Numerical Simulation of Flash Floods Routing Based on Improved Leap-frog Method

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***Abstract*—In the 2-d numerical simulation of flash flood disaster, due to flood often occurred in the steep terrain and water flow rapidly changed, lead to that the calculated value is unstable and even the calculation diverge in the simulation. This paper presents a grid outflow correction method, which is based on the leap-frog finite difference format, through modifying the outflow rate of the grid circularly, to ensure the mass conservation in the whole process of computing. In the local dam bursting model, the simulated result comparison of the grid outflow correction method and the algorithm of implicit alternating direction on the mass conservation shows that, the new method can ensure the simulation accuracy and the numerical stability under the condition of steep terrain and moving boundary. According to the proposed method, the simulation analysis in the process of extreme flash flood disasters which happened in 2010 Zhouqu county in Gansu province was carried out. The comparison of simulation results and remote sensing estimation results shows that the deviation of the flood evolution time, speed and impact height are within 5%, and the consistency of evolution path is good, which verifies the validity of the algorithm.**

Keywords- Flash Floods; Numerical Simulation; Plane Shallow Water Wave Equation; Grid Outflow Calibration; Numerical Stability.

I. INTRODUCTION

The flash flood disaster is often caused by the influence of heavy rainfall or a larger peak flow, in the hilly areas [1]. China is located at the East Asian monsoon region, the mountainous and hilly areas account for two-thirds of the country's land area. Among them, the prevention area of the flash flood disaster is 46.3 million square kilometers, covering a population of 5.6 million. According to the statistics, in the period of 1950 to 2000, the death toll of flood was 263000, in which the massif's was 180000, accounting for 68.4 percent of the total deaths. In recent years, the proportion which the flash flood disaster caused the death of population accounts for the national flood disaster deaths has been improving and is more than 70 percent and it has been becoming the main casualties of disaster. With the development of society and economy, the prevention work of the flash flood disaster has been taken seriously more and more.

Earlier, the predicted technique of the flash flood disaster is that takes a field sampling on the trench and the mouth of the trench, and according the possible disasters types and levels to sure dangerous index [2]. The most representative is H. Aulitzky who put forward the torrent classification and hazard zone mapping index method [3], through the collection of the 51 specific factors of the nine kinds of indicators divided into different levels of danger zone. With the rapid development of modern science and technology, such as technology of geographic information systems, digital elevation models, remote sensing and satellite telemetry, Plane shallow water wave equation-based simulation method is widely used in forecasting and quantitative analysis of the phenomenon of convection within the floods, flash floods and landslides and other disasters. The method is not the similarity theory of the model experiment restrictions can be used to quickly and accurately reveal the cause of the disaster and process, thus greatly improving the flash floods and other emergencies flood forecast period. The most commonly used a 2-d numerical simulation method including the finite element method [4], the finite volume method and finite difference method [5]. According to the flood simulation, Jin W.L. and others proposed to use the discrete momentum equations of second -order upwind format nonlinear convection item and second-order leap-frog format discrete linear convection item to simulate the overbank process of Nakdong River [6]. Roger AF, who proposed a MacCormack +TVD format that is borderline and fitted the numerical model [7]. By detecting the water depth in each time step whether reaches the dry critical value to determine the dynamic boundaries. DHI water &environment (Denmark) instituted the implicit alternating direction algorithm to develop water dynamics simulation software Mike21 [8]. Furthermore, Simple method and the CIP method [9] are representative of the numerical format.

But in the 2-d numerical simulation of the debris flow disaster, Due to the river terrain steep, water flow changed rapidly, the traditional method often leads to negative water depth, which is in the process of moving boundary under the large computing time step outflow zero, resulting in the simulation of mass and momentum is not conserved [10], and ultimately leading to the calculation of numerical instability or even calculate divergently and can't be deprived of the results. At present, there is no one for the steep terrain, flash floods flat two-dimensional numerical simulation method. This paper presents an improved Leap-frog method, through the correction of finite difference leap-frog format grid outflow rate, making the whole calculation process of mass conservation, reducing the conservation of momentum error, to ensure the simulation accuracy and the numerical stability under the condition of steep terrain and moving boundary.

