Methods for Determining Differences of Attribute Weight between Evidences In Bridge Assessment

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***Abstract*—The attribute weights determined is a key issue in the decision-making. Attribute weights given in the form of distribution of the attribute value to language assessment and rating their trust completely unknown or partially unknown circumstances, the proposed method of weight determination of the two based on objective evidence of differences between rights. Method one calculated based on the evidence of conflict between the coefficient weights; method to calculate the mean and variance of the research evidence weights. Finally, through the application of specific cases to verify the effectiveness of the method.**

Keyword:multi-attribute decision;making conflicting evidence ;language assessment; Variance

I. INTRODUCTION

For problems on MCDM and MCGDM, both attribute weight and expert weight have direct impacts on decision results. So it's critical about how to determine scientifically the weight. So far, the determination method invented by reseachers falls into three categories[1]: subjective method, objective method and aggregrative method. The first solution is discussed the most and representatives are direct evalution method[2], eigenvector method, and mathematical programming solution[6]. Such methods are all dependent on subjective preferences of decision-makers. Objective method can better avoid influences by human factors with the advantage of objective decision-making matric information, e.g. entropy method, variance solution, and ideal point method, and distance method as well. Aggregative method attempts to integrate merits of both subjective and objective methods, such as, Ma. J et al. employed the multi-objective teaching planning model to aggregate two methods as above; yet, Xu[3] pointed out that the obtained objective weight was not desirable.

In the paper, the author defines based on the theory of evidence the mean value and variance between evidences, in the context of decision-making information expressed with linguistic evaluation scale and confidence distribution and by an evidence combining method, and thus proposed an attribute weight dermination method based on the degree of conflicts between evidences and the other based on attribute mean values and variances. Both methods can help get attribute weight from objective decision-making matrices, in avoidance of effects by subjective preferences of decision-makers, and easy to apply them for solving decision-making or group decision-making problems under the similar conditions[4].

II. KNOWLEDGE PREPARATIONS

A. D-S evidence theory

D-S evidence theory[5] is a method with regards to uncertainty reasoning. With weaker axiom system than probability theory and no need of priori probability, the method has remarkable advantages in the uncertain decision-making analysis. Here, we do not introduce its basic concepts.

Provide two basic probability assignments (BPA) m_1 and m_2 in Ω from different creditable evidence sources like $m_{12}(A) = m_1 \oplus m_2$, then, define

$$m_{12}(A) = (1-k)^{-1} \sum_{B,C \subseteq \Omega, B \cap C = A} m_1(B) m_2(C) \quad (1)$$

Here, k is conflict coefficient, $k = \sum_{B,C \subseteq \Omega, B \cap C = \Phi} m_1(B) m_2(C)$, $\forall A \subseteq \Omega, A \neq \Phi$.

For the reason that the combination rule would often reach conclusions contrary to intuition when synthesizing highly conflicted evidences, quite a few of researchers raised improved combination rules. Zhang Shenggang et al.utilized certainty factor to modify the original BBA and utimately obtained the combination method based on evidence certainty. It demonstrated favorable performance as to synthesizing highly and low conflicted evidences.

B. Description of decision-making problems

Suppose for one MCDM problem, there are M candidate assessment solutions $a_i(i = 1,2,, M)$ and N attributes $c_j(j = 1,2,\cdots,N)$; attribute weight is unknown as $w = (w_1, w_2, \cdots, w_N)^T$ and meets $\sum_{j=1}^{N} w_j = 1$, $w_i \in [0,1](i = 1,2,, M)$. For the sake of simplicity, decision-makers make assessment based on the uniform linguistic scale with T linguistic variables by each attribute, which is put as $\{s_t | t=1,2,\cdots,T\}$ to signify T rating levels since it does not mention linguistic variables. Evaluation made by decision-makers each time puts the generated decision-making matrix as $D=(B(c_j(a_i)))_{M\times N}$ in the form of joint reliability distribution of rating levels, in which,

$$B(c_j(a_i))=\{(s_t,\beta_{t,j}(a_i)) | \beta_{t,j}(a_i)\in[0,1],\sum_{t=1}^{T}\beta_{t,j}(a_i)\le 1,\sum_{t=1}^{T}\beta_{t,j}(a_i)+\beta_{\Omega,j}(a_i)=1,t=1,2,\cdots,T\}$$

$\beta_{t,j}(a_i)$ means decision-makers believe the solution a_i belongs to the level s_t in terms of its performance on attribute c_j. $B(c_j(a_i))$ is a_i 's BBA on attribute c_j. If $\beta_{\Omega,j}(a_i)=0$, the assessment is complete, or it's not.

