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So… what was the question?

Gérard G EMCH

Department of Mathematics, University of Florida, Gainesville, Florida, USA
E-mail:gge@math.ufl.edu

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Abstract

An overview of the lectures at the 2002 Białowieża Workshop is presented. The symbol * after a proper name indicates that a copy of the corresponding contribution to the proceedings was communicated to the author of this summary.

1 Introduction

The title of this overview is meant to convey a sense of the extraordinary range of the 2002 Białowieża Workshop.

In earlier years, the focus was often centered mathematically: on developments in differential geometry and representation theory of Lie groups; and physically: around quantization – mostly of the geometric persuasion. The question then would have been something like: "What are the latest progresses in formulating a theory that would encompass classical as well as quantum physics – mostly mechanics, but also field theories – and still would be powerful enough to allow predictive mathematical syntheses?"

While the official rallying theme of this XXIst Workshop on Geometric Methods in Physics was *Recent developments in Quantization*, there were more than 40 lectures, the combined scope of which was considerably broader, displacing in part the earlier drive towards a unity of discourse, although many of the lectures still stimulated the type of searching general discussions that are an integral part of the local tradition.

2 Non-commutative geometry

Physical applications of non-commutative geometry and quantum groups have been considered in earlier workshops, but never before was their grip on center–stage as firm as it was at the 2002 workshop.

The general landscape was reviewed with great enthusiasm in two lectures by Harald Grosse*: *Regularization* [1st lecture] *and Renormalization* [2nd lecture] *of Quantum Field Theory in Non-commutative spaces.* The paper in these proceedings is written with Raimar Wulkenhaar. They mention that much of the impetus for this line of research was initiated by John Madore* who also gave, at the Workshop, a series of lectures on its mathematical background, as well as some new physical directions in a non-commutative version of the

Kasner metric. Some references to his earlier papers on the subject are included in the Grosse-Wulkenhaar contribution. The original model was revisited by Giorgio Immirzi: *Schwinger Model on Fuzzy Sphere.* The mathematics is certainly intriguing, although none of these authors seemed willing to elaborate on the observable evidences for the physics that plays on the Planck scale. In this regard, Madore insisted nevertheless that whereas the short-scale physics is new and widely lacking in empirical grounding, the long-scale physical predictions of the new theory reduce to those of the old theory elaborated in the usual, ordinary commutative space. The contribution to these proceedings based on the material of these lectures *On the Resolution of Space-Time Singularities II* was written by M. Maceda and J. Madore.

Lech Woronowicz is appreciated for his relentless approaches to circumscribe the general mathematical structure of quantum groups. His emphasis this year was on *Quantum groups obtained by Rieffel deformation.* Amongst the novelties, he isolated a condition which he calls "manageability" which he found to be both mathematically powerful and axiomatically ubiquitous... except for the fact that, up to now, it seems to have gone unnoticed! Some of the main features of the theory were illustrated in concrete realizations by Paweł Kasprzak: *Quantum Lorentz-Heisenberg group;* and by Wiesław Pusz *On a quantum GL(2,C) group at roots of unity,* who also presented a helpful synthesis and review of this circle of ideas.

In a field seemingly dominated by quantum groups and fuzzy spaces, it was refreshing to hear Jiří Tolar* report in plain, mathematically precise, language *On Pauli graded contractions of* $sl(3, \mathsf{C})$*,* a Prague-CRM(Montreal) collaboration. All definitions and prerequisite results are succinctly presented, with special emphasis on the role of MAD-groups, i.e. maximal abelian groups of diagonalizable automorphisms of the Lie-algebra considered. Their knowledge of the allied machinery allows them to classify quite explicitly the contractions that preserve certain gradings. The paper for the proceedings mirrors the pristine flavour of the lecture.

3 Quantization

Yet, the original preoccupations of finding a mathematical formalism that could be used for the description of both classical and quantum theories did not fade away from Białowieża.

