# Distributed Fusion Kalman Self-turning Filter

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**Abstract.** This paper puts forward an optimal and distributed fusion Kalman filter based on the Riccati equation, optimal and distributed fusion The Kalman filter has fewer calculated dimensions and less calculated amount than the centralized global optimal Kalman filter. Therefore, it has greater effect in the practice. Present the distributed local self-correcting Kalman filter at the same time. The simulation examples showed its validity.

## Local Optimal Kalman Filter

Augmented matrices equation

$$x(t+1)=\Phi x(t)+\Gamma e(t) \tag{1}$$

$$y_i(t)=Hx(t)+v_i(t)\quad i=1,\ldots L \tag{2}$$

$$s(t)=H_s x(t) \tag{3}$$

$$H=\left(H_\alpha,\ H_\beta\right) \tag{4}$$

$$H_\alpha=\left(1,0,\cdots.0\right)\ H_\beta=\left(1,0,\cdots.0\right) \tag{5}$$

where $s(t)$ is the signal $y_i(t)\in R^{m_i}$, $y_i(t)$ is the $i$-th sensor signal output (observation), $\eta(t)$ is th e public colored noise, $v_i(t)$ is the sensor noise, and $X(q^{-1})$ is a polynomial matrix having the form $X\left(q^{-1}\right)=x_0+x_1q^{-1}+\cdots+x_{n_x}q^{-n_x}$ with the backward shift operator $q^{-1}$.

**Assumption 1.** $\omega(t)$, $v_i(t)$ and $\xi(t)$ are zero uncorrelated white noises with the variances $\sigma_w^2$, $\sigma_{v_i}^2$ and $\sigma_\xi^2$.

$$E\{\begin{bmatrix} w(t) \\ v_i(t)\end{bmatrix}\begin{bmatrix} w^T(t) & v_j{}^T(t)\end{bmatrix}\}=\begin{bmatrix}\sigma_w^2 & 0 \\ 0 & \sigma_v^2\delta_{ij}\end{bmatrix}\delta_{tk}$$

**Assumption 2.** $\Phi$ is a nonsingular matrix, ( $\Phi$, H) is a completely observable couple, $(\Phi,\Gamma)$ is a completely controllable couple.

**Assumption 3.** The observational data $y_i(t)$ is bounded.

**Theorem 1.** For the system ( 1 )-( 5 ), with the assumptions 1 - 3, the local optimal Kalman filter has the distributed local optimal Kalman filter $\hat{x}_i(t\,|\,t)$.

$$\hat{x}_i(t\,|\,t)=\Psi_i(t)\hat{x}_i(t-1\,|\,t-1)+K_i(t)y_i(t)$$

$$\Psi_i(t)=(I-K_i(t)H)\Phi$$

$$K_i(t)=\Sigma_i\left(t\middle|t-1\right)H^T[H\Sigma_i\left(t\middle|t-1\right)H^T+\sigma_{vi}{}^2]^{-1}$$

$$\Sigma_i\left(t+1\middle|t\right)$$

$$=\Phi[\Sigma_i\left(t\middle|t-1\right)-\Sigma_i\left(t\middle|t-1\right)H^T(H\Sigma_i\left(t\middle|t-1\right)H^T+\sigma_{vi}{}^2)^{-1}H\Sigma_i\left(t\middle|t-1\right)]\Phi^T+\Gamma Q_e\Gamma^T$$

$$\hat{s}_i(t\,|\,t)=H_s\hat{x}_i(t\,|\,t)$$

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$$H_s = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}$$

where the error variance matrix is

$$P_{ij}\left(t|t\right) = \Psi_i(t) P_{ij}\left(t-1|t-1\right)\Psi_j^T(t) + (I - K_i(t)H)\Gamma Q_e \Gamma^{-1}(I - K_j(t)H) + K_i(t)\sigma_{vi}{}^2 K_j(t)\delta_{ij}$$

