Modeling and Attitude Control of a Spinning Spacecraft with Internal Moving Mass

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***Abstract*—An attitude control system of a spinning spacecraft with internal moving mass is presented in this paper. This system consists of a rigid body and two internal radial moving masses. The mathematical model, including attitude kinematics and nonlinear dynamics equations, is established based on Newtonian mechanics. The control law is designed based on the linear-quadratic-regulator (LQR) theory. The performance of the controller is demonstrated in numerical simulation, and the response shows that the attitude control system is stable and effective.**

Keywords- spinning spacecraft; moving mass; LQR; attitude control

I. INTRODUCTION

Mass moment techniques produce maneuver via changing the positions of internal moving masses[1-3]. For a slow spinning spacecraft the misalignment between thrust and the center of mass of the vehicle, which is caused by changing the positions of internal moving masses, result in attitude control moment. Whereas, for a fast spinning spacecraft, the effects of principal axis misalignment, caused by the movement of internal moving masses, generate attitude control moment. Some control law has been investigated to solve the problem of attitude control of spinning vehicle using internal moving mass.[4-9] Wang and Yang[7] chose the hybrid PID method as the control law, and a GA training block was employed to obtain the optimal coefficients. In reference [8], an attitude control system is developed via utilizing sliding mode control approach, and using fuzzy algorithm to overcome the traditional chattering problem. Shen and Zhang[9] develop the control system via utilizing nonlinear predictive control approach whose coefficients were optimized by ant colony genetic algorithm.

A slow spinning spacecraft with two internal radial moving masses is investigated in this paper. The mathematical model, including attitude kinematics and linear dynamic equations, is established based on Newtonian mechanics and small perturbation assumption. The control system is further designed based on LQR theory. The numerical demonstrations show that the proposed control system can effectively respond to the commands of both pitch channel and yaw channel.

II. MATHEMATICAL MODEL OF SPINNING SPACECRAFT WITH INTERNAL MOVING MASS

A. Coordinate systems

To describe the motion of the airframe, three relevant coordinate systems, datum coordinate system, body coordinate system, and non-spinning body coordinate system, are defined in the following.

The datum coordinate system *Axyz* is assumed to be an inertial coordinate frame. *A* is located at the intersection of the earth's surface and the line determined by the center of mass of the vehicle and the earth's core, *Ax* is in the horizontal plane towards to target, *Ay* points upward in the vertical plane containing *Ax*, and *Az* is defined by the right-hand rule.

The body coordinate system $Ox_1y_1z_1$ is a spinning frame and is fixed to the spinning vehicle. The origin O is at the center of mass of the vehicle, Ox_1 is coincident with the longitudinal axis pointing to the nose, Oy_1 orthogonal to Ox_1 in the symmetrical plane of the vehicle, and Oz_1 defined by the right-hand rule. The body coordinate system $Ox_1y_1z_1$ can be obtained by rotating Axyz an angle ψ about Ay and then an angle ϑ about Az, here ϑ and ψ are pitch and yaw angle respectively.

The non-spinning body coordinate system $Ox_4y_4z_4$ can be obtained by rotating $Ox_1y_1z_1$ an angle γ about Ox_1, here γ is the roll angle.

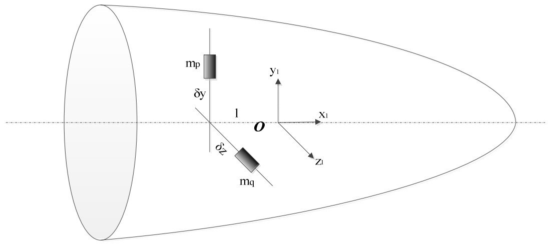

Figure 1. System configuration

B. Description of the spacecraft

The spinning spacecraft of a cone-shaped rigid body with two radial moving masses as actuator is shown in Fig.1.

(1) m_b is the mass of rigid body.

(2) m_p, m_q are the masses of actuator, respectively.