C. Mean values and variances between evidences

To consider the evaluation set $B(c_j)=(B(c_j(a_1)),B(c_j(a_2)),\cdots,B(c_j(a_M)))^T$ in $c_j(j=1,2,\cdots,N)$, we can regard it as an evidence set. In order to gain the average value of the attribute, i.e. average evaluation level, we can firstly attempt to aggregate M evidences, which may be of low or high conflict. Define distance matrix between evidences (assessment) $dif(c_j)=(d_{kl})_{M\times M}$, in which,

$$d_{kl}=\begin{cases}0, & k=l\\ \sqrt{\sum_{t=1}^{T}[\beta_{t,j}(a_k)-\beta_{t,j}(a_l)]^2}, & k\neq l\end{cases} \quad (2)$$

Obviously, $\max\{d_{kl}\}=\sqrt{2}$ makes $dif(c_j)$ standardized as $\tilde{d}_{kl}=d_{kl}/\sqrt{2}$ and the result matrix is the same put as $dif(c_j)$. Therefore, the mean squared Euclidean distance from evidence $B(c_j(a_i))$ to the set $B(c_j)$ is $sod_i(c_j)=\sum_{k=1}^{M}\tilde{d}_{kl}^{\,2}/M$. The size of $sod_i(c_j)$ refers to the degree of difference between $B(c_j(a_i))$ and other evidences.

Definition 2[20]: the certainty factor of evidence $B(c_j(a_i))$ is $\varepsilon_i(c_j)=(1-sod_i(c_j))e^{sod_i(c_j)}$.

Use $\varepsilon_i(c_j)$ to modify $B(c_j(a_i))$ for the original BBA. The outcome is written as $\tilde{B}(c_j(a_i))$:

$$\tilde{\beta}_{t,j}(a_i)=\beta_{t,j}(a_i)\cdot\varepsilon_i(c_j), \quad \tilde{\beta}_{\Omega,j}(a_i)=1-\sum_{t=1}^{T}\beta_{t,j}(a_i) \quad (3)$$

in which, $\tilde{\beta}_{t,j}(a_i)$ and $\tilde{\beta}_{\Omega,j}(a_i)$ stand respectively for the certainty degree which is assigned and not assigned to the level t after modification. Next, apply Dempster combination rule, i.e. equation (1) to combine and the result goes:

$$\tilde{B}(c_j)=\{(s_t,\beta_{t,j}) | \beta_{t,j}\in[0,1],\sum_{t=1}^{T}\beta_{t,j}\le 1,\sum_{t=1}^{T}\beta_{t,j}+\beta_{\Omega,j}=1,t=1,2,\cdots,T\} \quad (4)$$

Definition 3: call $\tilde{B}(c_j)$ in (4) the average evaluation of each scheme on attribute $c_j(j=1,2,\cdots,N)$, which is also called the evaluation mean and put as $E(c_j)=\tilde{B}(c_j)$.

Definition 4: name the quadratic sum between evidence of evaluation (evidence) set $B(c_j)$ and the mean value $E(c_j)$ the evaluation level variance $sd(c_j)$ on attribute c_j, that is:

$$sd(c_j)=\sum_{i=1}^{M}\|B(c_j(a_i))-E(c_j)\|^2\Big/M=\sum_{i=1}^{M}\sum_{t=1}^{T}[\beta_{t,j}(a_i)-\beta_{t,j}]^2\Big/M \quad (5)$$

III. EIGHT DETERMINATION METHOD BASED ON DIFFERENCES BETWEEN EVIDENCES

A. Method based on conflict coefficient (method 1)

In the process of combining evidences, conflict coefficient k indicates the scale of difference between two evaluations. If evaluation value discrepancy of each solution of one attribute is little, it oughts to give a smaller weight; otherwise, it should be a bigger weight. According to the idea, we conclude weight computation method as follows: the matrix constituted by conflict coefficient k of modified evidences among both $\tilde{B}(c_j(a_i))$ and $\tilde{B}(c_j(a_{i_2}))$ is put as $K=(k_j(i_1,i_2))_{M\times M}$, where, $k_j(i_1,i_2)$ is the coefficient between $\tilde{B}(c_j(a_{i_1}))$ and $\tilde{B}(c_j(a_{i_2}))$, $i_1,i_2=1,2,\cdots,M$. Consider $k_j^2=\sum_{i_1=1}^{M}\sum_{i_2=1,i_2>i_1}^{M}k_j^2(i_1,i_2)$ as conflict quadratic sum between evidences on $c_j(j=1,2,\cdots,N)$. The attribute weight can be obtained through (6):

$$w_j=k_j^2\Big/\sum_{j=1}^{N}k_j^2, \quad j=1,2,\cdots,N 。 \quad (6)$$

B. Method based on variances between evidences (method 2)

In the definition 4, variance reflects quantitatively the differentiation of evaluation value of each solution on the attribute. Apparently, if evaluation values of various solutions on one attribute are the same, then, the attribute will not affect making decisions and should be given the weight 0; if expert evaluation variance is big on one attribute, then it is decisive to make decisions and should be given a bigger weight. Based on that principle, a method for computing the objective weight of an attribute is provided under two circumstances.

