Compressive sensing-based angle estimation for MIMO radar with multiple snapshots

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***Abstract*—The issue of angle estimation for multiple-input multiple-output (MIMO) radar is studied and an algorithm for the estimation based on compressive sensing with multiple snapshots is proposed. The dimension of received signal is reduced to make the computation burden lower, and then the noise sensitivity is reduced by the eigenvalue decomposition (EVD) of the covariance matrix of the reduced-dimensional signal. Finally the signal subspace obtained from the eigenvectors is realigned to apply the orthogonal matching pursuit (OMP) for angle estimation. The angle estimation performance of the proposed algorithm is better than that of estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm, and reduced-dimension Capon. Furthermore, the proposed algorithm works well for coherent targets, and requires no knowledge of the noise. The complexity analysis and simulation results verify the effectiveness of the algorithm.**

Keywords- multiple-input multiple-output (MIMO) radar; angle estimation; compressive sensing; OMP

I. INTRODUCTION

Multiple-input multiple-output (MIMO) radars use multiple antennas to simultaneously transmit diverse waveforms and utilize multiple antennas to receive the reflected signals, and they have many potential advantages over conventional phased-array radars [1-4]. MIMO radar systems have more degrees of freedom than other systems with a single transmit antenna, and these additional degrees of freedom can overcome fading effect, enhance spatial resolution, strengthen parameter identifiability and improve target detection performance [5-8]. Angle estimation is a key issue in MIMO radar, and several algorithms for angle estimation have been established, which contain estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm [9], Capon algorithm [10], multiple signal classification (MUSIC) algorithm [11] and parallel factor analysis algorithms [12-13]. Reduced-dimension (RD) ESPRIT algorithm [14] and RD Capon [15] have been proposed for angle estimation in monostatic MIMO radar, and they reduce the complexity and improve the performance compared to conventional methods.

Compressive sensing [16-17] has attracted a lot of attention in the last decade, and it has been applied to a variety of problems, including image reconstruction and restoration, wavelet denoising, feature selection in machine learning, radar imaging and penalized regression [18]. The super-resolution property and ability of resolving coherent sources can be achieved when apply it in the source location [19]. Lots of the location methods using compressive sensing just use one snapshot and are very sensitive to the noise, for multiple snapshots, ℓ_1-SVD method [18] employs ℓ_1 norm to enforce sparsity and singular value decomposition to reduce complexity and sensitivity to noise, and sparse recovery for weighted subspace fitting (SRWSF) [19] improved the ℓ_1-SVD method via the weight to the subspace. However, both the method in [18] and [19] have a common problem, which is the choice of the regularization parameter, so a prior knowledge of the noise may be known.

In this paper, we propose a compressive sensing-based method for angle estimation in MIMO radar. The reduced-dimension transformation is utilized to reduce the dimension of the signal, i.e. the dimension of the dictionary, and then the eigenvalue decomposition (EVD) is employed to reduce the sensitivity of the noise. Finally, according to the relationship between the direction matrix and signal subspace, the signal subspace is realigned to apply the orthogonal matching pursuit (OMP) [20] for angle estimation. The angle estimation performance of the proposed algorithm is better than that of RD ESPRIT algorithm, and RD Capon. Furthermore, the proposed algorithm works well for coherent targets, and requires no knowledge of the noise.

Notation: $(.)^T$ 、 $(.)^H$ 、 $(.)^{-1}$ and $(.)^+$ denote transpose, conjugate-transpose, inverse, pseudo-inverse operations, respectively; diag(**v**) stands for diagonal matrix whose diagonal element is a vector **v**; $D_n(.)$ is to take the nth row of the matrix to construct a diagonal matrix; $\mathbf{I}_K$ is a $K \times K$ identity matrix; $\otimes$, $\circ$ and $\odot$ are the Kronecker product, Khatri–Rao product and Hadamard product, respectively; Re(.) is to get real part of the complex; min(.) is to get minimum elements of an array; $E[.]$ is expectation operator and $vec(\cdot)$ denotes an operator stacking the columns of a matrix on top of each other

