Circular Arc Gear Reliability Study Based on Modified FOSM Method

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Abstract--**As different limit state functions are used to analyze reliability, there is a great distinctness among the calculated results. In this paper an improved LOSM method is proposed, namely, checking point method. The circular arc gear case is employed to demonstrate this method. In contrast to the results of Monte Carlo simulation, this method can greatly improve reliability calculation's precision.**

Key words：circular arc gear; modified FOSM method; reliability

INTRODUCTION

In practical engineering application, FOSM method is used widely. It has become a basic method for structure reliability analysis and calculation and also been the simplest method to calculate approximation reliability. However, for the same problem, as limit state equations are different, reliability indices are various. Aiming at this problem, a modified FOSM method, checking point method, has been put forward.

1. MODIFIED FOSM METHOD

1.1 Weakness of FOSM Method

The traditional FOSM Method expands nonlinear limit state equation at checking point. For abnormal random variables, the equivalent normalizing method recommended by the International Structure Safety Joint Committee is adopted. Therefore, it can be seen that the traditional FOSM Method possesses two kinds of calculation errors.

- (1) The error is caused by Taylor expansion when the limit state equation of nonlinear degree is a little higher.
- (2) The error is caused when abnormal random variable is considered as equivalent normalizing.

The improved FOSM Method can eliminate these errors.

1.2 Checking Point Design

For multiple random variables, the limit state equation is

$$Y = f(x_1, x_2, \cdots x_n) = f(X) = 0 \tag{1}$$

Introducing standard normal variable $x_i = (x_i - \mu_i)/\sigma_i, i = 1,2,\cdots n,$ and substituting $x_i = \sigma_i x_i + \mu_i$ for the limit state equation, Formulas (2) can be obtained:

$$Y = f(X) = f(\sigma_1 x_1 + \sigma_2 x_2, \cdots \sigma_n x_n) = 0 \tag{2}$$

Based on the related theory of the space analytic geometry, the cosine between the normal from the origin o' to limit state surface (plane) and coordinate vector direction is

$$\cos\theta x_i = \frac{-\frac{\partial f(x)}{\partial x_i}\Big|_{x_i} \cdot \sigma_i}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial f(x)}{\partial x_i}\Big|_{x_i} \cdot \sigma_i\right)^2}} \tag{3}$$

in which $\frac{\partial f(x)}{\partial x_i}\Big|_{x_i}\cdot$ signifies assignment of the point p for the partial derivative of function f (x) with xi

Thus, the coordinates of p* are

$$x_i^* = x_i\sigma_i + \mu_i = op^* \cos\theta x_i \sigma_i + \mu_i \tag{4}$$

As p* is one point on the limit state surface, it must satisfy the limit state Equation, namely:

$$Y = f(x^*) = f(x_1^*, x_2^*, \cdots x_n^*) = f(X) = 0 \tag{5}$$

When the distribution parameters μ_i and σ_i of the basic variable $x_i = (i = 1,2,\cdots,n)$ are known, from formulas (4) and (5) the reliability coefficient value β can be obtained.

2. DETAILED SOLVING STEPS OF CHECKING POINT

- (1) The limit state equation $Y = f(x_1, x_2, \cdots x_n) = f(X) = 0$ is known. The mean and variance of the every random variable xi is μ_i and σ_i, respectively.
- (2) Choose the coordinates initial value of check point. Generally the mean point $x_i = \mu_i$ is selected and then $\frac{\partial f(x)}{\partial x_i}\Big|_{x_i}\cdot$ is calculated. According to Formulas (3), $\cos\theta x_i$ can be solved.

- (3) According to formulas (4), the relation between xi* and β can be gotten. Thus, β can be solved by formulas (5). Then through formulas (3) and the value of β the new value of xi* can be obtained.
- (4) Repeat steps (2)~(4), β can be solved again and then is taken away from the last value. If this result is less than permitted error, the reliability coefficient is gotten. Otherwise, the iteration solution will be adopted repeatedly until the error requirement is satisfied.