Tudor Ratiu: *Banach Lie Poisson Spaces and reduction [Part I]* introduced the category of Banach Lie-Poisson spaces, and Anatol Odzijewicz: *Banach Lie Poisson Spaces and reduction [Part II]* showed how the category of W^*-algebras appears as one of its subcategories. The beauty of the concepts presented in these two communications is that they generalize naturally to infinite dimension the dual structures of finite-dimensional Lie-algebras and Lie-Poisson spaces. The price to pay for this generalization is to forego the convenience that, in the finite-dimensional case, the duality defines an isomorphism. Hence the appearance of W^*- rather than just C^*-algebras. After a protracted search by diverse researchers for unifying formalism, it appears that the present two authors have hit on the right concepts; as powerful evidences, on the theoretical side, they presented their generalized version of quantum reduction and momentum maps; and on the model side, they indicated how Toda lattices and/or Lax systems ought to be viewed in the new light provided by their formalism.

The geometry of finite-dimensional Poisson manifolds was however not forgotten at this year's Białowieża workshop. Considering the remark by Weinstein (1987) according to which "every symplectic leaf of a Poisson manifold carries an intrinsic Lie algebroid which controls the infinitesimal Poisson geometry around the leaf," Yuri Vorobiev*: *On the Poisson realizations of transitive Lie algebroids* shows that this route can be traveled in the opposite direction.

In his lecture, *Egorov theorem,* Ondrej Lev sketched a formulation of the problem of associating a quantum evolution to a classical flow, with special concern for situations akin to the Kepler problem and the quantum equivalent to planar motion.

As a preliminary to attacking the problem of giving a covariant formulation of quantum theory on a curved space-time, a first step may be to consider theories with an absolute time. Hence the title *Covariant quantization of Galilean theories* of the exploratory lecture by Josef Janyska on his work with M. Modugno.

Deformation quantization – bridging the classical Poisson bracket and the quantum Moyal bracket – demands an analysis of the *Convergence of star-products* which was the object of the lecture by Akira Yoshioka.

An allied talk was presented by Martin Schlichenmaier* on *Deformation quantization for almost Kähler manifolds;* the new development is the consideration of "almost complex" structures allowing connections with torsion and a classification of the corresponding star-products up to equivalence classes.

Ihor Mykytyuk: *Invariant Kähler structures on the cotangent bundle of spheres* claims to have found a way to avoid obstructions that had hindered the quantization program on such manifolds. The originality of his approach transcends the mere fact that his toolbox contains coherent states technology, although he put some emphasis on the latter in the lecture he delivered at Białowieża.

Two lectures on mathematical structures motivated by (geometric) quantization programs were presented by Marco Bertola: *Wigner transform on (rank one) non-compact symmetric spaces;* and by Twareque Ali: *Wigner transforms using Plancherel's theorem.*

The symmetry considerations behind the geometric quantization on closed compact manifolds (Spheres, Willmore tori, Dupin cyclides, surfaces of constant negative curvature) involve a considerable amount of classical analysis that is ultimately linked to a description of the very geometry of the objects considered; a typical example is the Selberg trace formula, and the link between the length of geodesics and the spectrum of the Hamiltonian. Having stated such motivations in his lecture, Ivailo Mladenov* proceeded to describe some *New geometric applications of the elliptic integrals.* For illustration purposes, his contribution to the proceedings focuses on the so-called Mylar balloon; this is interesting reading for its own sake, independently of the announced initial motivation.

4 Quantum theory

In the course of its long history, quantum mechanics has prompted innumerable developments in functional analysis, particularly in the spectral analysis of self-adjoint operators, and in the theory of distributions.

"Schrödinger operators with magnetic fields" is the title of a sequence of three papers by J. Avron, I. Herbst and B. Simon (1978). After such a formidable onslaught, the question is naturally of what can be done next. The key to the success of the investigation pursued by Mikhael Shubin: *Spectral properties of magnetic Schrödinger operators* is a systematic refinement of the concept of Wiener capacity, building on some early work of H. Weyl (1910) and K. Friedrichs (1934), revisited by A.M. Molchanov (1953) who introduced the function that bears his name; these were revived recently by M. Shubin, V. Kondratiev and V. Mazya.

