

# Backstepping Based Adaptive Control of Magnetic Levitation System

Wei Zhou<sup>1,2, a</sup>, Baobin Liu<sup>3,b</sup>

<sup>1</sup> Jiangsu Institute of Economic & Trade Technology

<sup>2</sup>Zhejiang University

<sup>3</sup> Jiangsu Engineering and Technology Center of Circulation Modernization and Sensor Network

<sup>a</sup>pink\_20020351@163, <sup>b</sup>jsliubin@sina.com.cn

**Keywords:** adaptive control; magnetic levitation; nonlinear control; backstepping

**Abstract.** In view of parameter uncertainty in the magnetic levitation system, the adaptive controller design problem is investigated for the system. Nonlinear adaptive controller based on backstepping is proposed for the design of the actual system with parameter uncertainty. The controller can estimate the uncertainty parameter online so as to improve control accuracy. Theoretical analysis shows that the closed-loop system is stable regardless of parameter uncertainty. Simulation results demonstrate the effectiveness of the presented method.

## Introduction

Magnetic levitation is a new technology, which use magnetic force to make an object, such as a magnetic ball, stay in a fixed position or move regularly in the magnetic field. For the suspended object is not in contact with support force directly, the energy consumption caused by the friction would be greatly reduced and the speed of suspend objects can be greatly improved. Magnetic levitation technique has been widely used in fast trains, vibration isolation systems, and magnetic bearings because of high efficiency. And it is concerned with electromagnetics, electronic technology and control engineering as a whole. For the controller is generally required in magnetic levitation system, much interest has been focused on it recently. There are several methods to design the controller of magnetic levitation system, such as proportional integral differential control [1,2] and generalized proportional integral control [3]. However, it is nearly always desired to produce a stable state in the presence of variations in the parameter uncertainty and nonlinear system, which be ignored in these approaches. This motivates us to consider an adaptive controller of the magnetic levitation system. Unlike previous results, nonlinear adaptive control can avoid deleting too many uncertain parameters to lead deterioration of the controlled system, so it can improve the accuracy of the control [4-6].

## The Model of the Magnetic Levitation System

Sensor, controller and drive constitute a magnetic levitation system usually. Fig. 1 illustrates a schematic diagram of the system. An electromagnet powers the drive with a control current. The current generates a force to equalize the weight of the magnetic ball, thus the magnetic ball can be in a suspended state. Equilibrium of the magnetic levitation ball is unstable. If a slightly disturbance happened, it would cause the ball deviate from the original location. So the control target is that the desired height of magnetic ball can be obtained even if disturbances happened. Hence, we obtain the equation of motion of the ball through stress analysis.

$$m\ddot{y} = -k\dot{y} + mg - \frac{L_0 i^2}{2a(1 + y/a)^2}. \quad (1)$$

Where  $m$  is the mass of the ball,  $y \geq 0$  is the vertical position,  $k$  is a viscous friction coefficient,  $i$  is the drive current and  $a, L_0$  are positive constants.

## Controller Design

In the above system, it is easy to know that viscous friction coefficient  $k$  will changes with environment. The field of controller design is concerned with the problem of model uncertainties nonlinear system. Now we apply backstepping method to solve the nonlinear problem. Let  $x_1 = y, x_2 = \dot{y}$  and  $\mu = i^2$ . The differential equations is given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m}x_2 + g - \frac{k_1}{2m(a+x_1)^2}\mu \end{cases}, \quad (2)$$

where  $k_1 = aL_0$ .

In the section, we will consider the adaptive control of the nonlinear magnetic levitation system. The stabilizer  $\mu$  will be designed in a backstepping way. Virtual control and parameter estimation will be determined according to Lyapunov theory.

