial(j)\backslash a} \hat{v}_{b\to j}^{(t)}(\omega_j) \quad (8)$$

$v_{(j\to a)}$ means the message send from the function node to the user in its cellular.

4. The criterion to maximize the sum data rate is:

$$u_j^{(t+1)}(\omega_j) \propto \exp(\beta E_{a_j}^{(1)}(\omega_j)) \prod_{a \in \partial(j)} \hat{v}_{a\to j}^{(t)}(\omega_j) \quad (9)$$

It can be proved that

$$\max_{\omega_j} u_j(\omega_j) \Leftrightarrow \max_{\omega_j} E(W) \quad (10)$$

5. For obtaining the suitable ω_j to maximize u_j, use the interactive method.

A. Interactive method

1. Define $\hat{v}_{j\to a}^{(0)}(\omega_j) = \frac{1}{|F|}$,t=0; do the loop: from t=0 to t_{max}.
2. Update the value of $v_{(a\to j)}^{(t)}$ of each user a by using the formula (7).
3. Update the value of $v_{(j\to a)}^{(t+1)}$ of each function node by using the formula (8).
4. Update the value of $u_{j\to a}^{(t+1)}$ by using the formula (9).
5. Detection step, check the value of Δ:

$$\Delta = \left| u_{j\to a}^{(t+1)} - u_{j\to a}^{(t)} \right| \quad (11)$$

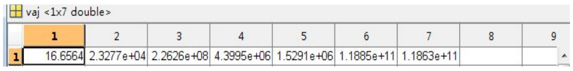
6. When $\Delta < \varepsilon$ or $t = t_{max}$, we end the loop.

After the loop, the vectors of ω are determined, they will lead to the maximum $u_{j\to a}^{(t+1)}$, it equals to the optimization solutions of the E (W).

B. Problem of the algorithm

In each step of the loop, when a new $v_{(a\to j)}^{(t)}$ is updated, it needs the new value of ω_j to calculate the value of $v_{(j\to a)}^{(t+1)}$ by using formula (8).

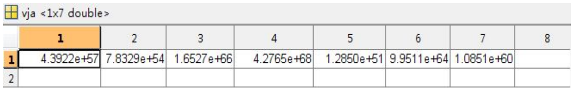
Test in the MATLAB, when t=0, the value of each $v_{(a\to j)}^{(t)}$ is shown in Fig.6.Then we can find out the value of $v_{(j\to a)}^{(1)}$, which is shown in Fig.7.



vaj <1x7 double>

	1	2	3	4	5	6	7	8	9
1	16.6564	2.3277e+04	2.2626e+08	4.3995e+06	1.5291e+06	1.1885e+11	1.1863e+11		

Figure 6 The value of the vriable $v_{(a\to j)}^{(t)}$



vja <1x7 double>

	1	2	3	4	5	6	7	8
1	4.3922e+57	7.8329e+54	1.6527e+66	4.2765e+68	1.2850e+51	9.9511e+64	1.0851e+60	
2								

Figure 7 The value of variable $v_{(j\to a)}^{(1)}$

During the test, we have found that there isn't the one to one correspondence between $v_{(a\to j)}^{(t)}$ and ω_j, which just disables the loop.

For solving the problem, we suppose to use the order to present the relation between $v_{(a\to j)}^{(t)}$ and ω_j. Take the order of $v_{(a\to j)}^{(t)}$, the node with greatest $v_{(a\to j)}^{(t)}$ will be assigned a vector ω, and the node with second greatest $v_{(a\to j)}^{(t)}$ will be assigned another ω.

There are 7 nodes but 4 codeword in this case, so this method can't be used very well. This idea will be used in the next part for power control problem.

VI. POWER CONTROL

A. Margin analysis

For user m and n in the system, the power allocation and the system data rate has the following relation.

$$E_m = \frac{\partial E}{\partial m} \quad (12)$$

$$E_m = \frac{\partial E}{\partial n} \quad (13)$$

As we know, the two partial differentials are inequality, we can suppose $E_m > E_n$. We can reduce the power of user n, and add this part of power to user m, and it can increase the system data rate.

$$E(P') = E(P) + \frac{\partial E}{\partial P_m} \cdot \delta - \frac{\partial E}{\partial P_n} \cdot \delta + O(\delta) > E(P) \quad (14)$$

For each user's power control, the margin effect (partial differential) is always positive and decreased; the system can allocate the power of each user and make sure that:

$$\frac{\partial E}{\partial P_1}=\frac{\partial E}{\partial P_2}\cdots\frac{\partial E}{\partial P_m}=\ldots=\frac{\partial E}{\partial P_z} \tag{15}$$

When all the partial differentials are equality, the stable point can be achieved, the system achieve the maximum data rate.