In their contribution to the proceedings: *von Neumann Quantization of Aharonov-Bohm Operator with δ Interaction: Scattering Theory, Spectral and Resonance Properties*, Gilbert Honnouvo and Norbert Hounkonnou* present an exactly solvable model in quantum mechanics, in which a singular radial delta potential at the origin is added to the Aharonov-Bohm Hamiltonian. To their already long descriptive title, they might yet have added the key role the resolvent equation plays in their investigation.

In the first of two lectures *On multiplication of distributions,* Anatolij Antonevich described the basic mathematical tenets of a program, a physical application of which he discussed in his second lecture *Schrödinger equation with point interactions.* While the proposed non-linear theory of generalized functions seems to have been particularly tailored for the study of stochastic processes and stochastic integrals, applications were also mentioned to the description of infinitely narrow solitons; to non-linear differential equations such as the Hopf and Burgess equations; and, further down the path, he conjectured that his regularization construction might also be of use in QFT. As these problems are notoriously hard, the author framed his own contributions with references to previous works; namely for the first lecture, by J.F. Colombau (1984, 1992), V.P. Maskov and V.A. Tsupin (1979, 1986); and for the second lecture, by Berezin, Fadeev (1961), Friedman (1972), Albeverio, Gesztesy, Hoegh-Krohn, Holden and Simon (1988, 1995, 1997).

Vasyl Kovalchuk* contributes to the proceedings with a paper entitled *Green functions for Klein-Gordon-Dirac equation,* which is self-explanatory and self-contained, thus requiring no commentary here.

The contribution to the proceedings by Marek Czachor* is *Reducible representations of CAR and CCR with possible applications to field quantization.* Perhaps as a result of its more catchy title: *Non-canonical quantization of electrodynamics, or 1925 oscillators revisited,* the lecture presented at the Workshop raised more controversy than most. The main bone of contention may have been the unqualified claims that "the resulting field operators are indeed operators and not operator-valued distributions", and that the theory has no IR or UV divergences; such contentions would perhaps have been less abrasive had the author announced earlier that, in his very own views, his "representations have the status of toy models." Controversial toys may sharpen the mind or just the conversation, depending on the circumspection – or lack thereof – with which they are handled. Eager readers, read, but beware!

Roman Gielerak* reported on work done with Robert Rałowski: *Statistical mechanics of a class of anyonic systems. Rigorous approach;* the lecture's title added: *through Wick algebras.* In the last 25 years or so, as the dexterity of experimentalists has given access to the physics of systems constrained to two-dimensional space, new theoretical problems have appeared. The authors view this part of the story as "well-known" and thus do not spend more than a few lines (with some references) on exploring the phenomenological direction. Their contribution concentrates on one mathematical aspect of the problem; the proceedings paper reflects the strict, matter-of-fact style of the lecture: it gives all the necessary definitions; formulates concisely the main questions; illustrates them with a few examples; and announces the results in the form of two sharp propositions and one theorem. Some ideas for the proofs are sketched, the full details being deferred to a subsequent publication. As to be expected when dealing with the formalization of a reasonably fashionable physical subject, the presentation at the Workshop was followed by some conflicting claims of priority; some of these are taken into account in the proceedings; Gerald Goldin claimed that his papers with David Stapp should have been mentioned; for references, see their contribution to the proceedings of the 1994 Białowieża Workshop.

At first sight, the problem posed by Yurii Samoilenko*: *When is the sum of projections equal to a scalar operator?* could be dismissed as a wayward – non-commutative – generalization of the concept of a spectral family. It is therefore somewhat curious that its motivation includes the following diverse sources: graph-theoretical problems associated with regular planar lattices, considered by Temperly and Lieb (1971) in connection with the relations between percolation and coloring problems; papers by Baxter (1982) on two-dimensional lattice models in statistical mechanics; or the work of Jones on index of subfactors (1983). The proceeding paper gives further references – the most recent ones being the reviews by Klyachko (1998) and by Fulton (2000) – with further motivations in analysis, algebraic geometry, representation theory (mostly of C^*-algebras), and allied aspects of mathematical physics. This sophisticated motivation was alluded to in the title under which Yurii originally announced his lecture: *Algebras of projections depending on a parameter, their representations and applications.* The answer to the simple question is as surprising and/or complex as its motivation: for instance, let n be the number of members of the family of projectors considered in $\sum_{k=1}^{n} P_k = \alpha I$; the parameter α may take only discrete values when $n \leq 4$ whereas for $n \geq 5$ the set of all allowed values of α contains a non-empty interval. These informations obtain already from the first page of the rich – if somewhat dry – 12-page survey the author wrote for these proceedings.

