

# *Reflections on Mathematics Core Literacy*

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**Abstract**—The core literacy of the six major mathematics is reflected. Two conclusions are drawn. A new formulation of mathematical core literacy is given. The new definition of core literacy is introduced. The relationship between two kinds of mathematical core literacy formulation is analyzed. Finally, two questions about the definition of the new mathematical core literacy are explained.

**Keywords**—*Mathematical core literacy; Level mathematics; Mathematical problem solving; Reflections*

## I. INTRODUCTION

### A. Research Status of Mathematics Core Literacy in China

Nowadays, core literacy has become a new hotspot in the field of basic education in China. It has become a key factor in deepening the reform of basic education curriculum and implementing the goal of quality education. The connotation of "core literacy" in the field of basic education can be grasped from three levels: the bottom "double-base orientation", with basic knowledge and basic skills as the core; the middle "problem-solving orientation", with the basic methods obtained in the process of solving problems as the core; and the top "scientific (broad) thinking". "Direction" refers to the relatively stable thinking, problem-solving thinking methods and values gradually formed through the process of experience, understanding and internalization in systematic learning. In essence, the world outlook and methodology of understanding and reforming the world are preliminarily obtained.

In 2014, the Ministry of Education issued its Opinions on Fully Deepening the Curriculum Reform and Implementing the Fundamental Task of Establishing Virtue and Fostering People. It calls for "research and formulation of the core literacy system for students' development and academic quality standards". It calls for highlighting the core literacy of disciplines, and defines core literacy as "mainly referring to what students should possess and be able to adapt to Necessary character and key ability for lifelong development and social development." Professor Kong Fanzhe, Professor Ma Yunpeng, Professor Wang Shangzhi and Professor Shi Ningzhong all put forward their own views on the core

literacy of mathematics. Among them, Professor Wang Shangzhi and Professor Shi Ningzhong have the highest recognition.

Professor Wang Shangzhi and Professor Shi Ningzhong divide the core literacy of mathematics into six aspects: mathematical abstraction, logical reasoning, mathematical modeling, visual imagination, mathematical operation and data analysis. Based on these, the Ministry of Education classifies the core literacy of mathematics in Mathematics Curriculum Standards for Senior High Schools (2017 Edition). In view of this classification, there are still some experts who believe that its existence needs to be improved [1]. How does Professor Xiaoya reflect on the six core literacy of mathematics one by one, and gives the revised framework of core literacy of mathematics [2-5].

### B. International Research Status of Mathematics Core Literacy

International research on core literacy is much earlier than domestic research. In Britain, economist Clausser coined the term mathematical literacy in 1956.

Today, the connotation of mathematical literacy can be divided into three levels: fluent use of mathematical knowledge, mathematical reasoning, and application of mathematical knowledge to solve complex problems.

Australia and Finland focus on Applied Mathematics in a variety of situations, so interdisciplinary courses have become a way to cultivate students' mathematical literacy; Denmark and the United States highlight the integration of mathematical capabilities. In the 2012 and 2015 PISA tests, mathematical literacy was defined as "the ability to transform situational problems into mathematical problems, to use mathematics and to interpret mathematics" [6].

## II. REFLECTIONS

Nowadays, we have been discussing "core literacy of mathematics" rather than "mathematical literacy", so what we should pay attention to most is the word "core". "Mathematics core literacy" should be refined enough on the basis of

reflecting the literacy of mathematics discipline. In this way, the definition of mathematics core literacy should be a process of being refined and perfected constantly.

In 2011, "Mathematics Curriculum Standards (Revised Draft)" clearly put forward ten mathematical concepts (Professor Zheng Yuxin preferred to call them the core qualities of mathematics). They are perception, symbol consciousness, operation ability, model idea, space idea, geometric intuition, reasoning ability, data analysis idea, application consciousness and innovation consciousness. In 2017, Mathematics Curriculum Standards for Senior High Schools (2017 Edition) condenses the core qualities of mathematics into mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, mathematical operation and data analysis.

Comparing the ten concepts of mathematics in 2011 with the six core qualities of mathematics in 2017, we can easily find that the core qualities of mathematics abandon the sense of number, innovation and application, which makes people more acceptable.

Conclusion: The six mathematical core literacy still need to be refined in order to embody the word "core".

The international definition of mathematical core literacy emphasizes the integration of mathematical competence and the application of mathematics in various scenarios. These two points are not clearly reflected in the core literacy of mathematics in our country. The six core qualities of mathematics in our country embody the parallel relationship, and should they be improved? Firstly, the "four abilities" are mentioned in the curriculum objective of "General Senior High School Mathematics Curriculum Standards (2017 edition)", which refers to the ability to discover and propose problems, analyze and solve problems from the perspective of mathematics. The goal of senior high school curriculum is to discover and solve problems in mathematics, which requires the integration of various abilities. Secondly, mathematics comes from reality. We should pay attention to the relationship between mathematics and the real world and the application of mathematics in various situations. Only in this way can we make mathematics have vitality and let students stop asking "What's the use of learning mathematics?"

Conclusion: The core literacy of mathematics should emphasize the integration of ability and the relationship between mathematics and the real world.