$$P_{Sij}\left(t|t\right) = H_s P_{ij}\left(t|t\right) H_s^T$$

$$\delta_{ii} = 1, \delta_{ij} = 0 (i \neq j)$$

## Weighted Distributed Fusion Kalman Filter

**Theorem 2.** With the assumptions 1-3, system (1)-(5) has the component weighted fusion Kalman filter:

If there is $\hat{x}_i,\ i = 1, \cdots L$ is the L-th unbiased estimation of the random vector $x \in R^n$ , the covariance matrix $P_{Sij}\left(t|t\right), i, j = 1, \cdots L$ whose error of estimation is $\tilde{s}_i\left(t|t\right) = s\left(t|t\right) - \hat{s}_i\left(t|t\right)$ and $\tilde{s}_j\left(t|t\right) = s\left(t|t\right) - \hat{s}_j\left(t|t\right)$ is known, the weighted optimal fusion unbiased estimation in the linear minimum variance sense will be

$$\hat{s}_0\left(t|t\right) = \sum_{i=1}^{L} w_i\left(t\right)\hat{s}_i\left(t|t\right)$$

where the optimal weighting coefficient is

$$w\left(t\right) = [w_1\left(t\right), \cdots, w_L\left(t\right)]$$

$$w\left(t\right) = \frac{e^T P_S^{-1}\left(t|t\right)}{e^T P_S^{-1}\left(t|t\right) e}$$

where the column vector $e$ that define the $L \times L$ matrix $P_S\left(t|t\right)$ and $L \times 1$ is

$$P_S\left(t|t\right) = (P_{Sij}\left(t|t\right)) = \begin{bmatrix} P_{S11}(t|t) & \cdots & P_{S1L}(t|t) \\ \vdots & & \vdots \\ P_{SL1}(t|t) & \cdots & P_{SLL}(t|t) \end{bmatrix}, i, j = 1, \cdots, L \quad e = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

The corresponding optimal fusion estimation error variance matrix

$$P_{S0}(t|t) = (e^T P_S^{-1}\left(t|t\right) e)^{-1}$$

And they have the relation

$$P_{S0}(t|t) \leq P_{Sii}(t|t) \quad i = 1, \cdots, L$$

## Distributed Fusion Kalman Self-turning Filter

For the system whose model parameters and noise variance are partially unknown, first, identify the model parameters and noise variance with related methods, substitute the results into distributed local optimal Kalman filter. Then substitute the results into the weighted distributed fusion Kalman filter in order to get the distributed fusion self-turning Kalman filter. The *i*-th subsystem( sensors) local self-turning condition filter has the following steps:

**Step 1.** Use the recursion auxiliary variable alogrithm and the G-W-CLS algorithm to get model parameters and noise variance fusion valuations of $\hat{A}_f(q^{-1}), \hat{C}_f(q^{-1}), \hat{P}_f(q^{-1})\hat{\sigma}_{\xi f}^2(t)$, $\hat{\sigma}_{v_i f}^2(t), \hat{\sigma}_{wf}^2(t)$, $Q_e = \begin{pmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_\xi^2 \end{pmatrix}$;

**Step 2**. Then we can get the local Kalman self-turning filter of the *i*-th sensor subsystem;

The local Kalman self-turning filter of the *i*-th sensor subsystem is

$$\hat{x}_i^s(t\,|\,t)=\hat{\Psi}_i(t)\hat{x}_i(t-1\,|\,t-1)+\hat{K}_i(t)y_i(t)$$
$$\hat{\Psi}_i(t)=(I-\hat{K}_i(t)H_i)\hat{\Phi}$$
$$\hat{K}_i(t)=\hat{\Sigma}_i\left(t\middle|t-1\right)H^{\mathrm{T}}[H\hat{\Sigma}_i\left(t\middle|t-1\right)H^{\mathrm{T}}+\hat{\sigma}_{vif}{}^2]^{-1}$$
$$\hat{\Sigma}_i\left(t+1\middle|t\right)$$
$$=\hat{\Phi}[\hat{\Sigma}_i\left(t\middle|t-1\right)-\hat{\Sigma}_i\left(t\middle|t-1\right)H^{\mathrm{T}}(H\hat{\Sigma}_i\left(t\middle|t-1\right)H^{\mathrm{T}}+\hat{\sigma}_{vif}{}^2)^{-1}H\hat{\Sigma}_i\left(t\middle|t-1\right)]\hat{\Phi}^{\mathrm{T}}+\Gamma\hat{Q}_e\Gamma^{\mathrm{T}}$$
$$\hat{s}_i^s(t\,|t)=H_s\hat{x}_i^s(t\,|t)$$