3. CIRCULAR ARC GEAR RELIABILITY ANALYSIS

The common failure modes in circular arc gear drive are varied such as bending and breaking, rippling and wear in gear teeth and so on. Therefore, the bending stress leading to tooth breaking and the contact surface stress causing tooth wear should be considered fully in the design. In the paper, the latter stress is studied in detail.

The calculation formula for the contact stress of circular arc gear's tooth surface[4,5] is

$$\sigma_H = \left(\frac{T_1 K_A K_V K_1 K_{H2}}{\mu_\varepsilon + K\Delta\varepsilon}\right)^{0.7} \frac{Z_E Z_\mu Z_\beta Z_\alpha}{Z_1 M_n^{2.1}} \quad (6)$$

Where T1 is input torque coefficient; KA is circumstance coefficient; KV is dynamic load coefficient; K1 is load distribution coefficient between the contact traces; KH2 is load distribution coefficient in the contact traces; μ_ε is integer part of overlap ratio; $K\Delta\varepsilon$ is contact trace part; Z_E is elasticity coefficient; Z_μ is gear ratio coefficient; Z_β is spiral angle coefficient; Z_α is contact arc length coefficient; Z1 is tooth number of pinion; M_n is Normal module of pinion.

The parameters μ_ε, M_n, Z_μ, Z_α, Z_β related to contact stress can be valued in permitted tolerance range of accuracy grade, thus technique is easy to get guaranteed. Such parameter is treated as determined value, and other parameters and coefficients are considered to obey normal distribution processing.

The strength of tooth surface contact is

$$[\sigma_H] = \frac{\sigma_{H\min} Z_N Z_L Z_V}{S_{H\min}} \quad (7)$$

in which $\sigma_{H\min}$ is gear contact fatigue limit; ZN is contact life coefficient; ZL is lubricant coefficient; ZV is speed coefficient; SH min is minimum safety coefficient.

In fatigue analysis process, $\sigma_{H\min}$ is to obey lognormal distribution or Weibull distribution, and coefficient ZN, ZL and ZV and to obey normal distribution.

Table1 Random variable mean and variance

Random variable	Rean μ_i	Variance σ_i
Z_N	1.3	0.03
Z_L	1.05	0.03
Z_V	1.12	0.02
$\sigma_{H\min}$	900	5
K_A	1.6	0.01
K_V	1.5	0.03
K_L	1.1	0.05
K_{H2}	1.38	0.01

4. CALCULATION EXAMPLE

In a circular arc gear drive system, pinion transmission rated power p =70 kW, gear number Z1 =20 and mn=3.5, spiral angle β = 25°44′20″,speed n＝700 r/min，transmission ratio 5.1. The pinion material is 35CrMn， annealing， the big wheel material is 35SiMn，annealing，calculate reliability coefficient β and reliability R。

Firstly, a limit state equation is set up,

$$Z = [\sigma_H] - \sigma_H$$

$$= \frac{\sigma_{H\min} Z_N Z_L Z_V}{S_{H\min}} - \left(\frac{T_1 K_A K_V K_1 K_{H2}}{\mu_\varepsilon + K\Delta\varepsilon}\right)^{0.7} \frac{Z_E Z_\mu Z_\beta Z_\alpha}{Z_1 m_n^{2.1}}$$

In this equation, random variable $X = [Z_N, Z_L, Z_V, \sigma_{H\min}, K_A, K_V, K_1, K_{H2}]$; others are constant. Secondly, every random variable's mean and variance is determined based on literature [3]. The analysis results are followed in Table 1.

For random variables are relatively more, the iterative calculation by the computer programming is needed and the final results are shown in Table 2.

Table 2 Result

Result	Advanced First Order Second Moment	Monte Carlo numerical simulation
Reliability coefficient β	2.78	2.75
Reliability R	0.9981	0.9980

5. CONCLUSION

According to the defect of a second order moment reliability in calculation method, the paper puts forward an improved first order second moment method with accuracy and high-efficiency. Compared with Monte Carlo numerical simulation results, this method can improve reliability calculation accuracy effectively, though the accuracy is not higher than secondary second moment method. But it is simple and practical.

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