Chaotic System Parameter Identification Based on Firefly Optimization

Wei-ming Gao [1], Zhi-cheng Zhang [2], Yao-hua Chong [3]
School Of Information Science and Engineering
Lanzhou University
Lanzhou, Gansu, China
E-mail: zzcc209@sina.com[1], gaowm10@lzu.edu.cn[2], chongyaohua1984@yahoo.com.cn[3]

***Abstract*—Firefly Optimization Algorithm (FA) is a novel heuristic stochastic algorithm based on swarm intelligence, which is inspired by the fireflies' biochemical and collective behavior. Due to the increment of attractiveness and the fixed step factor, the optimizing results are easily repeated oscillation on the position of local or global extreme value point, and the optimizing accuracy is reduced. Accordingly, this paper puts forward chaos firefly optimization algorithm (CFA), the improved algorithm can improve the diversity of population and the ergodicity of optimization, increase the ability of getting rid of trapped into local minima point. Chaos firefly optimization algorithm is used for the identification of chaotic system parameter; the results show the high accuracy of the algorithm parameter identification.**

Keywords-Swarm Intelligence; Firefly Algorithm; Inertia Weight; Parameters Identification

I. INTRODUCTION

There are about two thousand firefly species, and most fireflies produce short and rhythmic flashes. The pattern of flashes is often unique for a particular species. The flashing light is produced by a process of bioluminescence where the exact functions of such signaling systems are still on debating. Nevertheless, two fundamental functions of such flashes are to attract mating partners (communication) and to attract potential prey. Based on flashing behavior of fireflies, Xin-She Yang[1] proposed the Firefly Algorithm (FA) for solving multimodal optimization problem. This algorithm is relatively simple in theory and implementation and it is very effective in solving some optimization problems. Moreover, it can be better than other traditional algorithms. As a new tool of optimization algorithm, it has been successfully used for collective robot, multi-modal function optimization, image processing [2,3],engineering structural optimization[4], complicated nonlinear constrained programming[5], job scheduling[6], economic dispatch[7,8] and so on.

II. STANDARD FIREFLY ALGORITHM

Inspired by the luminous behavior of fireflies, standard firefly optimization algorithm has been proposed. At the same time, luminescence properties were idealized treated to make the algorithm simple and effective. It has three idealized constraints which are derived from firefly features.

- All fireflies are unisex so that one firefly is attracted to other fireflies regardless of their sex;
- Attractiveness is proportional to their brightness, so any two flashing fireflies, the less bright one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If no one is brighter than a particular firefly, it moves randomly;
- The brightness or light intensity of a firefly is affected or determined by the landscape of the objective function to be optimized.

A. Algorithm Principle

The Firefly Algorithm simulates the fireflies' individuals by the keys in the search space, the process of optimization is simulated with the attraction and the position changing of the fireflies' individuals, through an iterative computation, the fitness of the optimization problem is determined by the advantages and disadvantages of the fireflies' location, the process of finding good feasible solutions is presented by the process of the fireflies searching the better locations in the sky from iterative process. In the algorithm, two important factors are involved: the variation of light intensity and formulation of the attractiveness. For simplicity, suppose that the attractiveness of a firefly is determined by its brightness which in turn is associated with the encoded objective function. The higher of the brightness and the better of the location, and the more fireflies will be attracted to the direction, if the brightness is equal, the fireflies will move random. As light intensity and thus attractiveness decreases as the distance from the source increases, the variations of light intensity and attractiveness should be monotonically decreasing functions.

B. Some related definitions

In order to implement the algorithm, we need to define the concepts.

Definition 1: the variation of light intensity;

$$I(r) = I_0 e^{-\gamma r} \tag{1}$$

Where I_0 is the original light intensity(r=0) and γ is the light absorption coefficient. r is the Cartesian distance,

$$r_{ij} = \left\| x_i - x_j \right\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})} \tag{2}$$

d is space dimension, $x_{i,k}$ is the k-th component of the spatial coordinate x_i of the firefly.

Definition 2: formulation of the attractiveness

$$\beta(r)=\beta_0 e^{-\gamma r^m},(m\geq 1) \quad (3)$$

Where β_0 is the attractiveness at $r = 0$. γ is light absorption coefficient, the value of m is usually 2.

Definition 3: formulation of location moving

$$x_i(t+1)=x_i(t)+\beta(x_j(t)-x_i(t))+\alpha\varepsilon_i \quad (4)$$

Where $x_i(t+1)$ is the position of x_i after $t+1$ times movements; α is the step parameter which varies between [0,1] ; ε_i is a random factor conforming Gaussian distribution between[0,1].

The basic steps of the FA are summarized by the pseudo code listed in Table I.

