The Application of dynamic programming in the system optimization of environmental problem

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Abstract. Based on the application in the system optimization of environmental problem, the solution procedures of dynamic programming are introduced. Combining with some typical problems, such as the shortest path problem, the optimum scheme problem of water treatment and the water resources allocation problem, reliability analyses of the solution procedures by LINGO software is processed. The results show that the LINGO software can effectively solve this kind of dynamic programming problem and is the effective tool to solve the environmental problems and resource problems.

Introduction

Environmental problem is one of the biggest issues faced by mankind. Environmental science, aimed at solving the environmental problems and promoting the effective utilization of environmental resources, is an emerging multi-disciplinary and intersectional subject. Computer technology plays a very important role in promoting the rapid development of this discipline. Practical environmental problems are often analyzed and solved by the method of establishing model[1]. After establishing a mature model, the solution of the model still takes a lot of time. This is because that the variants involved are generally many and the types and forms of constraint are complicated, therefore it is difficult to obtain satisfactory result using general mathematics method. Nevertheless,a variety of computer software can help researchers solve the model, and LINGO software is one of them. As one of the most widely used software for solving optimization problem in aspect of teaching, scientific research and industry, LINGO has many advantages: powerful function of model expresses and model solution, fast operation, friendly interface, easy operation and so on. It is mainly used to solve the problems of linear programming, nonlinear planning, transportation problem, quadratic programming, integer programming, dynamic programming, etc. Meanwhile, it can also solve linear equation, nonlinear equation and algebraic equation[2].

Combining with the study of environmental planning, this paper introduces the steps for establishing mathematical model of dynamic programming. Aiming at some typical problems, such as the shortest path problem, the optimum scheme problem of water treatment and the water resources allocation problem, reliability analyses of the solution procedures by LINGO software is processed.

The solution procedures of dynamic programming

Dynamic programming is a branch of operational research which is an important mathematics programming method to resolve multi-stage policy process optimizing. The characteristic of this method is that it can transform an n-dimension decision problem to several one-dimension optimization problems, and solve them one by one. It must be pointed out that dynamic planning is a method to solve some kinds of problems but not a kind of algorithm. Since different character of the problems and different conditions of the optimal solution, it should give concrete analysis to the concrete conditions. In other words, corresponding model is established and solved by the theory and methods of dynamic programming. Since dynamic programming Dynamic came out, it had been widely used in the economic management, production scheduling, engineering technology and

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optimal control and so on. The fact has proved that it is more convenient to use the dynamic programming than other method in solving some practical problem, such as the shortest path problem, inventory management, resource allocation, renewal of equipment, sorting problem, loading problem, etc[3-6].

The steps for establishing mathematical model of dynamic programming can be categorized as follows:

The stage division. The problem should be properly divided into several stages according to the time and space of things development. Some static problem can also be divided into several stages by introducing the concept of 'time'.

The variable selection. The Selected variables can not only exactly describe the process evolution but also meet the requirements of non-aftereffect, and the state variables of all stages should be determined. Generally, the state variables are found from the characteristics of the process evolution.

The choice of decision variable and the determination of admissible decisions set. Generally, key variable of the solving problem is selected as decision variable and the scope of decision variables should be given, that is, the determination of admissible decisions set.

The correctly writing of state transition equation. According to the k stage state variable and decision variable, k+1 stage state variable is written. The state transition equation should have the recurrence relation.

The determination of target function and the establishment of dynamic programming basic equation. These five steps are the general steps to establish a dynamic programming mathematical model. The dynamic programming model, different from linear programming model, has no unified mode. Establishing a dynamic programming model should give concrete analysis to the concrete conditions. Only by continuous practice and summary can methods and skills of establishing a model be mastered. After building a model, the reversed order method or sequence method can be used to solve the problem. What draws special attention is that a method of simple solution procedure should be flexibly selected according to the characteristics of the problem.

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The shortest path problem. There is a network of 6 zones. The drains are planed to construct from zone 1 to zone 6. The expenses between each two zones are already known (see Fig. 1). Please determine the economical path from zone 1 to zone 6.