B. Single user case

In the single user case, the problem can be described by:

$$E(P)=\sum_{j=1}^{J}\log(1+\frac{P_j\|H_{j,j}\omega_j\|^2}{\frac{1}{\|H_{j,j}\omega_j\|}\sum_{k\in\Phi(j)}P_k|w_j^*H_j^*H_{k,j}\omega_k|^2+Z_0}) \tag{16}$$

The formula (4) can be transformed to the following line:

$$E(P)=\sum_{j=1}^{J}\log(p_j\|H_{j,j}\omega_j\|^4/\Phi(j))+\sum_{(j,k)}\log(p_k\log|\omega_j^*H_{j,j}^*H_{k,j}\omega_k\||\omega_k^*H_{k,j}^*H_{j,k}\omega_j|) \tag{17}$$

Calculate the partial differential of each user j, Then we'll find the power allocated to each user should be equal to maximize the E(P).

C. Multiple user case

In each cellular, there is not only one user. They will be allocated to different values of power.

Suppose there're I users in a cellular, their frequencies are $f_1, f_2, \ldots, f_I$. For the user i in the jth cellular, its allocated frequency is f_i, allocated power is p_{ji}. It will influence by the other variable cellular's allocated power on frequency i.The power it received is $p_{j,i}||H_{j,ji}\omega_j||^2$. Then account all the interference source, this part of power is $\frac{1}{\|H_{j,j}\omega_j\|}\sum_{k\in\Phi(j)}P_{k,i}|w_j^*H_{j,ji}^*H_{k,ji}\omega_k|^2$.

In this case, each user has a main channel $H_{j,ji}$ with its functional node , and other interference channel $H_{k,ji}$. The problem can be described as formula (18).

$$E(W)=\sum_{j=1}^{J}\sum_{i=1}^{I}\log(1+\frac{p_{j,i}\|H_{j,ji}\omega_j\|^2}{\frac{1}{\|H_{j,j}\omega_j\|}\sum_{k\in\Phi(j)}p_{k,i}|w_j^*H_{j,ji}^*H_{k,ji}\omega_k|^2+Z_0}) \tag{18}$$

$p_{k,i}$ means the allocated power to the user using frequency i in the kth cell.

D. Graphical method of Power control

Here we build a model to simplify the problem. There're I users in one cellular and there are I groups of power to allocate to them. The power $\rho_1, \rho_2, \ldots, \rho_I$ has the orders: $\rho_1 \geq \rho_2 \geq, \ldots, \geq \rho_I$, our model is to allocate the power to different users on different frequencies. Then we can use the graphical method to solve the power allocation problem in order to get a higher sum data rate.

1. Transform the formula of the sum data rate.

$$E(W)\approx\sum_{j=1}^{J}\sum_{i}\log(p_{j,i}\|H_{j,ji}\omega_j\|^4/|\Phi(j)|)+\sum_{j}\sum_{k\neq j}\sum_{i}\log(p_{k,i}|\omega_j^*H_{ji,ji}^*H_{k,ji}\omega_k\||\omega_k^*H_{ki,ki}^*H_{ji,ki}\omega_j|) \tag{19}$$

2. Separate the formula into two parts:$E_a^{(1)}$ as the factor of the received power and $E_a^{(2)}$as the factor of the interference power.

$$E_a^{(1)}=\log(p_{j,i}\|H_{j,ji}\omega_j\|^4/|\Phi(j)|) \tag{20}$$

$$E_a^{(2)}=\sum_{k\neq j}\log(p_{k,i}|\omega_j^*H_{j,j}^*H_{k,j}\omega_k\||\omega_k^*H_{k,k}^*H_{j,k}\omega_j|) \tag{21}$$

3. Build the correlation coefficient about the algorithm. $v_{(a\to j)}$ is the message send from the function node to the user in its cellular

$$\hat{v}_{ai\to j}{}^{(t)}(p_{j,i})\propto\sum_{p_i}\exp(\beta E_{a_j}^{(2)}(p_{j,i}))\hat{v}_{k\to a}^{(t)}(p_{k,i}) \tag{22}$$

When all the value of $v_{(ai\to j)}{}^t$ has been calculated, for each j, take the order of $v_{(ai\to j)}{}^t$, and user on frequency i with the highest $v_{(ai\to j)}{}^t$ will be allocated to the highest power. For example, the order of the $v_{(ai\to j)}{}^t$ is

$$v_{(ai1\to j)}{}^t \geq v_{(ai2\to j)}{}^t \geq \cdots \geq v_{(aiI\to j)}{}^t \tag{23}$$

then the user i_1 will be allocated to power ρ_1, the user i_2 will be allocated to power ρ_2, the user i_I will be allocated to power ρ_I.

$v_{(j\to a)}$ means the message send from the function node to the user in its cellular.

$$\hat{v}_{j\to ai}{}^{(t+1)}(p_{j,i})\propto\exp(\beta E_{a_j}^{(1)}(p_{j,i}))\prod_{b\in\partial(j)\backslash a}\hat{v}_{bi\to j}^{(t)}(p_{j,i}) \tag{24}$$

When all the value of $v_{(j\to ai)}{}^t$ has been calculated, for each j, take the order of $v_{(j\to ai)}{}^t$ like formula (20), and user on frequency i will be re-allocated to power ρ following the order of with $v_{(j\to ai)}{}^t$, the highest $v_{(j\to ai)}{}^t$ will be allocated to the highest power ρ_1, the rest can be done in the same manner.

