Method of Mathematical Modeling Based on PSO Algorithms

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***Abstract*— In order to set up universal and non-linear map of variables, a full binary tree is constructed as mathematical model. Leaf nodes of the full binary tree are linear combination of input variables, and used as inputs of next nodes. On the basis of weighting two inputs by selector for inner node, the inputs are again linearly combined and used as output for next node. The inputs and outputs of all the inner nodes are constructed in turn as the same, and the output of root node is the output of mathematical model, implementing segment-linear approximation. With the means of machine learning of particle swarm optimization for data from some areas, all the coefficients of mathematical model are achieved for the special. The mathematical model is applied to seismic inversion to interpret stratum by seismic data, approving it very practical.**

Keywords: Mathematical modelling, Linear combination, Particle swarm algorithm, Seismic inversion

I. Introduction

Mathematical Modeling is widely applied in production area, with a variation of mathematical knowledge to abstract mathematical models which reflect the amount variables and space shapes of the real world[1].With the help of the model, we can interpret the objective phenomena, or forecast the law of development, or afford the optical strategy and decision on basis of the development of some phenomena. The entire process of mathematical modeling includes selecting variables, ascertaining variables' relations in the mathematical function, revising formula structure, and computing parameters in the formula.

In some applications of mathematical modeling such as seismic inversion and well-logging interpretation, the processing of mathematical modeling is usually based on physical modeling accurately, and then to build relevant mathematical model[2]~[4]. It is utmost important to select which variables, ascertain the amount of variables and model structure. However, the physic modeling is greatly affected by the factors such as our experience. At present, there are some drawbacks of mathematical modeling by means of the analysis of a variation of mathematical models as follows:

a) The complexities of objective problems results in the mathematical models are difficult to extend to other areas with same problems because of different variables and coefficients.

b) By means of physic models, main information (namely variables) is only adopted, ignoring some other potential information and its associations to bring up the low availability of the mathematical models.

c) On the basis of physical models, the mathematical modeling by adding revised-coefficients brings up almost the same structure of mathematical models. Because the physical models are not usually universal for objective problems, the mathematical models are not adapted to interpret all objective problems.

d) The variability of the amount and non-linear relation of variables etc. results in the difficulty to certain mathematical structure with the relation of variables, and then the precision faultiness of mathematical models.

In order to conquer the drawbacks above, and implement mathematical modeling universally, a method of mathematical modeling by means of segment-linear approximation is presented based on particle swarm optimal algorithms.

II. Segment-linear approximation model

The solution of objective problem can be expressed continuous distribution in multi-dimension space. Mathematical model of segment-linear approximation can be expressed to segment and approximate the multi-dimension space by means of some hyperplanes. The key process of the modeling includes determining the amount and sizes (namely edges) of hyperplanes.

A. Segment-linear mathematical model

Segment-linear mathematical model(SLMM) is shown by Figure 1. Let input vector $X=\{x_1, x_2 \ldots x_m\}$, output vector $Y=\{y_1, y_2 \ldots y_n\}$, the structure of SLMM is that of full binary tree, and leaf nodes of SLMM are linear combination of all elements of input vector.

Let node number be n, layer number be h in the SLMM, and $h = \lfloor \log_2 n \rfloor + 1$. Leaf node i is linear calculator i.（i $\in \{2^{h-1}, ..., 2^h - 1\}$）：

$$Y_i = \omega_i X + \theta_i^T \quad (1)$$

where Y_i is the output of calculator, ω_i is a coefficient matrix, θ_i is an offset vector.