II. DATA MODEL

We consider a monostatic MIMO radar system equipped with both of uniform linear arrays for the transmit and receive arrays, and the transmit array and receive array are both located in the y-axis with half-wavelength spacing between adjacent antennas, respectively. We assume that there are K targets in the y-z plane, and the output of the matched filters at the receiver can be expressed as

$$\mathbf{x}(t) = [\mathbf{a}_r(\theta_1) \otimes \mathbf{a}_t(\theta_1), \cdots, \mathbf{a}_r(\theta_K) \otimes \mathbf{a}_t(\theta_K)]\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where θ_k is the (Direction of arrival) DOA of the kth target with respect to the transmit array normal or the receive array normal; $\mathbf{s}(t)=[s_1(t),s_2(t),\cdots,s_K(t)]^T\in\mathbb{C}^{K\times1}$, $s_k(t)=\beta_k e^{j2\pi f_k t}$ with f_k being Doppler frequency and β_k the RCS; $\mathbf{n}(t)$ is an $MN\times1$ Gaussian white noise vector of zero mean and covariance matrix $\sigma^2\mathbf{I}_{MN}$; $\mathbf{a}_r(\theta_k)\otimes\mathbf{a}_t(\theta_k)$ is the Kronecker product of the receive and the transmit steering vectors for the kth target, and

$$\mathbf{a}_r(\theta_k)=\left[1,e^{-j\pi\sin\theta_k},\cdots,e^{-j(N-1)\pi\sin\theta_k}\right]^T \quad (2.a)$$

$$\mathbf{a}_t(\theta_k)=\left[1,e^{-j\pi\sin\theta_k},\cdots,e^{-j(M-1)\pi\sin\theta_k}\right]^T \quad (2.b)$$

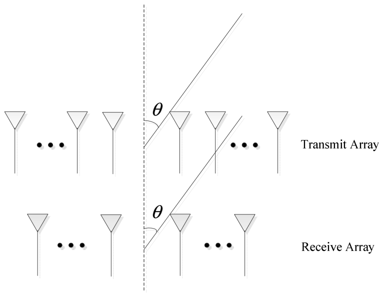

Figure 1. Array structure of monostatic MIMO radar

III. Compressive sensing-based method for angle estimation in MIMO radar with multiple snapshots

A. Reduced-dimension transformation

The length of $\mathbf{a}_r(\theta_k)\otimes\mathbf{a}_t(\theta_k)$ is MN, which costs high computation in the later recovery via OMP, so the reduced-dimension transformation is necessary. As

$$\mathbf{a}(\theta_k)=\mathbf{a}_r(\theta_k)\otimes\mathbf{a}_t(\theta_k)=\mathbf{G}\mathbf{b}(\theta_k) \quad (3)$$

where

$$\mathbf{G}=\begin{bmatrix} \left.\begin{matrix}1&0&\cdots&0&0&\cdots&0\\0&1&\cdots&0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots&\vdots&\ddots&0\\0&0&\cdots&1&0&\cdots&0\end{matrix}\right\}M \\ \left.\begin{matrix}0&1&\cdots&0&0&\cdots&0\\0&0&1&0&0&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots&\ddots&0\\0&0&\cdots&0&1&0&0\end{matrix}\right\}M \\ \vdots\;\vdots\;\vdots\;\vdots\;\vdots\;\vdots\;\vdots \\ \left.\begin{matrix}0&\cdots&0&1&0&\cdots&0\\0&\cdots&0&0&1&\cdots&0\\\vdots&\ddots&\vdots&\vdots&\vdots&\ddots&\vdots\\0&\cdots&0&0&0&\cdots&1\end{matrix}\right\}M \end{bmatrix}\in\mathbb{R}^{MN\times(M+N-1)} \quad (4)$$

$\mathbf{b}(\theta_k)=[1,\exp(-j\pi\sin\theta_k),\cdots\exp(-j\pi(M+N-2)\sin\theta_k)]^T$.

Then we define $\mathbf{W}\triangleq\mathbf{G}^H\mathbf{G}$,

$$\mathbf{W}=diag(1,2,\cdots,\underbrace{\min(M,N),\cdots,\min(M,N)}_{|M-N|+1},\cdots,2,1) \quad (5)$$