(3) $m_s = m_b + m_p + m_q$ is the mass of the system.

(4) $u_p = m_p / m_s, u_q = m_q / m_s$ are the mass ratio.

(5) $\mathbf{r}_y = \begin{bmatrix} l & \delta_y & 0 \end{bmatrix}^T, \mathbf{r}_z = \begin{bmatrix} l & 0 & \delta_z \end{bmatrix}^T$ are the position vectors of the actuator masses in body frame. l=const is a specified value. δ_y and δ_z are the displacements of the masses.

(6) $\dot{\mathbf{r}}_y = \begin{bmatrix} 0 & \dot{\delta}_y & 0 \end{bmatrix}^T$, $\dot{\mathbf{r}}_z = \begin{bmatrix} 0 & 0 & \dot{\delta}_z \end{bmatrix}^T$ are the relative velocities of the actuator masses in body frame.

(7) $\boldsymbol{\omega}_1 = \begin{bmatrix} \omega_{x1} & \omega_{y1} & \omega_{z1} \end{bmatrix}^T$ is the angular velocity vector in body frame.

(8) $\boldsymbol{\omega}_4 = \begin{bmatrix} \omega_{x4} & \omega_{y4} & \omega_{z4} \end{bmatrix}^T$ is the angular velocity vector in non-spinning body frame.

(9) $\boldsymbol{\omega}_m$ is the spinning rate of the vehicle.

(10) $\mathbf{P} = \begin{bmatrix} P & 0 & 0 \end{bmatrix}^T$ is thrust vector.

(11) $\mathbf{M} = -(\mathrm{u}_p \mathbf{r}_y + \mathrm{u}_q \mathbf{r}_z) \times \mathbf{P}$ is the moment.

C. Kinematics and dynamic equations

The angular vector in non-spinning body frame can be expressed as

$$\boldsymbol{\omega}_4 = \dot{\boldsymbol{\vartheta}} + \dot{\boldsymbol{\psi}} \tag{1}$$

The angular vector in body frame can be expressed as

$$\boldsymbol{\omega}_1 = \dot{\boldsymbol{\vartheta}} + \dot{\boldsymbol{\psi}} + \dot{\boldsymbol{\gamma}} = \boldsymbol{\omega}_4 + \boldsymbol{\omega}_m \tag{2}$$

The vector in Eq. (2) is projected to non-spinning body frame, one has

$$\begin{bmatrix} \omega_{x4} + \omega_m \\ \omega_{y4} \\ \omega_{z4} \end{bmatrix} = \begin{bmatrix} cos\vartheta & sin\vartheta & 0 \\ -sin\vartheta & cos\vartheta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\psi} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\vartheta} \end{bmatrix} + \begin{bmatrix} \dot{\gamma} \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

Then, the kinematics equations can be expressed as

$$\begin{cases} \dot{\gamma} = \omega_{x4} + \omega_m - \omega_{y4} tan\vartheta \\ \dot{\psi} = \omega_{y4} / cos\vartheta \\ \dot{\vartheta} = \omega_{z4} \end{cases} \tag{4}$$

The momentum of the system can be expressed as

$$\mathbf{L}_s = J_o \boldsymbol{\omega}_1 - m_p \times \left(\dot{\mathbf{r}}_y + \boldsymbol{\omega}_4 \times \mathbf{r}_y \right) - m_q \times \left(\dot{\mathbf{r}}_z + \boldsymbol{\omega}_4 \times \mathbf{r}_z \right) \tag{5}$$

where J_o is the moment of inertial tensor.