**Step 3.** Substitute into the weighted distributed fusion Kalman filter;

$$\hat{s}_0^s\left(t\middle|t\right)=\sum_{i=1}^{L}\hat{w}_i\left(t\right)\hat{s}_i^s\left(t\middle|t\right),i=1,\cdots,n_a$$
$$\hat{w}\left(t\right)=\frac{e^{\mathrm{T}}\hat{P}_s^{-1}\left(t\middle|t\right)}{e^{\mathrm{T}}\hat{P}_s^{-1}\left(t\middle|t\right)e};j=1,2,\cdots,L,i=1,2,\cdots,n_a$$

where $\hat{w}\left(t\right)=[\hat{w}_1\left(t\right),\cdots,\hat{w}_L\left(t\right)]$

$$\hat{P}_S(t)=\begin{bmatrix}\hat{P}_{S11}(t) & \cdots & \hat{P}_{S1L}(t)\\ \vdots & & \vdots\\ \hat{P}_{SL1}(t) & \cdots & \hat{P}_{SLL}(t)\end{bmatrix}$$

The definition of the cross covariance matrix $\hat{P}_{ij}(t)$ is

$$\hat{P}_{ij}\left(t\middle|t\right)=\hat{\Psi}_i(t)\hat{P}_{ij}\left(t-1\middle|t-1\right)\hat{\Psi}_j^T(t)+(I-\hat{K}_i(t)H)\Gamma\hat{Q}_e\Gamma^T(I-\hat{K}_j(t)H)+\hat{K}_i(t)\hat{\sigma}_{vif}{}^2\hat{K}_j(t)\delta_{ij}$$
$$\delta_{ii}=1,\delta_{ij}=0(i\neq j)$$

The three steps repeat at the *t* moment.

## Simulation Examples

Consider the multi-sensor invariant linear discrete stochastic systems

$$(1-1.5q^{-1}+0.56q^{-2})s(t)=(1+0.4q^{-1})w(t)$$
$$y_i(t)=s(t)+\eta(t)+v_i(t),i=1,2,3,4,5$$
$$(1-0.7q^{-1}+0.5q^{-2})\eta(t)=q^{-1}\xi(t)$$

**Assumption 1.** $v_i(t)$ and $\xi(t)$ are zero uncorrelated white noises with the variances are $\sigma_{vi}^2$, $\sigma_\xi^2$.

**Assumption 2.** $P\left(q^{-1}\right)$, $\sigma_\xi^2$ and $\sigma_{vi}^2$ are known.

**Assumption 3.** The observational data $y_i\left(t\right)$ is bounded.

**Assumption 4.** $A\left(q^{-1}\right)$, $\sigma_w^2$ and $C\left(q^{-1}\right)$ are unknown.

where $s(t)$ is the signal $y_i(t)\in R^{m_i}$, $y_i(t)$ is the *i*-th sensor signal output(observation), $\eta(t)$ is the public colored noise, $v_i(t)$ is the sensor noise. $w\left(t\right)$ and $v_i\left(t\right)$ are independent Gaussian white noises whose zero mean and variances are $\sigma_w^2$ and $\sigma_v^2$. Make $\sigma_w^2=0.8$, $\sigma_\xi^2=0.3$, $\sigma_{v1}^2=0.1$, $\sigma_{v2}^2=0.2$, $\sigma_{v3}^2=0.3$, $\sigma_{v4}^2=0.4$, $\sigma_{v5}^2=0.5$ in the simulation.

