

Research on the Scheduling Problems of Stand-alone Batch Processing in Production Logistics

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Abstract—Plenty of scheduling problems of production exist in the management of production logistics in industrial manufacturing enterprises. This field has a profound practice demand and an extensive application prospect. On the basis of analyzing three elements of scheduling problems, this paper has concluded up-to-date problems of scheduling optimization of stand-alone Batch Processing in production logistics, in order to provide reference for the production management of enterprises.

Keywords—*production logistics; stand-alone batch processing; scheduling problems*

I. INTRODUCTION

Scheduling problems are also called sequencing problem. Scheduling theory, as an applied science, has a profound basis of production practice and an extensive application prospect. The theory first originates from the production industry. Since the 1950s, problems of scheduling optimization have been extensively applied to fields such as management science, computer science, logistics transportation scheduling and engineering technology.

II. THE ESSENCE OF SCHEDULING PROBLEMS

The research on scheduling problems includes the allocation and optimization problems of resources and tasks, the distribution problem of Resource or Machine under the condition that Task or Job is given. In essence, it is to solve the problem of optimizing the scheduled target through allocating the limited resources to different work tasks according to the time sequence. In general, the optimization objectives are to realize the shortest time consumed, least cost and highest use ratio.

III. ANALYSIS ON THE THREE ELEMENTS OF SCHEDULING PROBLEMS

Scheduling problems are mainly comprised of three elements, namely Machine, Task and objective function. Scheduling problems originate from the manufacturing industry, so people often describe scheduling problems through terms of the manufacturing industry: Resource is also called Machine and sometimes called Processor, task is also called Job, and the Job composed by basic Tasks that have precedence order and interrelation is called Operation. Scheduling means reasonably allocating Machine and

Resource to Task to complete a given Task and optimize the objective function, under certain constraint condition.

The initial research and application on sequencing generally uses Machine, Job, Operation and processing time to describe the Task of a sequencing operation. That is to say, it is assumed that n Jobs will be processed through m sets of Machines according to a certain processing route. The processing route is determined by the craft process of Job processing, is the constraint of Job processing on technology and the sequence of manufacturing procedure required by Jobs. Sequencing means determining the processing sequence of n Jobs on m sets of Machines.

This paper has carried out sort description of Machine, Job and objective function [1]. The expressive methods of Machine, Job and objective function under different circumstances are as follows: It is assumed that the number of Job and Machine is limited. The number of Job or Task is denoted as n , and the number of Machine is denoted as m , the subscript j refers to a Job and the subscript i refers to a set of Machine, (i, j) refers to the operation of Job i on Machine j .

A. Machine

The sequencing problem only related to one set of machine is called single sequencing problem, or it is called multi-machine sequencing problem. Machine is divided into two major categories in the multi-machine problem: general parallel machine and special dedicated machine. The machines with the same functions are called uniform machine or parallel machine; dedicated machines mean machines have different functions and Jobs must be processed on different Machines.

A more complicated situation is the flexible flow shop, which is the combination and optimization of flow process and parallel machine. In addition, there is another kind of Machine called Batching Machine, which can process multiple Jobs simultaneously. It is different from the traditional situation that one set of Machine can only process on Job.

B. Job and Operation

Under the situation that interrupted processing is not allowed, the process that a Job ($J_j, j=1, 2, \dots, n$) is processed continuously on a set of Machine ($M_i, i=1, 2, \dots, m$) is called

operation. The sequencing can be divided into static and dynamic sequencing according to the difference of workshop where Jobs reach. When the sequencing is under progress, all the Jobs have arrived, and we can sequence them, this is the static sequencing problem; if Jobs are arrived in succession and we need to sequence them at any moment, this is the dynamic sequencing problem.

The constraint condition in sequencing mainly refers to the property of Job and their needs and limitation during the processing.

1) *Processing time*: The processing time of a Job: $P_j=(p1j,p2j,\dots,pnj)$, and the processing time of n Jobs is expressed through matrix:

$$P = \begin{pmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \dots & P_{mn} \end{pmatrix}$$

In this formula, p_{ij} refers to the processing time required by Job J_j on Machine i .

2) *Arrival time or ready time*: Arrival time or ready time r_j refers to the time that Job has already and can be processed immediately. If the ready time of all Jobs is the same, then $r_j=0, j=1,2,\dots,n$.
 $r=(r1, r2, \dots, rn)$

3) *Due date or deadline of Jobs*: Due date d_j means the time of completion defined for Job J_j . If people fail to complete on schedule, they will be punished. The time limit that delay is absolutely not allowed is called deadline.
 $d=(d1, d2, \dots, dn)$

4) *Weight of Jobs*: Weight of Jobs refers to the importance degree of the Job, compared with other Jobs.
 $W=(w1, w2,\dots, wn)$

Two important constraints exist during the process of Jobs: Preemptive or Non-Preemptive constraint and the precedence constraint between Jobs.

C. Objective Function

The completion time of Jobs is expressed via $C=(C1, C2,\dots, Cn)$. Generally speaking, it is necessary to minimize the function of the completion time C . In the sequencing problems, the objective function mainly includes the following kinds.