According to the theorem of angular momentum, the rotational motion of the airframe can be described as

$$\frac{\mathrm{d}\mathbf{L}_s}{\mathrm{dt}} = \frac{\partial \mathbf{L}_s}{\partial \mathrm{t}} + \boldsymbol{\omega}_4 \times \mathbf{L}_s = \mathbf{M} \tag{6}$$

The vector in Eq. (5) is projected to non-spinning body frame, one has

$$J_o \dot{\boldsymbol{\omega}}_1 + \boldsymbol{\omega}_4 \times J_o \boldsymbol{\omega}_1 + (1-u_p) m_p \mathbf{r}_y \times \mathbf{a}_y + (1-u_q) m_q \mathbf{r}_z \times \mathbf{a}_z = \mathbf{M} \tag{7}$$

where,

$$\mathbf{a}_y = \ddot{\mathbf{r}}_y + \dot{\boldsymbol{\omega}}_4 \times \mathbf{r}_y + 2 \times \boldsymbol{\omega}_4 \times \dot{\mathbf{r}}_y + \boldsymbol{\omega}_4 \times \left(\boldsymbol{\omega}_4 \times \mathbf{r}_y \right)$$

$$\mathbf{a}_z = \ddot{\mathbf{r}}_z + \dot{\boldsymbol{\omega}}_4 \times \mathbf{r}_z + 2 \times \boldsymbol{\omega}_4 \times \dot{\mathbf{r}}_z + \boldsymbol{\omega}_4 \times \left(\boldsymbol{\omega}_4 \times \mathbf{r}_z \right)$$

III. LQR CONTROLLER DESIGN

The linear dynamical equations of motions are derived from the nonlinear equation. And the simplification is performed under the following assumptions:

The rotation rate keeps constant in the flight $\dot{\gamma} = \omega_m$, and roll rate in non-spinning body frame (ω_{x4}) is equal to zero. Thus $\ddot{\gamma} = 0$.

The moments of inertial with respect to radial axes are the same, i.e., $J_o = diag\{J_{xx}, J_{yy}, J_{yy}\}$.

ω_{y4} and ω_{z4} are small.

The masses of actuators are the same, $m_p = m_q = m$, thus $u_p = u_q = u$. and $u \ll 1$.

The actuators are located at the center of mass of the vehicle, i.e., $l = 0$.

Under these assumptions and ignore nonlinear terms, the equation of motion becomes

$$J_2 \dot{\omega}_{y4} + J_1 \omega_{z4} \dot{\gamma} = -\mu \delta_z P \tag{8}$$

$$J_2 \dot{\omega}_{z4} + J_1 \omega_{y4} \dot{\gamma} = \mu \delta_y P \tag{9}$$

where $J_1 = J_{xx}$, $J_2 = J_{yy}$.

The tracking error in pitch and yaw can be expressed as

$$e_y = \psi_c - \psi \tag{10}$$

$$e_z = \vartheta_c - \vartheta \tag{11}$$

where ψ_c and ϑ_c are the constant input commands, then, the differential equations of Eq.(9) and (10) can be written as

$$\dot{e}_y = -\dot{\psi} = -\omega_{y4} / cos\vartheta \tag{12}$$

$$\dot{e}_z = -\dot{\vartheta} = -\omega_{z4} \tag{13}$$

The linear dynamics are composed by Eq. (8), (9), (12) and (13).

Define $X = \begin{bmatrix} \omega_{y4} & \omega_{z4} & e_y & e_z \end{bmatrix}^T$, $u = \begin{bmatrix} \delta_y & \delta_z \end{bmatrix}^T$, one has

$$\dot{X} = AX + Bu \tag{14}$$

where

$$A=\begin{bmatrix} 0 & -J_1\dot{\gamma}/J_2 & 0 & 0 \\ J_1\dot{\gamma}/J_2 & 0 & 0 & 0 \\ -1/\cos\vartheta & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & -mP/J_2 \\ mP/J_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For a continuous-time linear system as described in Eq. (14), according to the LQR theory, we can define the objective function as

$$J=\int_0^\infty \left(X^T QX + u^T Ru \right) dt \tag{15}$$

where Q is a symmetric positive semi-definite matrix, R is a symmetric positive definite matrix. By minimizing the objective function, the optimal feedback control is

