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Comparison of the improvement of directional stability for Anti-lock Brake control algorithm

Li Jing[1,a] ,Wang Zihan[1,b], Guo Li-shu[2],Shi Zheng-tang[2]

[1]State Key Laboratory of Automotive Simulation and Control, Jilin University, Changchun 130022, China

[2].Zhejiang Asia-Pacific Mechanical and Electronic Company Limited, Hang Zhou 311203, China

[a]liye1129@163.com [b]wangzihan_20071008@126.com

Key words: Anti-lock braking system; control algorithm; three-axle vehicle; phase-plane.

Abstract: A nine freedoms nonlinear three-axle vehicle dynamic model was constructed by Matlab/Simulink to analyze the control effect of directional stability for different Anti-lock Brake control algorithm. Algorithms analyzed here contain PID, logic threshold and sliding mode variable structure control algorithm. Then comparing and analyzing the control effect from the aspects of tire/whole vehicle parameters and the phase-plane of slid slip rate and its rate.

Introduction

ABS (Anti-lock Brake System) has been applied to vehicles since the beginning of 20th century. There exist many kinds of algorithms for ABS, such as logic threshold [1], PID [2-3], sliding mode variable structure [4-5], fuzzy control algorithm [6] and so on. Some researches [7] have been made to analyze the difference between PID control algorithm and logic threshold control algorithm, but those researches mostly based on the single wheel model and only analyze the parameters of wheel, which revealed the control effect of different ABS control algorithm limited. In this paper, the dynamic model was built on the basis of whole vehicle and nonlinear GIM tire model, and the analysis theories not only contain traditional parameters, such as slip rate, but also take the whole vehicle motion parameters into consider.

In this paper, ABS with PID controller, logic threshold controller and sliding mode variable structure controller were designed. Through simulating the three-axle dynamic model with different contro 1 ller, then analyzed the brake directional stability form the aspect of tire sliding rate, braking torque, slide slip angle and phase-plane [8-9] which has been used in some control system of vehicle.

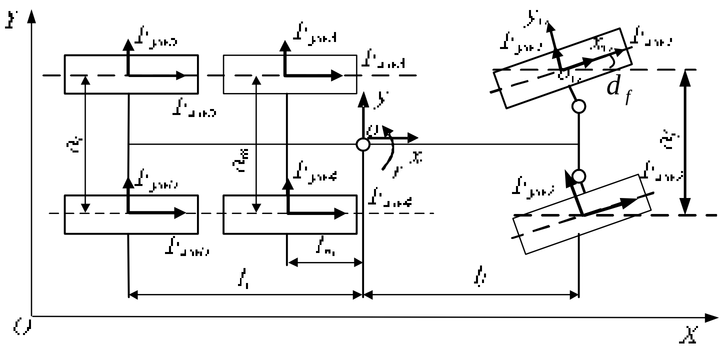


Fig.1 The mechanical analyze graph of three-axis vehicle

Dynamic models

Before constructed three axles model, we assumed that no air resistance was considered; vehicle only act planar movement parallel to the ground; the tire rolling resistance moment was constant and had no aligning torque; load transfer haven't taken into account and the brake was an ideal one-step inertial element.

The mechanical analyze graph of three-axle vehicle shown in figure 1. Here contains three coordinate systems: earth coordinate system *OXYZ*; vehicle coordinate *oxyz* and wheel coordinate system $o_w x_w y_w$. The origin point *o* of vehicle coordinate system located at the centroid of vehicle, the axis *x* is in the longitudinal symmetry plane and its positive direction point to forward, the positive direction of axis *y* point to the left of the driver and the positive of axis *z* point to the up. As to the tire coordinate system, the axis x_w is the intersection line between the tire longitudinal symmetry plane and ground, its positive direction point to the forward, the axis y_w lying in the ground plane, and its positive direction point to the left, the positive direction of axis z_w point to the up.