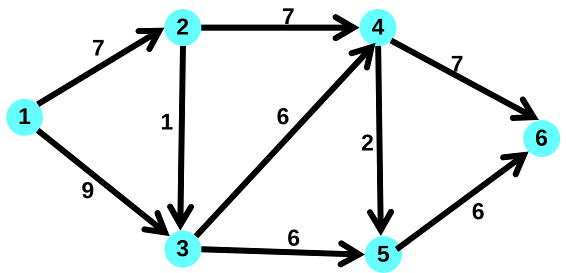

Fig. 1 the shortest path problem

Analysis;This is a typical dynamic programming problem and can be solved by dynamic programming method.

f(i) is defined as the shortest path from Pi to PN. The basic equation is derived by optimization principle as follows:

$$\begin{cases} f(i) = \min\limits_{j}\{c_{ij} + f(j)\}, \quad i = 1, 2, \cdots, N-1 \\ f(N) = 0 \end{cases} \tag{1}$$

The optimal path and the shortest path are achieved by reversed order method. The optimal path is 1-2-3-5-6 and the length of the shortest path is 20.

This problem can also be easily solved by LINGO software.

The programming is as follows:

```
model:
sets:
  p/1..6/:f;
  r(p,p)/1,2 1,3 2,3 2,4 3,4 3,5 4,5 4,6 5,6/: d;
Endsets
data:
d=7 9 1 7 6 6 2 7 6;
Enddata
f(@size(p))=0;
@for(p(i)|i #lt# @size(p):
f(i)=@min(r(i,j):d(i,j)+f(j)));
End
```

Solving the model, we get the shortest distance from city 1 to city 6, gives us the distance of the shortest path of 20.The results agree with the manual analysis of reverse method. It shows the reliability of LINGO software.

The water resources allocation problem. The water of a reservoir is assumed to have 7 units and allocated to 3 users (A, B, C).The benefits of each user with different amount of water are shown in Table 1. Please determine the optimal allocation scheme of the water resources.

Table 1 the benefit data

units	0	1	2	3	4	5	6	7
A	0	5	15	40	80	90	95	100
B	0	5	15	40	60	70	73	75
C	0	4	26	40	45	50	51	53

This problem can be solved by the reverse method and optimal solution can be gradually calculated. Otherwise Lingo software can simplify the solving process. The resolving program and results of Lingo software are as the following:

```
SETS:
! user;
USER/1..3/;
! water amount;
WATER_AMOUNT/1..8/;
! allocation scheme;
ARCS(USER,WATER_AMOUNT):BENEFIT,SELECTION,STATUS;
ENDSETS
MAX=@SUM(ARCS(I,J):BENEFIT(I,J)*SELECTION(I,J));
@FOR(ARCS:@BIN(SELECTION));
@FOR(USER(I):@SUM(ARCS(I,K):SELECTION(I,K))=1);
@SUM(ARCS(I,J):STATUS(I,J)*SELECTION(I,J))=7;
DATA:
! benefit;
BENEFIT=0 5 15 40 80 90 95 100
        0 5 15 40 60 70 73 75
        0 4 26 40 45 50 51 53;
! Undetermined distribution amount;
STATUS=0 1 2 3 4 5 6 7 0 1 2 3 4 5 6 7 0 1 2 3 4 5 6 7;
ENDDATA
```

Solving the problem by lingo software, we can obtained the best results. The results show that we can achieve the largest profit value for 1.2 million yuan when 7 units water resource are allocated to these 3 users on the scale of 4:3:0. The results agree with the manual analysis of reverse method. It indicates the solution by lingo software is accurate and reliable.

The optimum scheme problem of water treatment. The daily sewage drainage of an enterprise is about 3000 t/d, BOD_5 concentration of sewage is 1500 mg/L (1.5 t / 1000 t). If the treatment effect of three kinds of processing measures adopted are 65%, 85% and 90% respectively. Please answer:

(1) How to treat sewage at the lowest cost on the premise of pollution abatement?

(2) If pollution charges are calculated by the expression as. 10 * W / 0.1 and expression (W / 0.1) should returns the next highest integer value by rounding up value. Please give a new optimum scheme. (W represents the quality of BOD_5 / t)

Table 2 The benefit data

BOD_5 quality of Daily drainage /t	Daily drainage charge /yuan	BOD_5 quality of Daily drainage /t	Daily drainage charge /yuan
0.00-0.10	10	1.01-2.00	50
0.11-0.30	15	2.01-3.00	60
0.31-0.50	20	3.01-4.00	100
0.51-1.00	30	4.01-5.00	150

Table 3 The effect, operation cost and equipment cost of different processing scheme

treatment scheme	BOD_5 elimination effect	operation cost /(yuan/1000t)	equipment cost /(yuan/1000t)
1	65	30	15
2	85	40	25
3	90	50	30

Analysis: Corresponding to the three processing scheme, each scheme has three processing level, which is respectively 1000 t, 2000 t, 3000 t. That is to look for processing sewage quantity for each scheme, making the minimum total cost every day.