For more than 20 years, the Bell theorem – in one or another of its avatars – has been viewed as a touch-stone in the Quantum Foundations establishment. Yet, a whole literature and counter-literature has developed in recent years, claiming various faults in the Bell theorem, regarding its assumptions, proofs, or interpretations. On one side, one finds such names as Luigi Accardi, Karl Hess and Walter Philipp; and on the other, Asher Perez, David Mermin or Richard Gill... to mention only a few. At Białowieża, Aloysius Kracklauer*: *EPR-B correlations, non-locality or geometry?* proposed during his lecture an alternate route: most – if not all – the experiments of the EPR family could be accounted for by his classical model, so that these experiments would not actually justify quantum mechanics.

5 Quantum and classical non-linear systems

Gerald A. Goldin: *Non-linear gauge transformations in quantum mechanics* proposed yet another variation on a theme he has been exploiting for several years, namely that quantum mechanics would be a richer theory had it not been constrained by several unwarranted linearizations that are widely accepted at face value and without further ado.

Under the brief title of *Non-linear subsystems in QFT,* Pavel Bona surveyed the top layer of a deep program he has pursued over the last few years. One of the physical motivations of this program is to describe in closed form the dynamical behavior of subsystems of infinite quantum systems, such as those described in non-relativistic field theory and statistical mechanics. Much of the mathematical beauty of the investigation derives from the symplectic structure of projective Hilbert spaces. A synthetic monograph seems to be in gestation there.

Kent Harrison reported on work with Estabrock, and presented an ambitious lecture, an abbreviated title of which could have been *Differential forms and symmetries in mathematical physics.* Longer versions of the title floated at the Workshop included mentions to integrable systems, differential equations and their symmetries, Bäcklund transformation and general relativity.

The contribution to the proceedings *Nonlinear wave equation in special coordinates* by Alexander Shermenev* was motivated initially by the study of non–linear waves in shallow water. Two particular applications were discussed in some detail: water waves on a gently sloping beach, and acoustic waves in gases; extensions were also mentioned, in particular to the description of nonlinear wave motion in wave guides.

The contributions by Horowski and Tereszkiewicz in Section 6 below could also have been listed in the present section.

6 Dynamical systems

The study of classical dynamical systems branches traditionally in two complementary directions: ergodicity and integrability. Both were represented at the meeting.

Anatoly Stepin: *Mixing billards* discussed the place of these dynamical systems in the ergodic hierarchy, with special attention to their structural stability; specifically the delineation of the topologies with respect to which generic properties – in the sense of Baire category – can be established. The question of whether a property is generic within a given theoretical framework is important on two accounts. *Firstly,* since models such as the ones discussed here are idealizations, it is important to know whether the properties they exhibit are only brought about by the peculiar features of the models, or whether they hold for situations that differ from the model by small but otherwise uncontrollable perturbations. In ergodic theory, the relevance of this question was recognized by Oxtoby and Ulam (1941), and pursued for Hamiltonian systems by Markus and Meyer (1974); it is however fair to say that these papers exhibit fundamental concerns that have not yet penetrated the main stream. *Secondly,* besides their intrinsic interest in a classical framework where they have been discussed, Stepin conjectured that the answer to such questions may serve as some guide into what should be expected when embarking into the quantization of these classical dynamical systems.

A different stability problem, this one presented in the quantum realm, was examined by Andrei Lebedev *Strange spectral effects, stability and strong perturbations.* In brief, the question is to determine what spectral properties of bounded (self-adjoint) operators on Hilbert space are either continuous or stable; much of the discussion was modeled on weighted shift operators, although this particular case was conjectured to be typical.