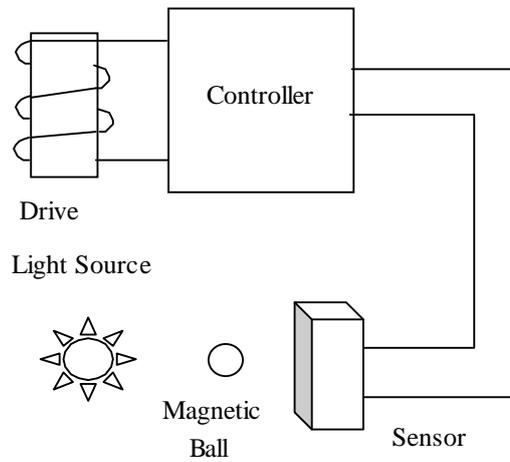


Fig.1 A schematic diagram of magnetic levitation system

First, define  $z_1 = x_1$ , and it is easy to obtain the differential equations  $\dot{z}_1 = x_2$ . Define  $x_2^d$  as the virtual control of  $x_2$ ,

$$x_2^d = -C_1 z_1. \quad (3)$$

Define the state variable  $z_2$ , which is the error between  $x_2$  and virtual control  $x_2^d$ . We can write

$$z_2 = x_2 - x_2^d = x_2 + C_1 z_1. \quad (4)$$

The time derivative of  $z_2$ , computed with Eq. 2 and Eq.4, is given by

$$\dot{z}_2 = -\frac{k}{m}z_1 + g - \frac{aL_0}{2m(a+x_1)^2}v + C_1(z_2 - C_1 z_1). \quad (5)$$

Define the actual control according to the stable theory Lyapunov function.

$$\mu = -(-C_2 z_2 - g + \frac{\hat{k}}{m}z_1 - C_1 z_2 + C_1^2 z_1 - z_1) \frac{2m(a+x_1)^2}{aL_0}. \quad (6)$$

Where  $\hat{k}$  is the parameter estimation of  $k$ .

Then, we can obtain from Eq.5 and Eq.6 that

$$\dot{z}_2 = (\hat{k} - k)z_1 - C_2 z_2 - z_1. \tag{7}$$

We select Lyapunov function of the system as

$$V = 0.5z_1^2 + 0.5z_2^2 + 0.5(\hat{k} - k)^2. \tag{8}$$

The time derivative of  $V$ , computed with Eq.7 is given by

$$\dot{V} = -C_1 z_1^2 - C_2 z_2^2 + (\hat{k} - k)(z_1 z_2 + \dot{\hat{k}}) / m. \tag{9}$$

So we obtain the update law of the uncertainty parameter

$$\dot{\hat{k}} = -z_1 z_2. \tag{10}$$

According to Barbalat lemma, we can obtain that when  $t \rightarrow \infty$ ,  $z \rightarrow 0$  [7-9]. The system is asymptotically stable.

### Simulations

Let  $m = 0.1$  kg,  $C_1 = 1$ ,  $C_2 = 2$ , the positive constants  $a = 0.05$  m and  $L_0 = 0.01$  H. The simulation results of status variable  $z_2$ , control variable and parameter estimation variable  $\hat{k}$  are shown in Fig. 2, Fig. 3 and Fig. 4.

It can be seen from the simulation results that:

- 1) State variable  $z_2$  converges to 0, which is consistent with theoretical analysis.
- 2) Control variable  $\mu$  is asymptotically stable and it converges to a constant.
- 3) Parameter estimation variable can precisely choice uncertain parameter, and the result is consistent with Reference [10], which indicates the approximation value of  $k$  is 0.001.