Whole vehicle dynamic model

Assuming the whole vehicle divided into four parts: sprung mass, front under spring mass, middle under spring mass and rear under spring mass. The model constructed here has nine freedoms: longitudinal velocity *u*, lateral velocity *v*, yaw velocity *r* and six wheels rolling freedoms. Here are the whole vehicle calculation equations

$$(m_f + m_m + m_r + m_b)(\dot{u} - rv) - r^2(m_f l_f - m_m l_m - m_r l_r) = Q_u \quad (1)$$

$$(m_f + m_m + m_r + m_b)(\dot{v} + ru) + \dot{r}(m_f l_f - m_m l_m - m_r l_r) = Q_v \quad (2)$$

$$(I_{zzf} + I_{zzm} + I_{zzr} + I_z + m_f l_f^2 + m_m l_m^2 + m_r l_r^2)\dot{r} + (m_f l_f - m_m l_m - m_r l_r)\dot{v} + (m_f l_f - m_m l_m - m_r l_r)ur = Q_r \quad (3)$$

Where, m_f、m_m、m_r: the quality of front/middle/rear under spring mass(kg);I_{zzf}.I_{zzm}.I_{zzr}: the rotation inertia of the front/middle/rear under spring mass(kg·m^2);I_z: the rotation inertia of spring mass (kg·m^2);*u*: the longitudinal velocity of whole vehicle centroid(m/s);*v:* the lateral velocity of whole vehicle centroid (m/s);*r*: the yaw velocity of whole vehicle centroid (rad/s);l_f.l_m.l_r: distance between centroid and front/middle/rear axle (m);m_b: the quality of spring mass (kg); Q_u.Q_v.Q_r are the generalized forces of *u*, *v* and *r*, the equations shown as follows

$$Q_u = (F_{xw1} + F_{xw2})\cos\delta_f - (F_{yw1} + F_{yw2})\sin\delta_f + \sum_{i=3}^{6} F_{xwi} \quad (4)$$

$$Q_v = (F_{xw1} + F_{xw2})\sin\delta_f + (F_{yw1} + F_{yw2})\cos\delta_f + \sum_{i=3}^{6} F_{ywi} \quad (5)$$

$$Q_r = [(F_{xw2} - F_{xw1})\cos\delta_f + (F_{yw1} - F_{yw2})\sin\delta_f]\frac{a_f}{2} + [(F_{xw2} + F_{xw1})\sin\delta_f + (F_{yw1} + F_{yw2})\cos\delta_f]l_f \quad (6)$$
$$+ (F_{xw4} - F_{xw3})\frac{a_m}{2} - (F_{yw3} + F_{yw4})l_m + (F_{xw6} - F_{xw5})\frac{a_r}{2} - (F_{yw5} + F_{yw6})l_r$$

Where, F_{xwi}: the longitudinal focus of wheel *i* (N);F_{ywi}: the lateral focus of wheel *i* (N);δ_f: steer angle of steering wheels (rad);a_f: wheel track of front axle (m);a_m: wheel track of middle axle (m);a_r: wheel track of rear axle (m).

Model of brake

Brake model reveals the relationship between brake torque and pressure, in this model, we assumed the brake as an ideal element and its brake equation is

$$M_{bi} = k_i \cdot P_i \quad (7)$$

Where, M_{bi}: the brake torque of wheel *i* (N·m);k_i: brake coefficient of wheel *i* (N·m/kpa);P_i: brake pressure of wheel *i* (kpa).

As to the lag result from transformation, here simplified it as an one-step inertia link, the transfer function *G* shown as follows

$$G = \frac{K_m}{T_m s + 1} \quad (8)$$

Where, K_m and T_m is the parameters of brake.

The model of wheel rolling kinetics

As shown in figure 2, constructed the wheel rolling dynamic model, the inputs of the model is the longitudinal focuses F_{xwi} and the brake torques M_{bi} of wheel i, the outputs of the model is the angular rates ω_i of wheel i,(i=1,2...6, is the number of wheel).

Through the Newton second law, the wheel dynamic functions is

$$\frac{dw_i}{dt} = \begin{bmatrix} \frac{r_{di}}{I_{wi}} & -\frac{1}{I_{wi}} \end{bmatrix} \begin{bmatrix} F_{xwi} \\ M_{bi} \end{bmatrix} \quad i = 1, 2, ..., 6 \qquad (9)$$

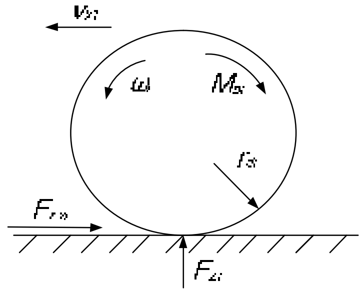


Fig.2 The mechanical analysis graph of wheel

Where, r_{di}: the rolling radius of wheel i; I_{wi}: the rotation inertia of wheel i(kg·m^2).