II. BASIC EQUATION

The evolution of the 2-d flood is usually described by shallow water wave equations, on the hydrostatics assumptions, the 2-d shallow water equations can through Navier Stokes equations for extended depth direction integral getting [11]. Quality continuity equation:

$$\frac{\partial h}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0\,, \tag{1}$$

Momentum continuity equation:

$$\frac{\partial M}{\partial t}+\beta\frac{\partial(uM)}{\partial x}+\beta\frac{\partial(vM)}{\partial y}=-gh\frac{\partial H}{\partial y}-\frac{\tau_{bx}}{\rho}\,, \tag{2}$$

$$\frac{\partial N}{\partial t}+\beta\frac{\partial(uN)}{\partial x}+\beta\frac{\partial(vN)}{\partial y}=-gh\frac{\partial H}{\partial y}-\frac{\tau_{by}}{\rho}\,, \tag{3}$$

where, M, N are respective for the x, y direction of the flow of flux; n is the roughness coefficient; g is the acceleration of gravity; β is the momentum correction factor; ρ is the density of water; τ_{bx}, τ_{by} are the bottom friction force in the x, y component in the direction.

$$\tau_{bx}=\rho g\frac{n^2 M\sqrt{M^2+N^2}}{h^{7/3}}\,, \tag{4}$$

$$\tau_{by}=\rho g\frac{n^2 N\sqrt{M^2+N^2}}{h^{7/3}}\,, \tag{5}$$

where, $H = h + \zeta$, H is the altitude of the free surface, ζ is respective for riverbed altitude, h is depth of the water.

III. NUMERICAL METHODS

A. Leap-frog format

It's very difficult to directly solve the basic equation, this paper adopts the finite volume method to 2-d shallow water wave equation for numerical discrimination. In order to improve the computational speed and stability, using staggered grid variable configuration. Variable h is water depth, which defined in the grid center, and flow velocity vector M and N are defined in the grid four sides of the midpoint, as shown in Figure 1.

In time, continuity equation in the quality is used to the forward difference scheme, and in space uses second-order central difference scheme. The time of the momentum equation of continuity, is used to the forward difference formats, nonlinear convection term is used to the first-order accuracy upwind difference format, and the pressure is used to the central difference scheme. In order to avoid the phenomena of Vasiliev numerical instability, friction is used to implicit difference scheme for the numerical dispersing.

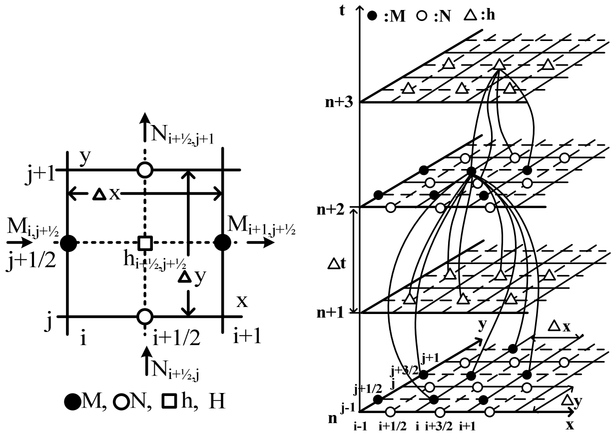

Figure 1. Computing Grid Definition

B. Grid outflow correction

Because of the terrain that the flash floods occurred is steep and rugged, the flood boundary, flow rate and water depth change suddenly. In the past, moving boundary calculation, it often happened to the water surface elevation below the ground elevation, in other words, the water depth calculated become negative. Resulting in quality can't guarantee conservation, stability becoming poor, and even divergent and result less. Therefore, the grid out of the correction method is used to correct grid outflow, thereby eliminating the negative depth, to ensure the conservation of mass and improve the stability of numerical calculation. The specific methods are as follows.

Numerical discrimination of the quality of the continuity equation as follows:

$$h^{n+3}_{i+1/2,j+1/2}(\Delta x\bullet\Delta y)=h^{n+1}_{i+1/2,j+1/2}(\Delta x\bullet\Delta y)+ 2\Delta t\left[\left(M^{n+2}_{i,j+1/2}-M^{n+2}_{i+1,j+1/2}\right)\Delta y+\left(N^{n+2}_{i+1/2,j}-N^{n+2}_{i+1/2,j+1}\right)\Delta x\right], \tag{6}$$

where, n+3 moment grid *(i,j)* of water = n+1 time grid *(i,j)* of water + 2Δt (unit time into the grid *(i,j)* of water-unit time outflow grid *(i,j)* of water).

When the grid depth appear negative, firstly judge whether the M, N inflow or outflow is, and calculate the grid inflows Q_{in} and outflow Q_{out}:

$$h^{n+3}_{i+1/2,\,j+1/2}=h^{n+1}_{i+1/2,\,j+1/2}+\frac{Q_{in}-\lambda Q_{out}}{\Delta x\cdot\Delta y}=0 \tag{7}$$

And then use the following formula to calculate the correction factor:

$$\lambda=(h^{n+1}_{i+1/2,\,j+1/2}(\Delta x\cdot\Delta y)+Q_{in})\,/\,Q_{out} \tag{8}$$

However, reducing inflows to the adjacent grid may cause the adjacent grid of depth becoming negative. In this case, we need to continue to double counting, until the entire negative depth grid corrected, and then transferred to the next time step.