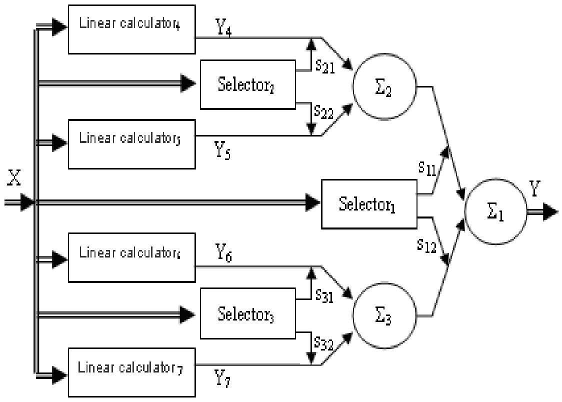


Fig. 1 Segment-linear Approximation Model

B. Selector

For each selector k（k$\in \{1,.., 2^{h-1} - 1\}$）, its output is s_{k1}, s_{k2}:

$$s_{kj} = \frac{\exp(\alpha_j X + \delta_j^T)}{\exp(\alpha_1 X + \delta_1^T) + \exp(\alpha_2 X + \delta_2^T)} \quad (2)$$

where j$\in \{1,2\}$, α_j is a coefficient matrix, δ_j is an offset vector. Obviously, $s_{k1}, s_{k2} \in [0,1]$,and $s_{k1}+s_{k2}=1$。

C. Output of combined node

For each combined node k（k$\in \{1,.., 2^{h-1} - 1\}$）,its output is Y_k:

$$Y_k = \sum_{j=1}^{2} s_{kj} Y_{2k+j-1} \quad (3)$$

where s_{k1} and s_{k2} are selectors。In fact, s_{k1} and s_{k2} are weights of linear combination. When one of them approaches to 1, the other approaches to 0.

It can be seen that this mathematical model of linear approximation is recursive definition. When k=1,and then Y_1 is output Y as entire model.

III. Optimization of Coefficients of SLMM Based on PSO

A. Presentation of coefficients in SLMM

After SLMM is constructed, precise determination of the coefficients of SLMM is utmost important. In this paper, PSO(Particle Swarm Optimization), a bionic algorithm of swarm intelligence[5]~[8], is applied to precisely determine the coefficients of SLMM. All the coefficients in SLMM make up a vector Pi=$<x_{i1},x_{i2},\ldots,x_{im}>$ as a particle, presenting a position in m-dimension space, and denoting a possible solution.

B. Learning optimization of SLMM

Each velocity v_i, corresponding to a particle, implements the update of particle. Since PSO is swarm algorithm, it searches in parallel for its possible solutions m- dimension space. On the basis of particle extremum P_{besti} and global extremum P_{gbest}, PSO avoids local minimum, and then obtains global optimization in m-dimension space. The update regulation of position and velocity of a particle is as follows:

$$v_{ij} = \eta \cdot v_{ij} + c_1 \cdot rand() \cdot (p_{bestij} - x_{ij}) + c_2 \cdot rand() \cdot (p_{gbestj} - x_{ij}) \quad (4)$$

$$x_{ij} = x_{ij} + v_{ij} \quad (5)$$

where i,j denote ith particle and jth component respectively. η is an inertial weight, c1 and c2 are acceleration constants, rand() is random function, generating a value $\in [0,1]$.

C. Learning goal of SLMM

For each data object（X,Y）, according to the coefficient vector Pi, SLMM is presented as Y*=f(X). The fitness function is defined as follows:

$$fit = \sum_{\forall X} (Y - Y^*)^2 \quad (6)$$

The learning goal of SLMM is that following formula is met for all the data objects:

$$\min\ fit = \min \sum_{\forall X} (Y - Y^*)^2 \quad (7)$$

namely obtaining the Pgbest when error is minimum.

D. Process of solving coefficients in SLMM

According to the requirement of calculation accuracy, the node number, input vector, coefficient number in SLMM have to be determined, and then the particles are defined. The process of solution of coefficients in SLMM is as follows:

Step 1: particles initialization: determining particle number, initialization of each particle position and velocity in random;

Step 2: particle fitness calculation: calculating fitness for each particle;

Step 3: particle position update with optimization: according to the particle fitness, determining update of the particle best position p_{best} or no update;

Step 4: global best position update: according to the fitness of each particle, determining update of the global best position p_{gbest} or no update;

Step 5: particle position and velocity update: according to formula （4）and （5）,updating particle position and velocity;

Step 6: solution end: obtaining the good enough fitness or getting to iterative number setting in advance,otherwise going to Step 2.