Using the reduced-dimension transformation $\mathbf{W}^{-\frac{1}{2}}\mathbf{G}^H$ for the receive signal $\mathbf{x}(t)$, we obtain

$$\begin{aligned}\mathbf{y}(t)&=\mathbf{W}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{x}(t)\\&=\mathbf{W}^{-\frac{1}{2}}\mathbf{W}[\mathbf{b}(\theta_1),\mathbf{b}(\theta_2),\cdots,\mathbf{b}(\theta_K)]\mathbf{s}(t)+\mathbf{W}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{n}(t)\\&=\mathbf{W}^{\frac{1}{2}}\mathbf{B}\mathbf{s}(t)+\mathbf{W}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{n}(t)\end{aligned} \quad (6)$$

where $\mathbf{B}=[\mathbf{b}(\theta_1),\mathbf{b}(\theta_2),\cdots,\mathbf{b}(\theta_K)]\in\mathbb{C}^{(M+N-1)\times K}$. $\mathbf{W}^{\frac{1}{2}}\mathbf{B}$ can be regarded as the new direction matrix which has lower dimension, and the reduced-dimension matrix is sparse, its transformation adds less computational load. The covariance matrix of $\mathbf{y}(t)$ in (6) is $\mathbf{R}_y(t)\in\mathbb{C}^{(M+N-1)\times(M+N-1)}$

$$\begin{aligned}\mathbf{R}&=E[\mathbf{y}(t)\mathbf{y}^H(t)]\\&=\mathbf{W}^{\frac{1}{2}}\mathbf{B}\mathbf{R}_s\mathbf{B}^H\mathbf{W}^{\frac{1}{2}}+\sigma^2\mathbf{I}_{M+N-1}\end{aligned} \quad (7)$$

where $\mathbf{R}_s=E[\mathbf{s}(t)\mathbf{s}^H(t)]$.

B. Compressive sensing-based method for angle estimation

The covariance matrix in (7) can be decomposed as

$$\mathbf{R}=\mathbf{E}_s\mathbf{D}_s\mathbf{E}_s^H+\mathbf{E}_n\mathbf{D}_n\mathbf{E}_n^H \quad (8)$$

where $\mathbf{D}_s$ denotes a $K\times K$ diagonal matrix formed by K largest eigen-values, and $\mathbf{D}_n$ denotes a diagonal matrix formed by the rest (M+N-1) – K smaller eigen-values. $\mathbf{E}_s$ and $\mathbf{E}_n$ represent the signal subspace and noise subspace, respectively, of which $\mathbf{E}_s$ stands for the eigenvectors corresponding to the K largest eigen-values, $\mathbf{E}_n$ consists of the rest eigenvectors. The equation between $\mathbf{E}_s$ and the direction matrix can be formulated as

$$\mathbf{E}_s=\mathbf{W}^{\frac{1}{2}}\mathbf{B}\mathbf{T} \quad (9)$$

where $\mathbf{T}$ represents a nonsingular $K\times K$ matrix.

Let $\tilde{\theta}_1,\tilde{\theta}_2,\cdots,\tilde{\theta}_L$ be a sampling grid of all target locations of interest. The number of potential source locations will typically be much greater than the number of sources K or even the number of sensors M+N-1. We construct a matrix composed of steering vectors corresponding to each potential target location as its columns: $\mathbf{\Theta}=[\mathbf{b}(\tilde{\theta}_1),\mathbf{b}(\tilde{\theta}_2),\cdots,\mathbf{b}(\tilde{\theta}_L)]$.

Then construct a matrix $\mathbf{Q}\in\mathbb{C}^{L\times K}$, and the rows of $\mathbf{Q}$ corresponding to the true DOAs keep the same with those of $\mathbf{T}$, with the other rows being all-zero. And

$$\mathbf{E}_s=\mathbf{W}^{\frac{1}{2}}\mathbf{\Theta}\mathbf{Q} \quad (10)$$

This implies that if $\mathbf{Q}$ can be recovered from $\mathbf{E}_s$, the DOAs can be determined by exploiting the positions of nonzero rows of $\mathbf{Q}$. Define $\mathbf{e}_s=vec(\mathbf{E}_s)$, which is the realigned form of $\mathbf{E}_s$, and it satisfies

$$\mathbf{e}_s=(\mathbf{I}_K\otimes(\mathbf{W}^{\frac{1}{2}}\mathbf{\Theta}))vec(\mathbf{Q})=\mathbf{\Phi}\mathbf{q} \quad (11)$$

where $\mathbf{\Phi} = \mathbf{I}_K \otimes (\mathbf{W}^{\frac{1}{2}}\mathbf{\Theta}) \in \mathbb{C}^{K(M+N-1)\times LK}$, and $\mathbf{q} = vec(\mathbf{Q}) \in \mathbb{C}^{LK\times 1}$ is a sparse vector with K^2 nonzero elements.