Total cost function can be written as,

$$Z= Ci\ Xi+ Bi\ Xi + Ai \tag{2}$$

Where Ci represents daily equipment cost for processing 1000 t, Bi represents daily operation cost for processing 1000 t, Ai represents the sewage charge of unprocessed BOD5.

Computing unprocessed BOD5 quality of each processing scheme and the results are shown in table 4.

Table 4 Unprocessed BOD_5 quality of each processing scheme

unprocessed BOD_5/t \ processing capacity /t	processing scheme		
	1	2	3
1000	$P_1=0.525$	$P_1'=0.225$	$P_1''=0.15$
2000	$P_2=1.05$	$P_2'=0.45$	$P_2''=0.3$
3000	$P_3=1.575$	$P_3'=0.675$	$P_3''=0.45$

Computing daily total cost with function 2 and the results are shown in table 5

Table 5 Processing cost of each scheme

processing cost /yuan \ processing capacity /t	processing scheme		
	1	2	3
0	0	0	0
1000	$m_1=75$	$m_1'=80$	$m_1''=95$
2000	$m_2=140$	$m_2'=150$	$m_2''=150$
3000	$m_3=185$	$m_3'=225$	$m_3''=260$

This problem is similar to the dynamic programming problem of resource allocation and can be solve by Lingo program.

```
model:
SETS:
! method;
method/1..3/;
! water;
WATER_AMOUNT/1..4/;
! allocation scheme;
ARCS(method,WATER_AMOUNT):cost,SELECTION,STATUS;
ENDSETS
```

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```
min=@SUM(ARCS(I,J):cost(I,J)*SELECTION(I,J));
@FOR(ARCS:@BIN(SELECTION));
@FOR(method(I):@SUM(ARCS(I,K):SELECTION(I,K))=1);
@SUM(ARCS(I,J):STATUS(I,J)*SELECTION(I,J))=3;
DATA:
! cost;
cost =0 75 140 185
        0 80 150 225
        0 95 150 260;
! Undetermined distribution amount;
STATUS=0 1 2 3 0 1 2 3 0 1 2 3;
ENDDATA
```

The similar resolution results can be obtained as the water resources allocation problem.

In SELECTION (i, j), i represents the processing scheme, j represents the processing capacity that is (j-1)*1000t. For instance, SELECTION (1, 4) = 1.000000 represents 3000t wastewater are treated by the first processing scheme. The results indicated that the optimal solution is only adopting the first processing scheme to treat 3000t wastewater and the minimum total cost is 185. The results agree with the manual analysis by sequence method. It indicates the solution by lingo software is accurate and reliable. Meanwhile, the results also show that companies would rather pay sewage charges than have a good effect of water treatment by the equipment of high operation cost and equipment cost.

To change the idea, we can consider increasing the collecting fees for discharging pollutants. If pollution charges is not based on table 1 but calculated by the expression as 10 * W / 0.1 and expression (W / 0.1) should returns the next highest integer value by rounding up value.

While daily total cost are shown in table 6.

Table 6 Processing cost of each scheme

processing costs /yuan \ daily processing capacity/t	processing scheme		
	1	2	3
0	0	0	0
1000	m_1=105	m_1'=95	m_1''=100
2000	m_2=190	m_2'=180	m_2''=190
3000	m_3=295	m_3'=265	m_3''=290

At this time, modifying the input part of Lingo model data, getting SELECTION (2, 4) = 1.000000, that is, 3000 t wastewater is treated by the second processing scheme. From the results, we know that the optimal solution is only adopting the second processing scheme to treat 3000t wastewater and the minimum total cost is 265. It shows that enforcing penalties of untreated wastewater discharge is beneficial to guide the enterprise independently treat wastewater.

Conclusion

This paper introduces the solution procedures of dynamic programming. Combining with some typical problems, such as the shortest path problem, the optimum scheme problem of water treatment and the water resources allocation problem, reliability analyses of the solution procedures by LINGO software are processed. The results show that the LINGO software can effectively solve this kind of dynamic programming problem and is the effective tool to solve the environmental problems and resource problems.

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