The tire model

This paper used GIM tire model which take the longitudinal velocity u_{xw} and the slid slip angular β_w of wheel as inputs, the longitudinal focus F_{xw} and the the lateral focus F_{yw} of wheel as outputs. The longitudinal velocity and slid slip angular calculated through the motion analysis of whole vehicle. First of all, the calculation functions of longitudinal and lateral slip rate s_x、 s_y and the combined slip rate s_{sa} as follows

$$s_x = 1 - \frac{w r_d}{u_{xw}} \quad (10)$$

$$s_y = \min(|tan b_w|, 1) \quad (11)$$

$$s_{sa} = \min(\sqrt{s_x^{\,2} + s_y^{\,2}}, 1) \quad (12)$$

The friction coefficient of ground can be presented approximately as

$$m = m_0 (1 - A s_{sa}) \quad (13)$$

Where $A = \frac{(m_1 - m_2)}{m_1 s_2 - m_2 s_1}, m_0 = \frac{(m_1 s_2 - m_2 s_1)}{s_2 - s_1}$

According to the principle of friction circle, the longitudinal adhesion coefficient μ_x and lateral adhesion coefficient μ_y shown as follows

$$m_x = \frac{m s_x}{\sqrt{s_x^{\,2} + s_y^{\,2}}} \qquad m_y = \frac{m s_y}{\sqrt{s_x^{\,2} + s_y^{\,2}}} \quad (14)$$

Where, μ_0: the static friction coefficient between tire and ground; A: characteristic coefficient of wheel; μ_1: the adhesion coefficient while the slip rate is 0.15; μ_2: the adhesion coefficient while the slip rate is 1. Based on those, the longitudinal focus F_{xw} and the the lateral focus F_{yw} can be calculated as follows

$$\begin{aligned} F_{xw} &= -[C_s s_x l_n^{\ 2} + m_x F_z (1-3l_n^{\ 2}+2l_n^{\ 3})\ ] \quad s_x < s_{sc} \\ F_{xw} &= -m_x F_z \quad\quad s_x \geq s_{sc} \end{aligned} \tag{16}$$

$$\begin{aligned} F_{yw} &= sign(b_w)[C_a s_y l_n^{\ 2} + m_y F_z (1-3l_n^{\ 2}+2l_n^{\ 3})] \quad s_y < s_{ac} \\ F_{yw} &= sign(b_w)[m_y F_z] \quad\quad s_y \geq s_{ac} \end{aligned} \tag{17}$$

Where, $l_n = 1 - \frac{1}{3mF_z}\sqrt{(C_s s_x)^2 + (C_a s_y)^2}$;

C_s: the longitudinal stiffness of tire(N/m); C_a: the lateral stiffness of tire (N/m).

As analyzed before, constructed the three axles vehicle model which is used to analyze the algorithms of ABS, figure 3 is the simulation model which constructed in Matlab/Simulink.

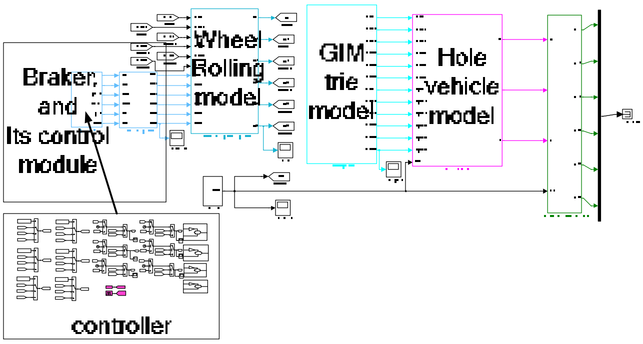


Fig.3 The diagram of simulation model

Controller of ABS with different algorithm

ABS with logic threshold controller

Logic threshold control algorithm is a classical algorithm and has been widely used. Based on the reference[1], built the ABS logic threshold controller which complete the chose of state through slip rate and wheel angular acceleration, the controller regulate the pressure of brake based on the state chosen before. There exist four states: pressure keeping, pressure rising, reduced pressure and pressure raised with litter step. Where pressure raised with litter step achieved by adding some pressure keeping state in the pressure raising state.

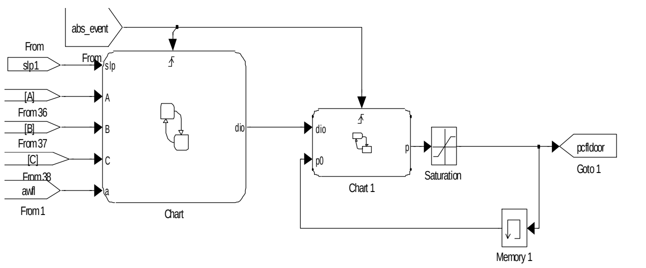


Fig.4 Simulink modle of logic threshold controller

As shown in figure 4, is the simulation model of logic threshold controller which constructed by the Matlab/Stateflow. The controller will regulate the brake pressure through comparing the slip rate and wheel angular acceleration with the threshold values, then change the signals of brake.