(i) If the weight is utterly unknown, we can calculate it as per:

$$w_j = sd(c_j) \Big/ \sum_{j=1}^{N} sd(c_j), \quad j = 1,2,\cdots,N \text{。} \quad (7)$$

(ii) If decision-makers have preferences for one attribute, or its weight is known, then, put it $w \in H$, where, It is set composed of by weight information of the following five cases:

（1） Weak order: $\{w_i \geq w_j\}$；

（2） Strong order: $\{w_i - w_j \geq \alpha_i (> 0)\}$；

（3） Times order: $\{w_i \geq \alpha_i w_j\}, 0 \leq \alpha_i \leq 1$；

（4） Interval order: $\{\alpha_i \leq w_i \geq \alpha_i + \varepsilon_i\}, 0 \leq \alpha_i < \alpha_i + \varepsilon_i \leq 1$；

（5） Differential order: $\{w_i - w_j \geq w_k - w_l\}, 其中 j \neq k \neq l$ 。

Construct the mathematical planning model below to achieve the optimal weight:

$$\min J = \sum_{j=1}^{N} \left(w_j - sd(c_j) \Big/ \sum_{j=1}^{N} sd(c_j) \right)^2$$

$$s.t. \quad w \in H; \sum_{j=1}^{N} w_j = 1 \quad (8)$$

$$0 \leq w_j \leq 1, j = 1,2,\cdots,N \text{。}$$

Once the weight is determined, it's probable to select the best solution or sort all candidate solutions with specific information aggregation technologies, e.g. combination method based on evidence certainty as in the work , and analysis ER algorithm in [5], to integrate values of all solutions on all attributes and sort those aggregation assessment values with the method like utility interval method in .

IV. ANALYSIS OF APPLICATIONS

A. *Experimentation of the proposed algorithm*

To validate the effectiveness of the solution discussed here, we chose three bridges on segments on Xiangwang Rd.-Yinmadi Rd., Xiangwang Rd.-Fazhan avenue, and Xiangwang Rd.-Fengfu Rd. for this test and analysis. Attribute values of those bridges are all different in terms of abrasive surface c_1, sidewalk c_2, upperlayer of roadway c_3, underlayer of roadway c_4, rail c_5 and expansion joint c_6. Details are shown in table I.

B. *Discussion of simulation process*

We discussed all attributes listed on the above table with solution 1 and 2, with steps as follows:

（1）Weight calculation

$$f_L(\mathbf{x}_j) = \sum_{i=1}^{L} \beta_i G(\mathbf{a}_i, b_i, \mathbf{x}_j) = \mathbf{t}_j, \quad j = 1,2,\cdots,N \quad (3.1)$$

where, β_i is output weight; $\mathbf{a}_i$ and b_i are learning parameters of nodes on hidden layers; $G(\mathbf{a}_i, b_i, \mathbf{x}_j)$ is the output of nodes on the i th hidden layer, which is relating to input $\mathbf{x}_j$.

For the additive hidden phase $G(\mathbf{a}_i, b_i, \mathbf{x}_j) = g(\mathbf{a}_i \cdot \mathbf{x}_j + b_i)$, $\mathbf{a}_i$ is input weight vector and $b_i \in R$ is divergence of the i th hidden layer. Put

$$\mathbf{H} = \begin{bmatrix} G(\mathbf{a}_1, b_1, \mathbf{x}_1) & \cdots & G(\mathbf{a}_L, b_L, \mathbf{x}_1) \\ \vdots & \ddots & \vdots \\ G(\mathbf{a}_1, b_1, \mathbf{x}_N) & \cdots & G(\mathbf{a}_L, b_L, \mathbf{x}_N) \end{bmatrix}_{N \times L},$$

$$\beta = \begin{bmatrix} \beta_1^T \\ \vdots \\ \beta_L^T \end{bmatrix}_{L \times m}, \quad \mathbf{T} = \begin{bmatrix} \mathbf{t}_1^T \\ \vdots \\ \mathbf{t}_N^T \end{bmatrix}_{N \times m},$$

then, equation (3.1) can be revised as $\mathbf{H}\beta = \mathbf{T}$.