After reduction, there are following augmented matrix

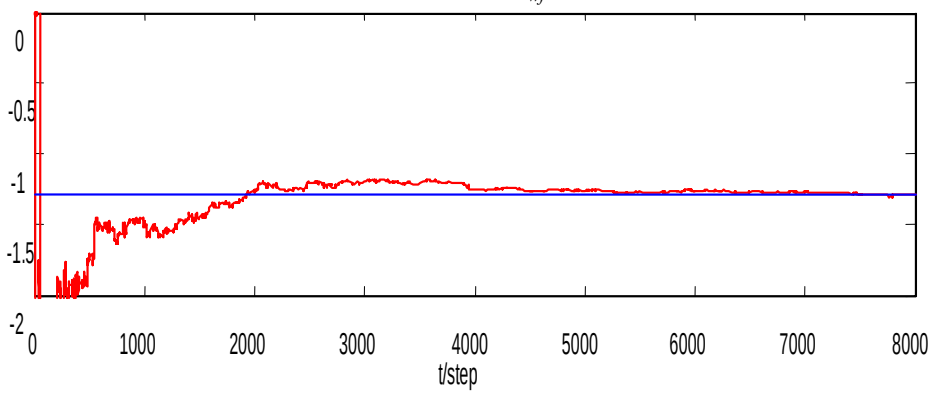
$$x(t+1)=\Phi x(t)+\Gamma e(t)$$
$$y_i(t)=Hx(t)+v_i(t),\quad i=1,2,3,4,5$$
$$s(t)=H_s x(t)$$
$$H_s=\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix}$$

where $x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$, $y_i(t)$ are system observed signal, $w(t)$ is the noise (when $t=T_0$),

$\Phi=\begin{pmatrix} 1.5 & 1 & 0 & 0 \\ -0.56 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 1 \\ 0 & 0 & -0.5 & 0 \end{pmatrix}$, $\Gamma=\begin{pmatrix} 1 & 0 \\ 0.4 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$, $T_0$ is the sampling period.

$H=\begin{pmatrix} H_\alpha & H_\beta \end{pmatrix}=\begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix}$ $Q_e=\begin{pmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_\xi^2 \end{pmatrix}=\begin{pmatrix} 0.8 & 0 \\ 0 & 0.3 \end{pmatrix}$

First, identify the unknown model parameters $A\left(q^{-1}\right)$ with the recursion auxiliary variable algorithm. The simulation results are as shown in the Figure1. When $t=8000$, the valuation model parameters. In the simulation figures , the straight lines say true value, and the curves say the fusion valuation. The convergence of the model parameter estimators are shown as Figure 2. The convergence of the noise variance estimators $\sigma_{wf}^2$ are shown as Figure 3.



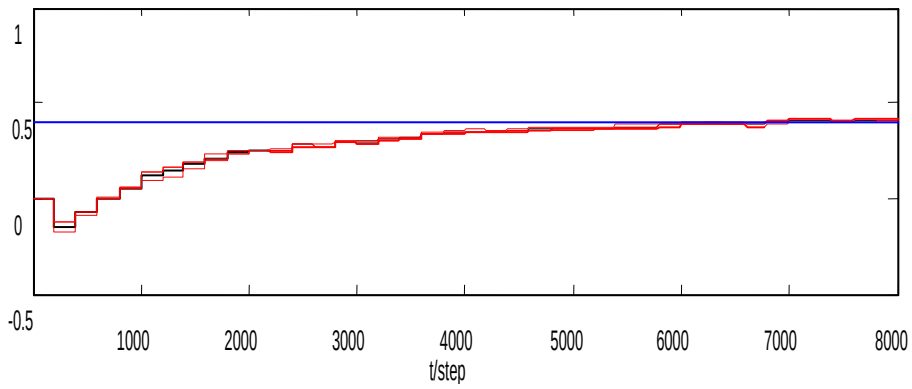

the fusion valuation of $\hat{a}_1$

Figure 1. The convergence of the model parameter estimators

The figure of $\hat{a}_2$ is omitted.