IV. SIMULATION EXAMPLES

A. The dam break model simulation

Barrier dam often occurs in the mountains and canyons, which is consisted of particles both gravel and a diameter greater than 1 m chunks of rock. Even in the turbulent flow effect, the remnants of the dam can be balanced which is in the resistance of the coarse particles and energy dissipation so that can't completely break. The paper quotes from Fraccrollo et al. people to dam local physical model, using the proposed method to experiment, and with the simulation results of implicit alternating direction algorithm for the conservation of mass comparison.

The physical model is of a 100m×100m square container, which is set a thickness of regardless of the tailgate in the 35m and at the upstream boundary from the container, so the central tailgate can in a very short period of time to open a width of 20m gap, and instantly forms the positive wave downstream promoted and the negative wave spread to the dam, so as to constitute a symmetric partial break model, shown in Figure 2(a). In order to verify the stability of the algorithm in steep terrain, the downstream slope of the flood plain is set to 0°, 10°, 25°and 45°, shown in Figure 2(b).

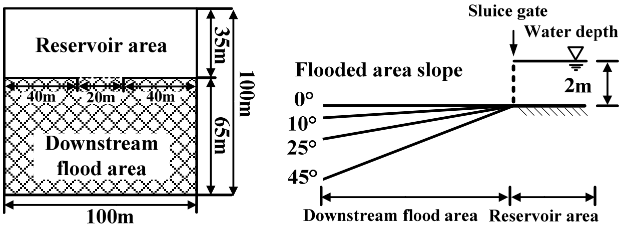

Figure 2. Dam break model of numerical simulation

Initial conditions: the initial upstream water depth of 2m, downstream water depth is 0m, the flow field in the initial velocity is 0. Boundary conditions: 3 sides of the baffle downstream open boundary, the container in addition to tailgate at the open boundary, the rest of the closed border, container of water remains the same.

Main parameters: drying depth (drying depth, water depth is less than this depth is not involved in the calculation) and flooding depth (Flooding depth, re-engage the depth of the calculation) is set to the minimum 0.01m and 0.02m. The computation time step is set to 0.0ls, 0.05s, 0.1s and 1s, excluding wind and bottom friction. It is concluded that the local dam break model, using the implicit alternating direction method for modeling and the quality of the relative error in Table I.

TABLE I. THE QUALITY RELATIVE ERROR OF DAM BREAK MODEL USING THE IMPLICIT ALTERNATING DIRECTION ALGORITHM SIMULATION (‰)

Time step	Quality relative error of simulation result at the 5th second				Quality relative error of simulation result at the 15th second				Quality relative error of simulation result at the 30th second			
	0 °	*10 °*	*25 °*	*45 °*	*0 °*	*10 °*	*25 °*	*45 °*	*0 °*	*10 °*	*25 °*	*45 °*
0.01S	0.02	0.06	0.06	0.05	0.08	0.07	0.05	0.05	0.17	0.08	0.06	0.04
0.05S	0.02	0.06	0.05	0.11	0.08	0.07	0.06	—	0.17	0.08	0.07	—
0.1S	0.02	0.06	0.05	1.71	0.08	0.07	—	—	0.17	0.08	—	—
1S	0.05	56.33	49.28	40.17	230.24	—	—	—	—	—	—	—

Through the local dam break model of flood routing simulation experiment can be seen, the implicit alternating direction algorithm will bring about the problem of inevitable unbalanced quality. Thus the greater the topographic slope to calculate more easily dissipate, the simulation can not continue. However, taken out with fixed grid, not only can eliminate the unbalanced quality during the simulation, but also computing robustness is greatly improved, which makes the simulation of the evolution process of flash floods under steep and complex terrain become possible.

B. Zhouqu large flash flood disasters visual simulation

Choose the debris flow disaster that happened in China's Gansu province Zhouqu County mountain, Sanyan valley ditch and Luojia valley ditch as an example, the application of the proposed method to join the sediment viscosity to simulate, and compared with the post-disaster remote sensing data.