ABS with PID controller

As shown in the figure 5, PID controller used the deviation between the target slip ratio s_0 and the actual slip ratio s_x which defined as e ($e=s_0-s_x$), to regulate the brake pressure. The target pressure value obtained form the proportional component, integral component and the differential component of e ,

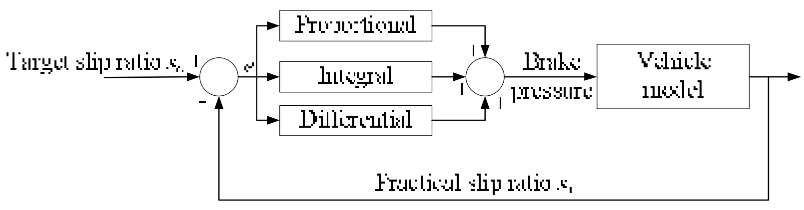


Fig.5 The principle of PID control

so as to accomplish the rational regulation of brake pressure. The relationship between brake pressure p_i and e is

$$p_i = [K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt}] \quad (18)$$

Where, K_p: proportional coefficient; K_i: integral coefficience; K_d: differential coefficient. Based on the model constructed before, got the proper coefficients through a lot of experiments, the controller simulation model shown in figure 6.

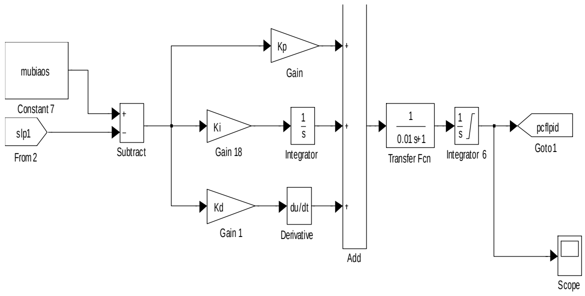


Fig.6 Simulink model of PID controller

ABS with sliding mode variable structure controller

Sliding mode variable structure control algorithm is a nonlinear control strategy based on the state of system. The controller distinguished whether the state of system deviates from the sliding mode plane and regulates the state of system. It owns many advantages, such as strong robustness and adaptable for handling the problem of uncertainty position and model.

This paper constructed the sliding mode plane of slip rate and its rate, after getting the sliding mode plane, the controller regulate the brake pressure to control the state of system in order to make the state coincidence with the sliding mode plane. Here defined the state deviation e is the same as PID control (e=s_0-s_x), the control parameter u_k is the brake pressure M_{bi} of wheel i. Through differential the equation (10), we can get

$$\dot{e} = \frac{1}{u_{xwi}}[-r_{di}\dot{w}_i + (1-e)\dot{u}_{xwi}] \quad (19)$$

Make it equate to 0 and combine the function (9) to get the control parameter M_{beq} as follows

$$M_{beqi} = F_{xi} r_{di} - \frac{I_{wi}}{r_{di}}(1-e)u_{xwi} \quad (20)$$

Here used the control rule of M_{bi}=M_{beqi}+k_h*sgn(e) to control the pressure and regulate the state of vehicle. The controller simulation model diagram shown in figure 7, the inputs are longitudinal focus F_{xwi}, wheel longitudinal acceleration u_{xwi} and wheel slip rate s_{xi}, the outputs of the controller are brake pressure M_{bi}.

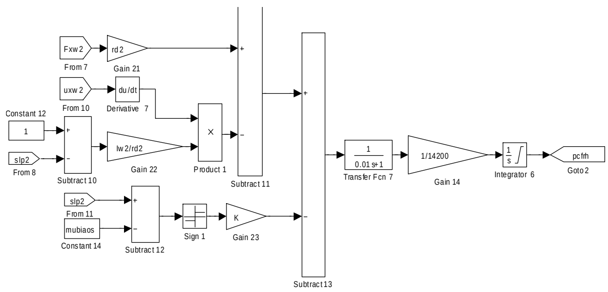


Fig.7 Simulink model of sliding mode variable structure controller

Analysis of simulation results

ABS can avoid wheel locking effectively, besides enough longitudinal force, tire can provide sufficient lateral adhesive force to guarantee the directional stability. Therefore, we choose braking while steering on low adhesion road as the simulation condition to analyze directional stability without and with different ABS controller.