IV. APPLICATION OF SLMM TO SEISMIC INVERSION

Seismic inversion is a process of mathematical modeling and its application, in which seismic data is used as input and stratum data as output[3][4]. In present, many mathematical models for seismic inversion are mainly constructed on the basis of mathematical methods, which have some limitation for their application. Following mathematical model for seismic inversion is built by means of SLMM.

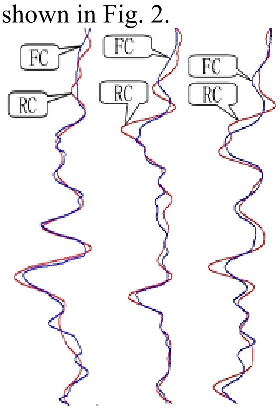
A. Model parameters

The layer number of SLMM is 5, the particle number is 100, learning factors $c_1= c_2=2.0$, inertial weight η =1.0, maximum velocity $V_{max}=0.08$, upper bound $X_{up}=3$ and lower bound $X_{down}=-3$ of particle flight position.

B. Experiments

1) Experiment by pre-stack seismic data

Pre-stack seismic data, consisting in seismic data as input and seismic waves as output, is from certain area of Daqing Oil Field. The schema of pre-stack seismic data is Envelope,Cosine_Pahse,Amp_Weight_Fre,Amp_Weight_Pahse,Derivative_Trace as input attributes, and Velocity as output attribute. There are 400 recorders, and then nearly divided into 3 average groups. Three cross-validation experiments are made by any one data group as testing data set and the other two as learning data set. The results of three experiments are shown in Fig. 2.



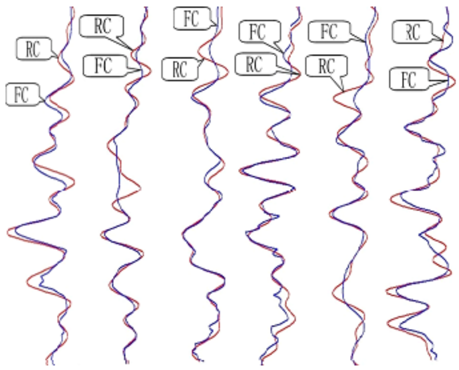
FC=Forecast Curve RC=Real Curve

Fig. 2 Pre-stack seismic inversion of velocity

According to cross- correlation coefficient method, the testing degrees of correlation are 0.8632, 0.8594 and 0.8456 respectively, and the average degree of correlation is 0.856067. All the results of experiments indicate that SLMM for seismic inversion is effective and useful for stratum interpretation.

2) Experiment by post-stack seismic data

Post-stack seismic data as input is from certain area of Daqing Oil Field and shale content as output from six wells in the area. The schema of pre-stack seismic data is Cube1_60Hz_trac, Cube1_60Hz_inte, Cube1_30Hz_trace, Cube1_30Hz_integ, Cube1_15Hz_trace and Cube1_15Hz_integ as input attributes,and Vshale as output attribute. 400 records of seismic data as a group are got from each well. Six cross-validation experiments are made by any one data group as testing data set and the other five as learning data set. The results of six experiments are shown in Fig. 3.



FC=Forecast Curve RC=Real Curve

Fig. 3 Post-stack seismic inversion of shale content

According to cross-correlation coefficient method, the testing degrees of correlation are 0.8627, 0.8556, 0.8486, 0.8515, 0.8603 and 0.8537 respectively, and the average degree of correlation is 0.8554. All the results of experiments indicate that SLMM for seismic inversion is effective and useful for stratum interpretation.

V. CONCLUSION

Segment-linear mathematical model(SLMM) is full binary tree. By means of selector with output near to 0 or 1 as branching selection, SLMM implements segment-linear approximation by leaf nodes. The structure of SLMM is very simple, and it is easy to obtain the coefficients in SLMM by optimization algorithm such as PSO. In particular applications, SLMM has some advantages such as generality, no need of physical modeling. Segment approximation by linear functions makes SLMM non-linear mapping, different from BP [9][10], which is made up by linear function and then mapped to non-linear by sigmod function. SLMM can be applied to statistical modeling. In fact, the linear calculator number and accuracy of SLMM are determined by acquaintance of objective problem and distribution of data in applications.

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