TABLE I. PSEUDO-CODE OF THE PROPOSED FA

```
Objective function f(x), x = (x1... xd)T
Initialize a population of fireflies xi (i = 1, 2... n)
Define light absorption coefficient
while (t <MaxGeneration)
  for i = 1 : n all n fireflies
    for j = 1 : i all n fireflies
        Light intensity Ii at xi is determined by f(xi)
        if (Ij > Ii)
            Move firefly i towards j in all d dimensions
        end if
        Attractiveness varies with distance r via exp [− γ r2]
        Evaluate new solutions and update light intensity
      end for j
  end for i
  Rank the fireflies and find the current best
end while
Post process results and visualization
```

III. CHAOTIC FIREFLY ALGORITHM

The Firefly Optimization Algorithm can improve the method of the global search and local optimization ability, but it is found that the optimizing results are easily repeated oscillation on the position of local or global extreme value point, and the optimizing accuracy is reduced. In order to solve this problem, further improving the algorithm of optimal accuracy, the chaotic firefly algorithm is proposed, in the improved algorithm, the search process is divided into two phases: the firefly algorithm has a global search; the highest degrees of 10% to 20% of the fireflies have a local search again [9].

The steps of Chaos Firefly Algorithm are shown below:

Step 1 x^k_j (j=1, 2… n) is mapped to cx_j^k ($x_j^k \in [0,1]$) , which can be calculated by the following formula:

$$cx_j^k=\frac{x_j^k-x_{\min,j}}{x_j^k-x_{\max,j}},\, j=1,2,...,n \quad (5)$$

Where $x_{min,j}$ is the minimum and $x_{max,j}$ is the maximum of the j dimensionality in the search space.

Step 2 cx_j^{k+1} is calculated based on (5);

Step 3 cx_j^{k+1} is translated to x_j^{k+1} by the following formula:

$$x_j^{k+1}=x_{\min,j}+c\bullet x_j^{k+1}(x_{\max,j}-x_{\min,j}),\, j=1,2,...,n \quad (6)$$

Step 4 The performance of the new solution is evaluate d based on the $x_j^{k+1}, j=1,2,...,n$

Step 5 If the new solution is better than $X^{(0)}=[x_1^0,...,x_n^0]$ or the state is situating the given number of the accuracy and the times of iteration, the new solution will be regard as the final result. Otherwise, we set k equals k+1 and loop to step 2.

The Chaos Firefly Optimization Algorithm's steps are:

Step 1 System initialization;

Step 2 Calculate the light intensity of each firefly;

Step 3 Update the location of each firefly according to (4), and the brightest firefly moves randomly;

Step 4 Calculate the light intensity of each firefly in the group after the location updating;

Step 5 Take the operation of chaos local searching in the chosen firefly group whose fitness is in top 10% in the group;

Step 6 Calculate the light intensity of each firefly in the group after the location updating;

Step 7 If the present fits for the conditions of end, terminate the loops and output the results. Otherwise, keep on.

Step 8 The search area is contracted by the two formula:

$$x_{\min,j}=\max\{x_{\max,j},x_{\max light,j}-r(x_{\max,j}-x_{\min,j})\},0<r<1 \quad (7)$$

$$x_{\max,j}=\min\{x_{\max,j},x_{\max light,j}+r(x_{\max,j}-x_{\min,j})\},0<r<1 \quad (8)$$

Where the $x_{\max light,j}$ is the value of the j th dimensionality of the brightest firefly.

Step 9 Generate 80% fireflies randomly in the contracted area and loop to step 3.

IV. EXPERIMENTAL RESULTS

A. *The test functions and environment settings*

The experimental conditions of testing improved inertia weight firefly optimization algorithm can be set as follows:

Hardware environment : CPU Intel® Core™2 Duo T5870/2.00GHz/1G;

Software platform: Windows XP, Matlab7.1;

The set of benchmark functions contains five functions and is shown in table II, the number of decision variables is fixed to be 2 for each function.

Experiment was conducted to compare two algorithms: the original FA and PSO [10]. Swarm size was taken to be 80, the maximum number of iterations equaled to 50; for PSO, the acceleration factors *c1* and *c2* were both 2.0; for IWFA, a decaying inertia weight ω starting at 1.1 and ending at 0.4 was used. The fitness functions of F3 and F5 were themselves, the others were $1/(F+0.1)$.

TABLE II. BENCHMARK FUNCTIONS

Fun	Expressions

F1	$\sum_{i=1}^{n} x_i^2$,(-4,4) n=2
F2	$\sum_{i=1}^{n-1} [100(x_{i+1}-x_i^2)^2+(x_i-1)^2]$ (-2.048,2.048) n=3
F3	$x_1*\sin(4\pi x_1)-x_2*\sin(4\pi x_2+\pi+1)$ (-1，2)
F4	$(x_1+2x_2-7)^2+(2x_1+x_2-5)^2$ （-10，10）
F5	$((x_1(+(x_2()e^{-0.0625(x_1^2+x_2^2)})$,(-5,5)

B. *Test Results*

Table III-V shows the optimized value and computation time for 20 iterations of each test function (time running on CPU. ）