On 7th August 2010, ZhouQuXian northeast mountainous dumped heavy rain for 40 minutes, hour rainfall of 96.77mm. Because the main channel, upstream and branch channel section of the Sanyan valley ditch is V shape, with an average longitudinal slope down to 300 ‰, the region's largest relative height difference of 2488m; and the Luo Jiayu valley cross-strait hillside slope angle averaged 50°, the ditch bed down an average of 334 ‰, the maximum relative height difference of 2474m, the steep topography complexity, there is no discovery can be applied to the numerical simulation algorithm of this type of flood disasters.

The topographic data of simulation is from the U.S. Space Administration (NASA) and Japan's economy, Trade and Industry (METI) jointly launched the Earth electronic terrain data in 2009, the vertical accuracy of up to 20m, the horizontal accuracy of 30m. Figure 3 shows that the simulation results of the evolution of Zhouqu flash flood disasters, the simulation time step of 0.1s, the bottom friction coefficient of 0.01, the mudslides viscosity coefficient μ= 0.028N•s/m^2.

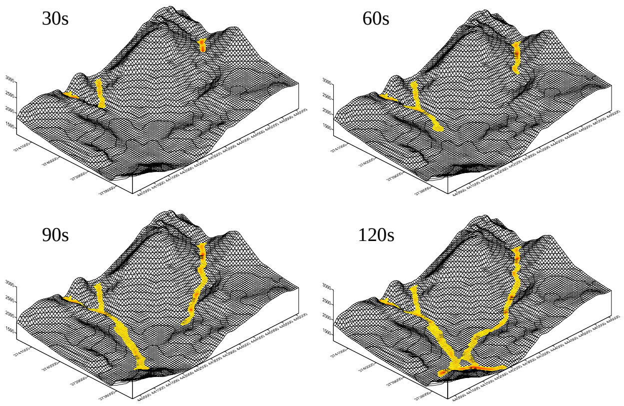

Figure 3. Zhouqu flash flood disaster routing process

Compare and analyze the results of post-disaster remote sensing and numerical simulation, Zhouqu flash flood disaster in the general characteristics:

(1) The short period of time precipitation into flood. Field analysis of the torrential striker out of the mountain pass to reach the county is about 2min [12], the simulation results for the 127s.

(2) The fast speed. According to the Zhouqu remote sensing image of flood disasters, the velocity value of the flash flood is approximate to the calculated value in several key positions.

(3) The impact height. According to the disaster site, measured the pass water level height of about 8m,the result of the proposed method of simulation is the 7.67m, and the deviation of with remote sensing estimates is 4.125%.

(4) The straight paths. The main part of flash floods is not the original drainage ditch along the west side of the direction of evolution after the mountain pass, but remained in the east side of a straight leap to the south. Zhouqu flash flood disaster images compared with the simulation results in Figure 4, where the highlight part is the remote sensing image after the disaster occurred on August 8, 2010.

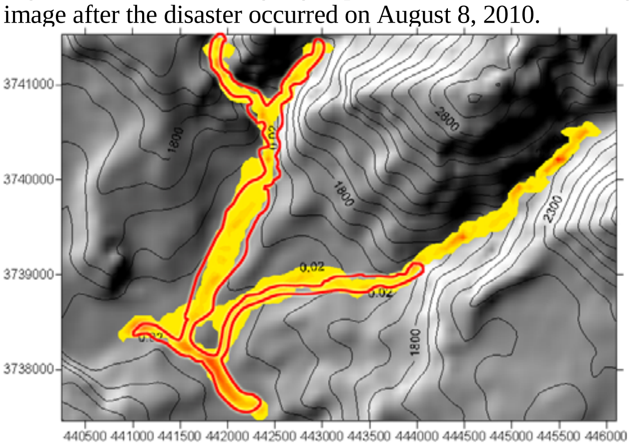

Figure 4. Zhouqu flash flood disaster remote sensing images compared with the simulation results (unit: m)

V. CONCLUSIONS

In the 2-d numerical simulation of flash flood disaster, due to the steep terrain watershed and water flow rapidly changed, the traditional method often leads to negative water depth, which is in the process of moving boundary under the large computing time step outflow zero, resulting in the simulation of mass and momentum is not conserved, and ultimately leading to the calculation of numerical instability or even calculate divergently and can't be deprived of the results. At present, there is not a suitable for steep terrain mountain flood plane 2-dnumerical simulation method. This paper presents a grid outflow correction algorithm, through amendment by the finite difference leap-frog format grid outflow rate, the calculation of mass conservation and reduces the momentum conservation error to ensure the stability of the moving boundary conditions of the steep terrain of simulation accuracy and calculation values.

With the in-depth study of the numerical simulation technology, developing more precise and more efficient numerical methods for flash floods becomes an important guarantee of a high-quality prediction and evaluating a process of flash floods power and harmfulness.

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