According to (11), $\mathbf{\Phi}$ can be regard as the recovery matrix or dictionary in compressive sensing, and the sparse vector $\mathbf{q}$ can be obtained by utilizing OMP recovery method [20]. The detailed recovery processing via OMP is shown in Fig.2. Then $\mathbf{Q}$ can be estimated by reshaping the vector $\mathbf{q}$ into the $L\times K$ matrix and the nonzero rows in $\mathbf{Q}$ will show the DOAs of the targets.

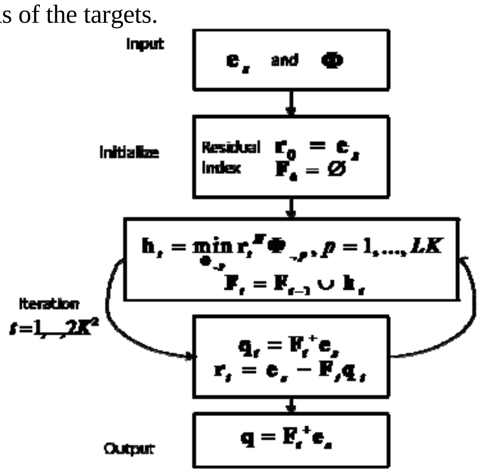

Figure 2. The OMP algorithm flow

With respect to (7), the covariance matrix $\mathbf{R} = E[\mathbf{y}(t)\mathbf{y}^H(t)]$ can be estimated with J snapshots by

$$\hat{\mathbf{R}} = \frac{1}{J}\sum_{t=1}^{J}\mathbf{y}(t)\mathbf{y}^H(t) \tag{12}$$

C. Complexity analysis and CRB

The proposed algorithm has higher complexity than RD ESPRIT, but has much lower complexity than RD Capon algorithm, which needs peak searching.

Fig.3 shows the run time of the three algorithms in computer versus the number of antennas, we choose M=N for simplify. From Fig.3, we find that our algorithm has much lower complexity than the RD Capon, and the change trend versus the number of antennas of the proposed algorithm is smaller than that of the other two algorithms.

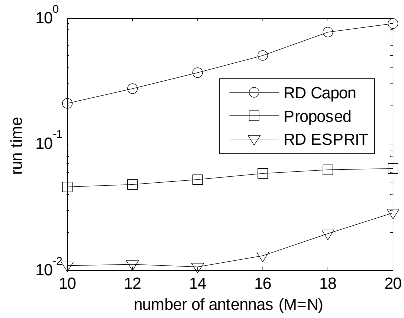

Figure 3. complexity comparison against number of antennas

According to Ref.[21], we can derive the CRB in bistatic MIMO radar

$$CRB = \frac{\sigma^2}{2J}\left\{\mathrm{Re}\left[\mathbf{D}^H\mathbf{\Pi}_{\mathbf{A}}^{\perp}\mathbf{D}\odot\hat{\mathbf{P}}_w^{\ T}\right]\right\}^{-1} \tag{13}$$

where $\mathbf{D} = \left[\frac{\partial\mathbf{a}_1}{\partial\theta_1}, \frac{\partial\mathbf{a}_2}{\partial\theta_2}, ..., \frac{\partial\mathbf{a}_K}{\partial\theta_K}, \frac{\partial\mathbf{a}_1}{\partial\phi_1}, \frac{\partial\mathbf{a}_2}{\partial\phi_2}, ..., \frac{\partial\mathbf{a}_K}{\partial\phi_K}\right]$ with $\mathbf{a}_k = \mathbf{a}_r(\theta_k)\otimes\mathbf{a}_t(\theta_k)$, and $\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_K]$; $\hat{\mathbf{P}}_w = \begin{bmatrix}\hat{\mathbf{P}}_s & \hat{\mathbf{P}}_s \\ \hat{\mathbf{P}}_s & \hat{\mathbf{P}}_s\end{bmatrix}$, $\hat{\mathbf{P}}_s = \frac{1}{J}\sum_{t=1}^{J}\mathbf{s}(t)\mathbf{s}^H(t)$; $\mathbf{\Pi}_{\mathbf{A}}^{\perp} = \mathbf{I}_{MN} - \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H$.