In a masterful series of lectures *Matrix models, isomonodromic deformations and duality,* John Harnad presented a vast panorama of mathematical techniques encompassing aspects of the classical theory of integrable systems, and the random matrices program – originally proposed by Wigner (1950s) for the description of nuclear spectra – that blossomed into a new kind of statistical mechanics [see Mehta (1991)], complete with canonical distribution on the space of $N \times N$-matrices with complex entries, partition function, correlation functions, and large $N-$limits. All that is the ancient history on which Harnad built a presentation that was fascinating in his far-reaching developments. A unifying feature cements together the different fields of mathematical physics that are encompassed in Harnad's structural analysis: the same group lurks behind every one of these motivating applications which now appears as consequences of universal properties.

The contribution to the proceedings by Vadim Serdobolskii*: *Limit spectra of random Gram matrices* is a piece of fine stochastic analysis; while he refers to applications in "random matrix physics and the solution of systems of empiric algebraic equations" the paper concisely describes general mathematical results.

It is widely recognized that to evade compactness requires a brave soul, already when dealing with integrable Hamiltonian systems with two degrees of freedom. Taking advantage of the pioneering work of Fomenko and his coworkers, one new member of the team, Galina Goujvina*, spoke at Białowieża 2002 on *Classification of the bifurcations emerging in the case of non-compact isoenergetic surfaces.* The classification presented is instructive, and by no means simple; nevertheless, it unfolds smoothly in her contribution to the proceedings.

Viktor Bachtin: *Perron-Frobenius dynamical systems* explored, with the help of models, mathematical structures that live at the juncture between ordinary, classical deterministic dynamical systems and systems that are usually described as stochastic processes. A conjecture was proposed according to which the category of Perron-Frobenius dynamical systems, some of the properties of which he delineated, may offer some insight into the behavior of Hamiltonian systems.

Analysis on real affine G-varieties bears the imprints left by the efforts of an impressive array of modern mathematicians, such as Grothendick, Gårding, Dixmier, Malliavin and several others. An episode of this epic was presented by Pablo Ramacher.

In *Some integrable systems in nonlinear quantum optics* Maciej Horowski set-up a model for quantum electromagnetic waves in nonlinear media, and outlined its solution in the interaction picture; taking advantage of a commuting set of integrals of the motion allowed him to reduce Fock space into irreducible dynamical subspaces. Special cases were studied with an abundance of analytical details in the related work of Agnieszka Tereszkiewicz: *Integrable systems related to classical orthogonal polynomials;* this was the occasion for a parade of special functions brought about through a powerful combination of classical analysis and representation theory.

7 Other contributions

For various reasons, having to do more with the limitations of this writer than with the speakers at the Workshop, a few contributions had to be omitted from this overview. The writer therefore present his apologies to Valery Boika: *Estimate for linear stochastic systems. The functional differential equations;* Jan Sławianowski: *Quantized systems on linear and affine groups;* Ivan Tsyfra: *Symmetry reduction of nonlinear heat wave equations;* and P. Zabreiko: *Focusing operators on ordered Banach spaces.*

8 Conclusions

Along with consolidations of old structures, several new blueprints were strongly advocated in the course of the Workshop. This diversity is reminiscent of some of the effervescence that was boiling in the mathematical physics of the 1960s. Yet, at Nordwijk am Zee (1967), rightly respected figures of the establishment uttered the following two sharp aphorisms. Mark Kac: "There are two generalizations of a bouillon; one is to add water, the other to add meat." George Uhlenbeck: "Show me something that you can do with your methods that I cannot do with mine." The context for these pronouncements may help us realize that, while the only oracles that are safe are those who keep silent, the scientific enterprise would not survive without the challenges uttered by these oracles. It is likely that some fateful oracles, as well as some in need of reinterpretations, spoke at Białowieża 2002; still unanswerable, a question remains as to what the enduring impact of our oracles will turn out to be, when another generation has passed. For a classical perspective compare Sophocles' *Oedipus the King* and his *Oedipus at Colonus,* allegedly written about forty years later.