（2）Computation of assessment ranking

The computation methods for certainty of evaluation results of decision-makers k against attribute value j are seen in 3.2, 3.3, 3.4, 3.5:

$$p_{ij} = (p_{ij1}, p_{ij2}, p_{il3}) \quad (3.2)$$

$$p_{ij1} = \frac{1}{m} \sum_{i=1}^{m} a_{ij} \quad (3.3)$$

$$p_{ij2} = \frac{1}{m} \sum_{i=1}^{m} b_{ij} \quad (3.4)$$

$$p_{ij3} = \frac{1}{m} \sum_{i=1}^{m} c_{ij} \quad (3.5)$$

The calculation formula adopted in the paper is shown in 3.6:

$$p_{ij}^k = (p_{ij} + 2p_{ij2+} p_{ij3}) / 4 \quad (3.6)$$

C. *C. Conclusion comparisons*

Take the case of some data, i.e. the first broad attribute, from the work [18] to demonstrate. Assess three bridges based on six aspects or attributes, with seven rating levels (see Table I). With the use of the proposed method, the weight obtained by method 1 (case 1), variance aquired by method 2 (case 3), e.g. $w_1 \geq w_5$, $w_3 \geq 0.1$, $w_4 \geq 0.05$, and the planning model (8) as well, optimal weights were achieved as observed in TableII.

Table I showed the distribution of certainties of such three bridges on each rating level by advantage of the method proposed in [18], taking case 1 for instance. The maximum and minimum expected utilities (similar to those in [18]) and ordering results were shown in Table IV.

V. CONCLUSION

Due to the complexity and uncertainties on practical problems, attribute weights are unknown or partially unknown and the evaluation is usually not on a particular linguistic scale. For problems on decision-making, several objective methods for weight determination were presented in the paper according to differences between attribute values. Those methods proved to be objective and reliable. They are easy to implement on computers and can make full use of known information.

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TABLE I. ORIGINAL BBA OF BRIDGE APPRAISAL

	Abrasive Surface C_1	**Sidewalk** C_2	**Upperlayer of Roadway** C_3	**Underlayer of Roadway** C_4	**Rail** C_5	**Expansion Joint** C_6
Bridge 1	$\{(s_5,1.0)\}$	$\{(s_5,0.2),(s_6,0.8)\}$	$\{(s_4,0.5),(s_5,0.5)\}$	$\{(s_4,0.8),(s_5,0.2)\}$	$\{(s_5,1.0)\}$	$\{(s_5,1.0)\}$
Bridge 2	$\{(s_2,0.6),(s_3,0.4)\}$	$\{(s_4,0.3),(s_5,0.6)\}$	$\{(s_3,0.7),(s_4,0.3)\}$	$\{(s_4,0.2),(s_5,0.8)\}$	$\{(s_3,0.1),(s_4,0.9)\}$	$\{(s_4,1.0)\}$
Bridge 3	$\{(s_4,0.7),(s_5,0.3)\}$	$\{(s_3,1.0)\}$	$\{(s_4,0.8),(s_5,0.2)\}$	$\{(s_3,0.4),(s_4,0.6)\}$	$\{(s_4,1.0)\}$	$\{(s_3,0.8),(s_4,0.2)\}$

TABLE II. ATTRIBUTE VARIANCES AND WEIGHT CALCULATION RESULTS

Attributes	C_1	C_2	C_3	C_4	C_5	C_6
Variances	0.70	0.71	0.3026	0.3474	0.94	0.8933
Weight（case 1）	0.21	0.14	0.13	0.127	0.17	0.223
Weight（case 2）	0.18	0.182	0.079	0.089	0.241	0.229
Weight（case 3）	0.222	0.112	0.1	0.05	0.221	0.295

TABLE III. COMBINED DISTRIBUTION OF CERTAINTIES OF EACH LEVEL (CASE 1)

Rating levels	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Ω
Bridge 1	0	0	0	0.1271	0.79	0.0829	0	0
Bridge 2	0	0.1169	0.1812	0.5262	0.1640	0	0	0.0117
Bridge 3	0	0	0.3503	0.5732	0.0765	0	0	0

TABLE IV. UTILITY INTERVAL VALUES AND ORDERING RESULTS

	Utility Interval (Case 1)	**Utility Interval (Case 2)**	**Utility Interval (Case 3)**	**Ordering Result (Case 1)**	**Ordering Result (Case 2)**	**Ordering Result (Case 3)**
Bridge 1	[0.5911,0.5911]	[0.6066,0.6606]	[0.6001,0.6001]	1	1	1
Bridge 2	[0.3568,0.3685]	[0.3702,0.3855]	[0.3459,0.3544]	2	2	2
Bridge 3	[0.3452,0.3452]	[0.3340,0.3340]	[0.3440,0.3440]	3	3	3