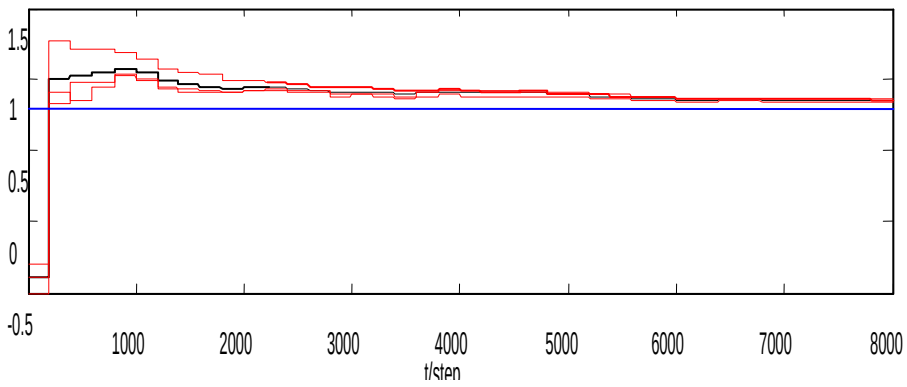
Next, use the G - W - CLS identification algorithm to identify $C\left(q^{-1}\right)$ and the noise variance $\sigma_w^2$.

Get the estimating valuation of $\hat{C}_f\left(q^{-1}\right)$.




$c_1, \hat{c}_{i1}(i=1,2,3), \hat{c}_{1f}$

Figure 2. The convergence of the model parameter estimators

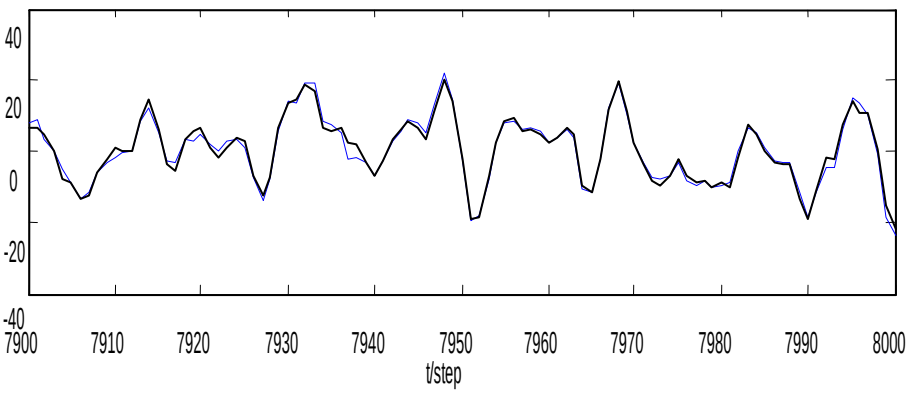



$\sigma_w^2, \hat{\sigma}_{wi}^2 (i = 1, 2, 3), \hat{\sigma}_{wf}^2$

Figure 3. The convergence of the noise variance estimators $\sigma_{wf}^2$

Substitute the estimating valuations $\hat{c}_1$, $\hat{a}_1$, $\hat{a}_2$ and $\hat{\sigma}_{wf}^2$ into the diatributed fusion Kalman self-turning filter.

The simulation results are as shown in the Figure 4. When $t = 8000$, the valuation model parameters. In the simulation figures, the straight lines say true value, and the curves say the fusion valuation.




Weighted distributed fusion self-turning filter $\hat{s}_0^s(t \mid t)$

Figure 4. The self-tuning information fusion Kalman filters

## References

[1] A. J. Roedker. C. D. Gillem: Comparison of two-sensor tracking methods based on state vector fusion and measurement fusion IEEE Transaction on Aerospace and Electronic Systems, Vol 24(1998). P. 447-449

[2] H. Z. Qiu H. Y. Zhang H. Jin: Fusion algorithm of correlated local estimates. Aerospace Science and Technology (2004) . P. 619-626

[3] Z. L. Deng: Information Fusion Filtering Theory with Applications. Harbin: Harbin Industrial University Press, 2007.

[4] Z. L. Deng: Identification of Noise Statistics Based on White Noise Estimators and Adaptive Kalman Filtering, Journal of Natural Science University, Vol. 8, no. 3, p. 67-72, 1991

[5] Z. L. Deng, Y. Gao, etc. New approach to information fusion steady-state Kalman filtering, Automatica, Vol. 41, no. 10, p. 1695-1707, 2005