1) *Maximum schedule length or make-span*: The make-

span can be defined as: $C_{max} = \max_{1 \leq j \leq n} \{C_j\}$, namely the completion time of the Job that is processed last.

2) *Weighted flow time and weighted completion time*: The flow time of a Job refers to the duration from the time when the Job arrives at the system to the time when the Job

is processed, including the waiting time and the processing time in the system: $Ft=Ct-rt$,

The average weighted flow time of the system:

$$F = \frac{\sum_{j=1}^n w_j F_j}{\sum_{j=1}^n w_j}$$

3) *Maximum lateness*: The lateness of a Job is defined as $L_j=C_j-d_j$, and the maximum lateness

$$L_{max} = \max_{1 \leq j \leq n} \{L_j\}$$

4) *Total weighted tardiness*: Tardiness means the completion time of a Job is longer than its deadline:

$$D_j = \{C_j-d_j, 0\} = \{L_j, 0\}$$

Total weighted tardiness:

$$D = \sum_{j=1}^n w_j D_j$$

Scheduling problems can be divided into two types according to whether the data of parameter before decision making is known: Deterministic Scheduling and Stochastic Scheduling. This paper mainly researches the Deterministic Scheduling.

IV. THE CLASSIFICATION OF BATCH SCHEDULING

A. The Classification and Expressive Method of Scheduling Problems (Sequencing)

Many methods exist to classify the sequencing problems. Different sequencing problems are formed because of the different features of Machine, Job, process parameter and objective function. This paper has adopted the three-parameter expressive method (Pinedo) to describe sequencing problems, namely $a/\beta/\gamma$.

Domain a expresses the quantity, type and environment of Machine: $a=a1a2$, $a1=\{.,P,Q,R,O,F,J,FF\}$, and the symbols respectively represent single machines, identical machines, uniform machines, free shop, flow shop, one-piece shop and flexible flow shop; $a2=\{.,m\}$, respectively express the number of Machines is uncertain and there are m sets of Machine.

Domain β expresses the property of Jobs, processing requirements and limitations, the type and quantity of Resources and its influence on processing. It can contain multiple items simultaneously, and the possible dominant items include: r_j (Jobs have different arrival time), S_{jk} (equipment adjustment time of sequential processing of Jobs), etc.

Domain γ expresses the objective function to be optimized: C_{max} (make-span), $\sum w_j C_j$ (total weighted completion time) and $\sum w_j D_j$ (total weighted tardiness).

B. Batch Scheduling

It is assumed in the classical sequencing problem that any Machine can only process one Job at any moment at most, but in reality, one set of Machine can often process multiple

Jobs at the same time. For example, an oven can bake many pieces of bread at a time. The Machine that can process multiple Jobs at the same time is called Batch machine, and the corresponding scheduling problems are the Scheduling Problems with batching machine. Meanwhile, the Jobs processed at the same time are called one batch. Generally speaking, compared with the single processing machine, the batching machine will give rise to a lower cost and a higher efficiency [2].

The scheduling problems of batching machine are described as follows: It is assumed n Jobs must be processed on one batching machine, and the submission date is r_j , the due date is d_j , the processing time is P_j , and the weight is ω_j . These Jobs can be divided into several batches (denoted as B_1, B_2, \dots, B_r). The batching machine can process b Jobs every time. The submission date of the batch is the maximum among the submission date of this batch, namely

the submission date is $\max_{1 \leq j \leq n} \{r_j\}$. The same completion time exists in the same batch of Jobs, and the readiness time s exists before the processing of every batch of Jobs. The batch scheduling problems must solve the problems such as how to divide Jobs in batches, how to arrange the processing sequence of each batch to optimize a certain objective function.

The scheduling of batching machine is divided into two kinds according to the different computing method of the processing time of a batch. One kind is that the processing time of a batch is defined as the maximum among the processing time of Jobs in this batch; another kind is that the processing time of a batch is defined as the sum of the processing time of all the Jobs in this batch. In general, the scheduling problems of the batching machine in the first kind are called Parallel Batching Problems, and the second kind is called Serial Batching Problems.

V. NEWEST RESEARCH ON THE SINGLE BATCH SCHEDULING PROBLEMS

This paper has introduced the latest achievement on the parallel batching scheduling problems and serial batching scheduling problems of the single batching machine.

A. Parallel Batching Scheduling Problems

In the parallel batching scheduling problems of single machine, the processing time of a batch is the maximum among the processing time of all Jobs, namely $\max_{1 \leq j \leq n} \{P_j\}$.