4. The criterion is to maximize the sum data rate.

$$u_{j,i}{}^{(t+1)}(p_{j,i})\propto\exp(\beta E_{a_{j,i}}^{(1)}(p_{j,i}))\prod_{a\in\partial(j)}\hat{v}_{ai\to j}^{(t)}(p_{j,i}) \tag{25}$$

It can be proved that

$$\max_{p_{j,i}}u_j\ (p_{j,i})\Leftrightarrow\max_{\omega_j}E(W) \tag{26}$$

5. For obtaining the suitable $p_{j,i}$ to maximize u_j , use the interactive method.

E. The interactive method in Graphical method of Power control

1. Define $v_{(j\to a)}{}^{(0)}=1$, t=0; Do the loop: t=0 to $t_{\max}$
2. Update the value of $v_{(ai\to j)}{}^{(t)}$ of each user a by usng the formula (22).
3. Update the new $p_{j,i}$ for each user in frequency i. Update the value of $v_{(j\to ai)}{}^{(t+1)}$ of each function node by using the formula (24).
4. Update the value of $u_{j\to ai}^{(t+1)}$ by using the formula (25).
5. Detection step, check the value of Δ.

$$\Delta = | u_{j\to ai}^{(t+1)} - u_{j\to ai}^{(t)} |$$

6. When $\Delta < \varepsilon$ or t= $t_{\max}$, end the loop.

After the loop, the vectors of ω are determined, they will lead to the maximum $u_{j\to a}^{(t+1)}$, it equals to the optimization solutions of the E (P).

VII. SIMULATION OF POWER CONTROL

We build a system to simulate the power control result is better than the throughput without power control.

In this case, we suppose the user in one cellular I=4, the allocated power is $\rho_1 = 0.4\, P_{\text{total}}, \rho_2 = 0.3P_{\text{total}}, \rho_3 = 0.2P_{\text{total}}, \rho_3 = 0.1P_{\text{total}}$. There are 4 user in each user, we calculate $4 \times 7 = 28$ users with 28 communication channels, and $4 \times 6 \times 7 = 168$ interference channels.

1. We do the beamforming by Maximum Signal-to-Generated-Interference-plus-Noise-Ratio Beamforming method, which is the best beamforming simulation result comparing to the others.
2. We do the random power allocation. Allocate the power to each user i randomly, then calculating the throughput by formula (19).
3. Use the graphical algorithm described in Section VI, allocate the power ρ to different user i. Then calculate the throughput of system.

Comparing the result of 2 and 3, the throughput of the communication of the system is much greater than the result without power control. As shown in Fig.8.

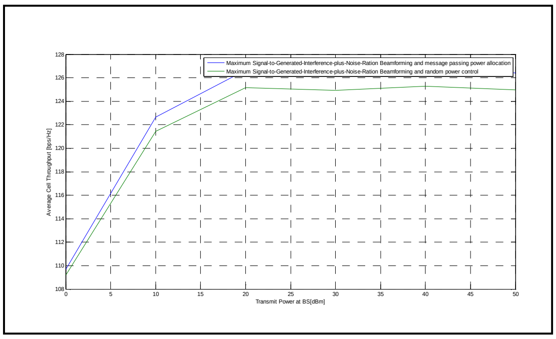


Figure 8 Comparison of the performance of 2 methods

VIII. CONCLUSION

The main problem we discuss is that how to deal with the beamforming in order to maximize the system sum data rate. First we had discussed some basic methods and analyze the simulation results. As we have found the difficulty to determine the relation $v_{(a\to j)}{}^{(t)}$, $v_{(j\to a)}{}^{(t)}$ and the beamforming vector ω_j , and to solve this problem, we propose a new method.

We found that if we can take the rank of the variables, we don't need to have the relationship between $v_{(a\to j)}{}^{(t)}$ and $v_{(j\to a)}{}^{(t)}$,which can reduce the complexity of the work. As they have the positive correlation with the $\sum_{p_i} \exp(\beta E_{a_j}^{(2)})\, \hat{v}_{k\to a}^{(t)}$ and $\exp(\beta E_{a_j}^{(1)}) \prod_{b\in\partial(j)\backslash a} \hat{v}_{bi\to j}^{(t)}$, so we can update the variables by the order of the value of $v_{(a\to j)}{}^{(t)}$and $v_{(j\to a)}{}^{(t)}$. The disadvantage of the method is that different $v_{(a\to j)}{}^{(t)}$ and $v_{(j\to a)}{}^{(t)}$ can't take the same variable. For this reason, we use it in the power control problem. Because the same frequency can't be allocated to different users at the same time, it's possible to allocate different power to different users. The disadvantage can be ignored and we can use this method in this case. And from the simulation result offered in Section VII, we know the power control did increase the sum of the data rate of the system.

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