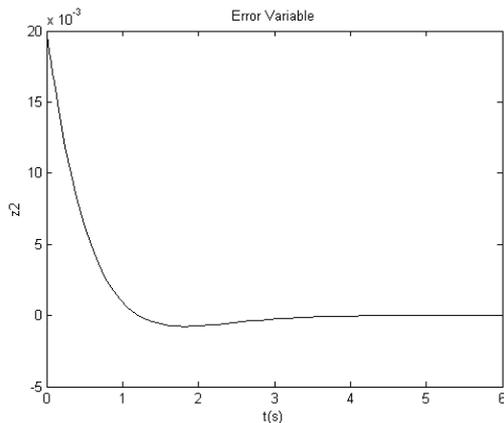


Fig.2 The Profile of Error Variable

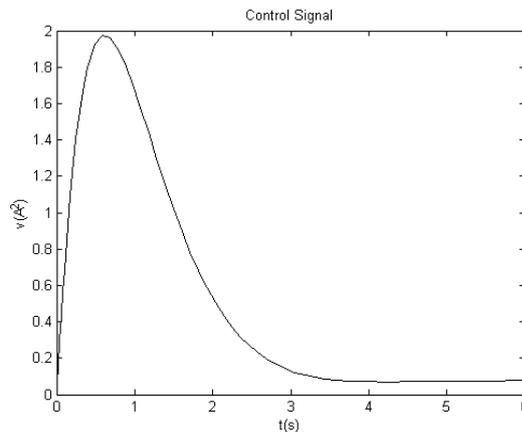


Fig.3 The Profile of Control Variable

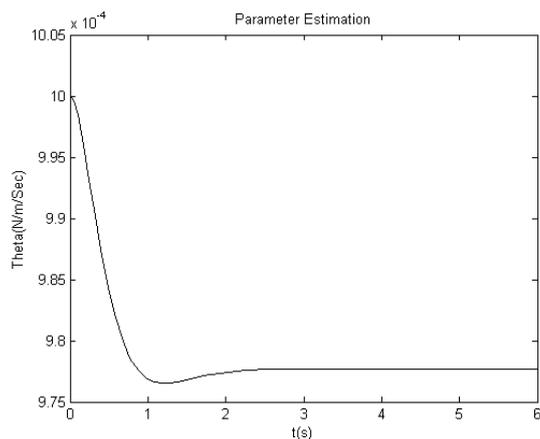


Fig.4 The Profile of Parameter Estimation

## Conclusions

Traditional GPI and PID control of the nonlinear system regard the uncertain parameters of the system model as a fixed constant, which decrease the accuracy of the control. The paper introduces the nonlinear adaptive control into magnetic levitation system and it can estimate the unknown parameters online. The approach is designed with backstepping method. And global asymptotic stability of the closed-loop system has been proved via Lyapunov function and Barbalat lemma. Therefore, the designed controller can effectively improve accuracy of the control, and simulation results show the effectiveness of the method.

## Acknowledgements

This research was financially supported by Jiangsu Provincial Education Department and Finance Department (SJBK [2010] 10#).

## References

- [1] Rong-Jong Wai, Jeng-Dao Lee and Kun-Lun Chuang: IEEE Transactions on Industrial Electronics, Vol. 58(2011), p.629-646.
- [2] Zhi-Guo Lv, Y. Deng and Z.P. Liu: Aviation Precision Manufacturing Technology, Vol. 44(2008), p.25-27.
- [3] R. Morales and H. Sira-Ramirez: International Journal of Control, Vol.83(2010), p.1155-1166.
- [4] D. Seto, A. M. Annaswamy and J. Baillieul: IEEE Transactions on Automatic Control, Vol. 39(1994), p. 1411–1427.
- [5] X.D.Ye and J.P. Jiang: IEEE Transactions on Automatic Control, Vol.43(1998), p.1617-1621.
- [6] Jean-Jacques E. Slotine and W.P. Li: *Applied Nonlinear Control*, (Machine Press, P. R. China 2006).
- [7] X.D. Ye: IEEE Trans. Automat. Contr., Vol.48(2003), p.169-173.
- [8] Y.G. Hong and D.Z.Cheng: *Nonlinear System Analysis and Control*, (Science Press, P. R. China 2006).
- [9] M. Krstic and P.V. Kokotovic: IEEE Transactions on Automatic Control, Vol.40(1995), p.426-440.
- [10] Hassan K. Khalil: *Nonlinear Systems* (Publishing House of Electronics Industry, P. R. China 2007).