Figure 8 is the contrasting diagram of slip rate with different ABS controller and without ABS, figure 9 is the contrasting diagram of brake torque. Form those diagrams, we can figure that without ABS, wheel will lock soon, but with ABS, though different controller, slip rate can be hold within an ideal range(here 0.2 is the target slip rate) which satisfied the requirements of ABS.

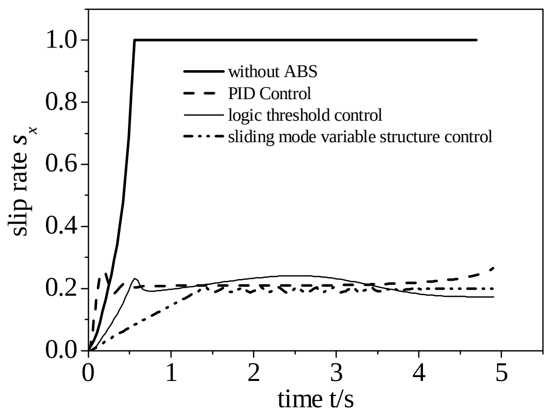


Fig.8 Contrasting diagram of slip rate

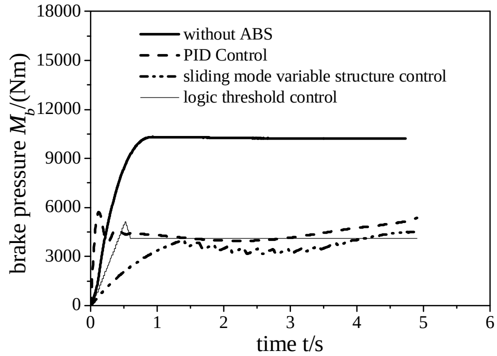


Fig.9 Contrasting diagram of brake torque

Though controller is different, the control target is consistent (brake torque). Combine figure 8 with figure 9 to analyze further, we can get those conclusions. Firstly, from the aspect of response time, with PID controller, ABS response fastest, logic threshold place the second and sliding mode variable structure is the slowest. Secondly, from the aspect of brake torque, compared PID with sliding mode controller, brake torque change little with logic threshold controller. As for the sliding mode controller, response slowest, but its slip rate control best and its stability is better, we can find that brake torque is fluctuating heavier, the reason is that: vehicle state can't hold on the slip mode plane always, the regulation is completing by making the state parameter going through the slip mode plane and the result is that the control target's fluctuation.

Based on the analysis before, analyzing further on the aspects of whole vehicle parameters and phase-plane of slid-slip angle and its' rate. Figure 10 and 11 are the contrasting diagram of trajectory and slid-slip angle.