The parallel batching scheduling problem is an important research direction in the job scheduling of production logistics. The decision making of this scheduling problem contains not only the sequencing of Jobs but also how to carry out batch setting. The batching machine can be divided into two types according to the capability of the batching machine or the capacity of the machine: One is that the capacity of machine is limitless. That is to say, the capability of batching machine is greater than the number of Jobs, namely $b \geq n$, which is also called unbounded model; another

is that the capacity of machine is limited. That is to say, the capacity of batching machine is restrained by a constant, namely $b < n$. At the same time, the number of Jobs processed cannot exceed b , which is called bounded model. In this paper, it is assumed that no readiness time exists before the processing of each batch of Jobs, namely $s=0$. The unbounded and bounded models of parallel batching and the fixed batch problems will be discussed next. To describe the capacity of Machine, when it is expressed through three-domain expressive method, $b < n$ or $b = \infty$ can be added in domain β .

1) *Unbounded model*: Because in unbounded model, $b \geq n$, and the batching machine can process Jobs of any quantity at the same time, the problem $B|p\text{-batch}, b = \infty|C_{\max}$ is ordinary. We only need to put all the Jobs in the same batch.

Given that the Jobs are sequenced through adopting the rule of Shortest Processing Time (SPT), namely the Jobs are sequenced according to the non-negative order of processing time, then $P_1 \leq \dots \leq P_n$. The algorithm of unbounded model bases on the following conclusion:

Lemma 1: for the minimized any regular objective function, an optimal scheduling exists B_1, B_2, \dots, B_r , and $B_v = \{j_v, j_v+1, \dots, j_v+1-1\}$, $1=j_1 < j_1 < \dots < j_r < j_r+1=n+1$. (SPT- batch scheduling)

Proving: to consider the optimal scheduling $\sigma: B_1, B_2, \dots, B_r$, and $k \in B_i, i \in B_q, l < q, P_k > P_j$. If Job j is transferred from batch B_q to batch B_l , the length of the two batches will not increase. Because the objective function is regular, the new scheduling is still optimal. The expected optimal scheduling will be obtained through repeating the above process for limited times. The theorem is proved.

2) *Bounded model*: In the bounded model, the optimal scheduling obtained through FBLPT (Full Batch Longest Processing Time) in proving of $B|p\text{-batch}, b < n|C_{\max}$, Lee[3] can be solved in the time of $O(n \log n)$. Given that n is the multiple of b , if that's not the case, virtual Jobs with the processing time of 0 can be added to meet the condition. The following method can be adopted to construct batches: to distribute b Jobs with the shortest processing time to the first batch B_1 , and b Jobs with the second shortest processing time to the second batch B_2 , and so on, until all the Jobs are distributed. It is easily to prove the distribution scheme obtained through this method is optimal.

3) *Scheduling problems of fixed batching*: The scheduling problems of fixed batch means the processing time of a batch on the batching machine is fixed and unrelated to the Job contained in it. It can be regarded as a special situation in the parallel batching model, which is first proposed by Ikura et al. They have given an efficient polynomial optimal algorithm for the minimization of the maximum completion time when dynamic Jobs arrive. Ahmadi et al. have researched the pipeline scheduling problems of two-machine and three-machine that include at least one batching machine, and analyzed four kinds of

optimal algorithm to minimize the maximum completion time under the environment of machine set with “batching machine or single machine”.

B. Scheduling Problems of Serial Batchin

In the scheduling problems of batching machine, it is called the serial batching problems when the processing time of a batch is the sum of the processing time of all Jobs in this batch. The serial batching scheduling has been applied in many industrial production fields. For example, in the production and continuous casting of steel, the continuous casting machine is batching machine. And each continuous casting machine requires the furnace of the same batch to use the same tundish to realize continuous casting. The total processing time of the furnace in this batch is equivalent to the sum of the processing time of Jobs in this batch and this period.

In terms of the scheduling problems of serial batching of single batching machine, Brucker [4] proposes: when the objective function is $\sum \omega_j C_j$, for the sequencing of n Jobs given, the batch setting of its optimal s - batch can be solved within the time of $O(n)$ according to the shortest path problem. For problems of $B|s\text{-batch}|\sum C_j$ and $B|s\text{-batch}, P=p|\sum \omega_j C_j$, the Jobs can be sequenced according to the non-descending order of processing time and weight, then to obtain the optimal scheduling order of the Jobs. The problem of $B|prec, s\text{-batch}, P_j=p|\sum \omega_j C_j$ can be solved through the topological sorting of Jobs and finding the sequence of Jobs that meets the precedence constraint and optimal batching.

VI. CONCLUSION

Scheduling optimization means reasonably allocating machine and resource to task under certain constraint condition in order to optimize the objective function. In the development of production practice in enterprises, because of the diversification of scheduling problems, the basic thought of solving scheduling problems should be: to apply or refer to methods related to the optimization problem of other combinations and take full advantage of the property and available condition of scheduling problems, to determine the optimal sequence that meets the constraint condition.

The decision optimization of scheduling production logistics of manufacturing enterprises makes for better optimizing configuration and taking advantage of enterprise resources, to ensure the organization of production logistics is reasonable and efficient and the production process of enterprises is at the best state. The optimization of production scheduling directly influences the production efficiency and operation cost of enterprises and the degree of being sensitive to customer demand. The scheduling optimization theory has a profound demand of production practice and an extensive application prospect and will be continuously deepened according to the practical development of production.

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