TABLE III. TEST RESULTS OF PSO

Fun	PSO	
	Optimized value	Cost time
F1	9.9787	0.1936s
F2	9.9994	0.1133s
F3	3.2079	0.1335s
F4	10.0	0.1148s
F5	2.4261	0.1179s

TABLE IV. TEST RESULTS OF FA

Fun	FA	
	Optimized value	Cost time
F1	9.8995	0.0844s
F2	9.9904	0.0833s
F3	2.9265	0.0816s
F4	10.0	0.0933s
F5	2.4160	0.0949s

TABLE V. TEST RESULTS OF CFA

Fun	CFA	
	Optimized value	Cost time
F1	10.0	0.0586s
F2	9.9990	0.0612s
F3	3.3099	0.0576s
F4	10.0	0.0626s
F5	2.4261	0.0610s

If the function's extreme point and optimization result error is less than 0.001, we consider that optimization value has been found. Data in Table III-V indicates that, with the same population size, the number of iterations and testing functions of the same dimensions, four algorithms are all able to successfully find the optimization value, but the CFA algorithm performed better in optimizing accuracy and efficiency.

C. *Identification of Chaotic System Parameters*

The research and application of chaos theory is a central issue at present. Chaos control and synchronization of nonlinear science has become one of the major research directions, a number of effective methods of control and synchronization has been proposed. However, the methods are based on known exact parameters of chaotic systems, if there is an unknown parameter in the system, they cannot effectively perform .In practice, due to the complexity of chaotic systems, it is difficult to measure or determine certain parameters; Or for some special reason, some of the parameters are not known (such as secure communication). If we want to control or synchronize chaotic systems, this approach has limitations because we must firstly estimate unknown system parameters of chaotic systems. In fact, parameter estimation should be implemented first in the chaotic system control and synchronization, it is of important significance estimated Lorenz parameter for chaotic system by chaos Ant Colony algorithm[11] which can achieve global optimization and get more relevant results with actual values, but the above mentioned methods only identificated one parameter of chaotic systems.

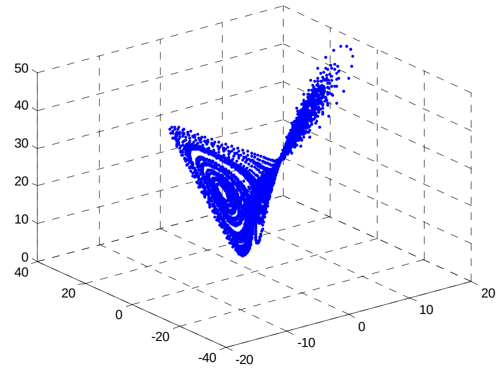

Fig. 1 Lorenz Chaotic attractor

Chaotic system is a strange attractor Dynamics system proposed by Lorenz in 1963[12].

$$\dot{x}_1 = a(x_2 - x_1) \quad (9)$$

$$\dot{x}_2 = (c - x_3)x_1 - x_2 \quad (10)$$

$$\dot{x}_3 = x_1 x_2 - b x_3 \quad (11)$$

With a = 10, b = 8/3, c = 3, Eq (9, 10, 11) show a Chaotic system. Fig. 1 shows the evolution process of the system.

Assume that parameters a, b, c of the Lorenz system is unknown, we will identificated them by inertia weight Firefly algorithm.

For all algorithms, iteration time is 50, run time is 30, and variable scope is:

FA: the light absorption coefficient $\gamma = 1$; the step parameter $\alpha = 0.2$.

PSO： w=0.729，C1=C2=1.49，VMAX = [10 30 1];

GA： Population size N=30；Crossover probability p_c=0.8；Mutation probability p_m change from 0.01 to 0.6; optimal strategies: the best values can be directly passed to the next generation.

D. *Simulation Analysis*

Table VI comparison of Lorenz system parameter identification results

TABLE VI OPTIMAL SOLUTION AND ERROR ANALYSIS

	PSO	GA	CFA
Mean error	0.0019379	0.215385	0.065673
Standard deviation	0.0045299	0.298406	0.0002466
a	9.999104	9.995112	9.999878
b	2.666868	2.665362	2.6666533
c	28.000317	28.000010	28.000000

Results are showed in Table VI. The optimal solution of derived by CFA is best which is almost close to the actual parameter value. Moreover, the mean error and standard deviation of CFA is much smaller than the other two algorithms. It illustrates that the algorithm parameter accuracy is very high.

V. CONCLUSION

In this paper, we proposed a new Firefly algorithm. Based on the standard Firefly algorithm, we involve chaotic search in the process of updating the location of the algorithm which effectively improve the optimization accuracy and the speed. The optimization accuracy and speed of the algorithm has been improved comparing with original algorithm. In experimental results, we compared the convergence speed and optimization accuracy. Results show that our algorithm performs better than PSO and FA. Our algorithm has a good prospect in engineering applications.

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