$$u = -R^{-1}B^T PX \tag{16}$$

where P is the algebraic matrix Riccati equation solution

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{17}$$

IV. NUMERICAL SIMULATIONS

The performance of the proposed control law is evaluated from the nonlinear dynamic equations based on Eq. (8), (9), (12) and (13). The parameters of the spinning spacecraft are given as: the thrust P=100N, the mass ratio u=0.05, the spinning rate $\omega_m = 4\pi$ rad/s, the moments of inertial $J_1 = 4.7\text{kgm}^2$ and $J_2 = 70\text{kgm}^2$. The inertial angular velocity $\omega_{y0} = \omega_{z0} = 0$. The inertial attitude angle $\psi_0 = -5°$ and $\vartheta_0 = 30°$. The input commands $\psi_c = \vartheta_c = 10°$. Set the weight matrix $Q = diag\{5,5,10,10\}$ and $R = diag\{0.1,0.1\}$.

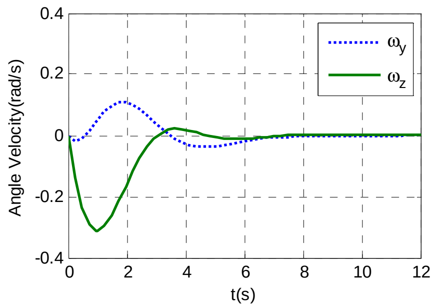

Figure 2. Response of the angular velocities without input command

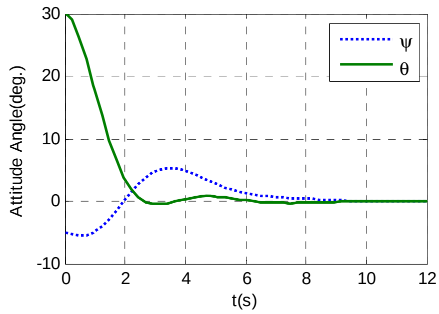

Figure 3. Response of the attitude angles without input command

Simulation of the control system results without input commands are shown in Fig.2 and Fig.3. Figure 2 shows the response of the angular velocities. Figure 3 shows the response of the attitude angles. It is observed that the control system force the angular velocities and the attitude angles to zero, thus the attitude control system with moving mass is stable. Simulation of the control system results with input commands are shown in Fig.4 and Fig.5. Figure 4 shows the response of the angular velocities. Figure 5 shows the response of the attitude angles. It is observed that the response time of the attitude control system to the input commands is less than 3s.

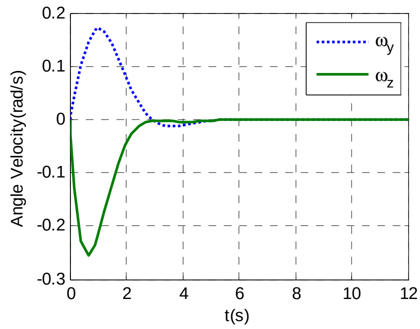

Figure 4. Response of the angular velocities with input command

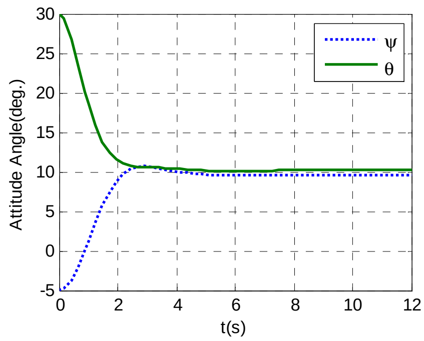

Figure 5. Response of the attitude angles with input command

V. CONCLUSIONS

In this paper, the mathematical model of a spinning spacecraft with two internal radial moving mass, including attitude kinematics and nonlinear dynamic equations, is established following Newtonian mechanics. With proper assumptions, the linear dynamics equations are further derived. The control system is designed based on LQR theory. The time response behavior of the control system is evaluated in numerical simulation, and the response of the controller shows that the attitude control system is effective and the response time is acceptable.

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