IV. SIMULATION RESULTS

Define root mean square error (RMSE) as $\frac{1}{K}\sum_{k=1}^{K}(\frac{1}{1000}\sum_{l=1}^{1000}[(\hat{\theta}_{k,l}-\theta_k)^2])^{1/2}$, where $\hat{\theta}_{k,l}$ is the estimate of DOD θ_k of the lth Monte Carlo trial. We assume there are K = 2 targets with angle $(\theta_1,\theta_2) = (5^\circ, 25^\circ)$. The RCSs and Doppler frequencies are $(\beta_1,\beta_2) = [e^{j\pi/5}, 0.8e^{j\pi/4}]$ and $(f_1, f_2) = [100Hz, 500Hz]$, respectively.

Fig.4 depicts angle estimation result of the proposed algorithm for all two targets with *M*=16, *N*=14, *J*=50, SNR(Signal-to-Noise Ratio)= 0dB, respectively. It is shown that the DOAs can be clearly observed.

We compare the proposed algorithm against the RD ESPRIT algorithm, RD Capon algorithm and CRB. Fig. 5 presents the comparison of the algorithms. From Fig. 5, we can find that the angle estimation performance of the proposed algorithm is better than that of RD ESPRIT algorithm, and has better performance than RD Capon algorithm when SNR is higher.

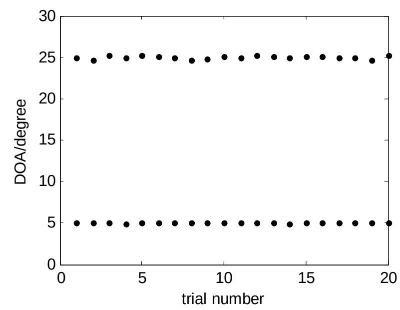

Figure 4. Angle estimation result of the proposed algorithm

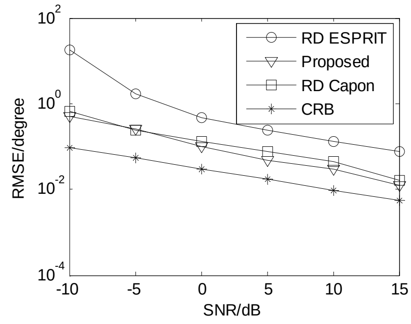

Figure 5. Angle estimation performance comparison

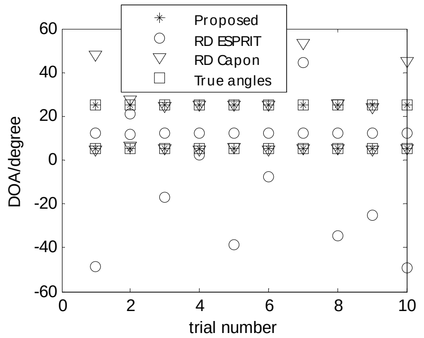

Figure 6. Angle estimation results for coherent targets

Fig.6 shows the estimation results of the three algorithms with two coherent targets ($(\beta_1,\beta_2)=[e^{j\pi/5},0.8e^{j\pi/5}]$ and $f_1=f_2=500Hz$), and SNR=10dB. It can be indicated from Fig.6 that the proposed algorithm works well for coherent targets, which make the other two algorithms fail to work or have performance degradation.

V. CONCLUSION

In this paper, we have proposed a DOA estimation algorithm in monostatic MIMO radar using compressive sensing and multiple snapshots. By using the reduced-dimensional transformation, EVD of the data and OMP for recovery, the angle estimation performance of the proposed algorithm is better than that of RD ESPRIT and RD Capon and the algorithm is effective due to the lower complexity. Furthermore, the proposed algorithm works well for coherent targets, and requires no knowledge of the noise.

ACKNOWLEDGMENT

This work is supported by China NSF Grants (61271327,61071164), Jiangsu Planned Projects for Postdoctoral Research Funds (1201039C), China Postdoctoral Science Foundation (2012M521099), Open project of key laboratory of underwater acoustic communication and marine information technology (Xiamen University), Hubei Key Laboratory of Intelligent Wire1ess Communications (IWC2012002), Open project of Key Laboratory of Nondestructive Testing (Nanchang Hangkong University)，Open project of Key Laboratory of modern acoustic of Ministry of Education (Nanjing University), the Aeronautical Science Foundation of China (20120152001), by Qing Lan Project and the Fundamental Research Funds for the Central Universities (NZ2012010, kfjj120115, kfjj20110215).

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