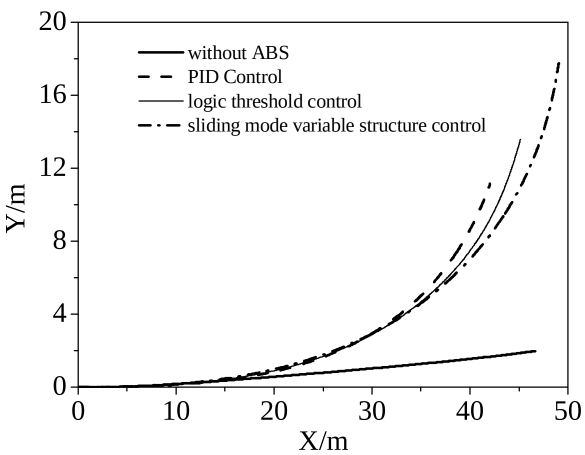


Fig.10 Contrasting diagram of trajectory

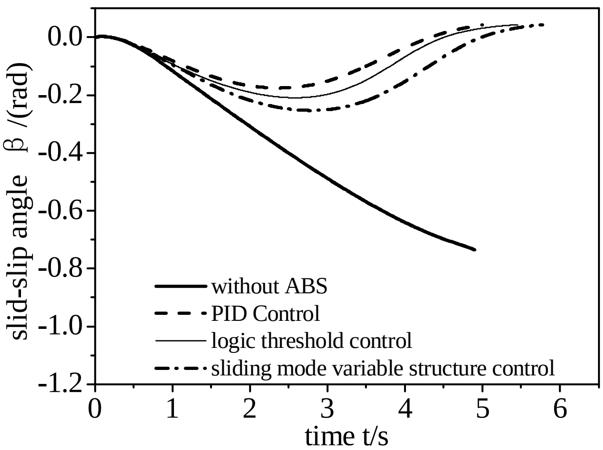
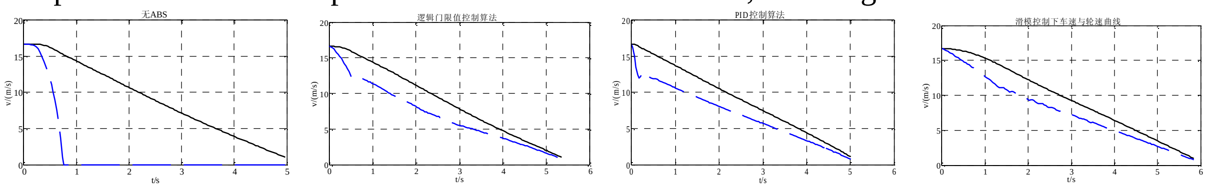


Fig.11 Contrasting diagram of slid-slip angle

From the trajectory and the slid-slip angle diagrams, we can know that vehicle can hardly steer without ABS and the slid-slip angle become very large, those phenomenon show that vehicle become instable and tail-flick on low adhesion road, but with ABS, though different controller, tire can provide enough lateral focus to hold the stability of vehicle while steering on low adhesion road. Through the contrasting patterns of slid-slip angle and trajectory, the control effects of PID controller is the best, it's slid-slip relatively smaller and turning radius relatively larger. Figure 12 compare vehicle speed with the wheel speed of different ABS controller, we can get the same conclusion.



(a) Without ABS control (b) Logic threshold control (c) PID control (d) Sliding mode control

Fig.12 Contrasting vehicle speed with wheel speed of different ABS controller

Those analyses take researches of ABS stability qualitatively not quantitatively. Based on the researches of Japan scholars Ken Koibuchi and Masaki Yamamoto [9], that slid-slip angle and its rate phase-plane can represent braking directional stability, its stability region shown as follows

$$\left|C_{b1} d b + C_{b2} b\right| < 1 \quad (21)$$

Where, C_{b1} and C_{b2} are constant values which be confirmed by sufficient simulate tests. Based on those theories, analyzed the braking directional stability of different ABS controller quantitatively.

As shown in figure 13, the phase track exceeds the stability boundary and become instable without ABS, however, with different ABS controller, the phase tracks can be hold within the stability region, but different controller have different characteristics. Like the figure show, phase track with PID can be hold in the smallest region; logic threshold control place the second, the sliding mode control is relatively larger. Combined analysis of wheel parameter such as slip rate and whole vehicle parameters such as slid-slip angle with the phase-plane analysis, we can get that brake directional stability with ABS of PID controller is the best.

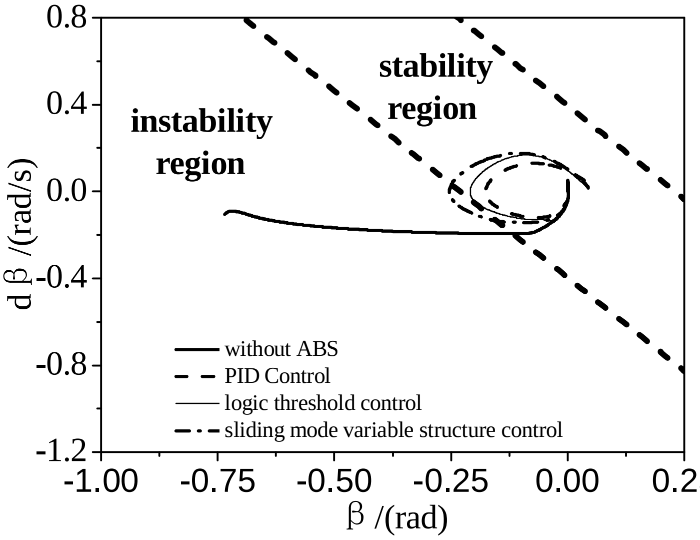


Fig.13 Contrasting diagram of slid-slip angle and rate phase-plane

Summary

In this paper, nine-freedom vehicle model of three-axles and ABS controller with logic threshold, PID and sliding mode variable structure control algorithm were designed. Based on those, take the off-line simulations. Through analyzing the parameters of tire, whole vehicle and phase plane, get those results: ABS with PID control response fastest but with larger overshoot, logic threshold controller with smallest fluctuations thus can be more easy to implement, ABS with sliding mode control response slowest but can hold the slip rate more accurately.

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