### III. A NEW APPROACH TO MATHEMATICAL CORE LITERACY

To sum up, we should pay attention to the following three points: first, we should refine enough. Secondly, we should emphasize the integration of core literacy. Thirdly, we should connect mathematics with the real world. Secondly, the Ministry of Education defines core literacy as: mainly refers to the essential qualities and key abilities that students should possess to meet the needs of lifelong development and social development. Therefore, it is more appropriate to divide the core mathematic literacy into two aspects:

Level mathematicization and mathematical problem solving, which are more refined than the six core mathematic literacy in terms of classification. Level mathematic and mathematical problem solving are more in line with PISA's definition of mathematical literacy, and have a more international perspective. At the same time, level mathematic and mathematical problem solving can better solve problems, so that learners can adapt to life-long development and social development.

#### A. *Level mathematically*

Mathematicalization is the process of sorting out reality, including all the organizational activities of mathematicians, where reality refers to the combination of the real world and the mathematical world [7,8].

Freudenthal divides Mathematicalization into two kinds: one is horizontal mathematicalization, that is, to discover the mathematical components in the problem and to deal with them accordingly. The other is vertical mathematicalization, i.e. formalizing the conformed mathematical materials within the mathematical scope. A complete mathematical thinking activity must include two stages: horizontal and vertical. The process of level mathematicization is the process of linking the "real world" with the "mathematical world", which emphasizes the connection between mathematics and the real world.

The goal of high school mathematics curriculum is to find and put forward the ability of problem solving, analysis and problem solving from the perspective of mathematics. After horizontal mathematicalization, the real problem has become a mathematical problem, so the problem we need to solve is mathematical problem solving, which is one of the reasons why I divide the core quality of mathematics into horizontal Mathematicalization and mathematical problem solving.

It is self-evident that mathematics problem solving plays an important role in mathematics teaching. Professor Luo Zengru believes that:

1. Problem solving is the core content of mathematics learning;
2. Problem solving is the basic way to master mathematics and learn to "think mathematically";
3. Problem solving is an important way to evaluate learning. Mathematician Polia believes that mastering mathematics means being good at solving problems.

In the teaching process of high school mathematics, we spend most of our time dealing with mathematical problem solving. Therefore, it is appropriate to regard mathematical problem solving as one of the core qualities of mathematics.

#### B. *Connotation of Mathematical Problem Solving*

Polia, a mathematician, divides mathematical problem solving into four parts: problem solving, plan formulation, implementation plan and review. The retrospective part is the least we can ignore. It includes four questions.

1. Can you verify this conclusion?

2. Can you derive this result in other ways? Can you see it all at once?

Can you apply this result to other problems? Review is the main difference between mathematical problem solving and mechanical problem solving.

Mathematical problem solving ability is not limited to mathematical problems. In *How to Solve a Problem*, Polia mentioned that the skills in "How to Solve a Problem Table" are not limited to any topic. Topics can be algebraic or geometric, mathematical or non-mathematical, theoretical or practical, all of which make no difference. These skills can help us solve problems.

In the process of solving mathematical problems, we need to apply the abilities of logical reasoning, mathematical modeling, intuitive imagination, mathematical operation and so on. Therefore, the core accomplishment of solving mathematical problems is not a complete negation of the original core accomplishment of mathematics, but a comprehensive negation of the original core accomplishment of mathematics on the basis of it. Integration, emphasizing the links between various capabilities, emphasizing the purpose of problem solving.

Review the whole process of solving problems. First of all, we turn the real world problem into a mathematical problem, which is a level mathematic problem. Secondly, we have solved this mathematical problem, that is, the process of solving mathematical problems.

### *C. Main Thoughts and Methods of Mathematical Problem Solving*

There are many ways of thinking in mathematics. Professor Gu Lingyuan mainly mentions abstraction and generalization, conjecture and refutation, deduction and transformation, calculation and algorithm, application and modeling, classification, combination of numbers and figures, and specialization in his book *Mathematical Thought and Method*. In the college entrance examination, we mainly study the thought of function and equation, the thought of combining numbers and figures, the thought of classifying and discussing, the thought of transformation and transformation [9]. *Limit Thought is Essential in Higher Mathematics and Mathematics Analysis and Higher Mathematics*

### IV. THE RELATIONSHIP BETWEEN TWO DEFINITIONS OF MATHEMATICAL CORE LITERACY

Level mathematicization and mathematical problem solving is a sequential relationship, and the six core mathematic literacy is a juxtaposition relationship. Mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, mathematical operations, data analysis exist in the process of Mathematicalization and mathematical problem solving. Mathematical abstraction is a part of mathematicalization. Data analysis is reflected in horizontal Mathematicalization and mathematical problem solving. Logical reasoning, mathematical modeling, visual

imagination and mathematical operation are all in the process of mathematical problem solving.

So there is a connection between the two definitions of mathematical core literacy.

Professor Wang Shangzhi and Professor Shi Ningzhong put forward six aspects of mathematical core literacy: mathematical abstraction, logical reasoning, mathematical modeling, visual imagination, mathematical operations, data analysis. The core qualities of mathematics put forward in this paper are horizontal mathematicization and mathematical problem solving.

The main differences between the two are as follows: first, mathematical abstraction is one of horizontal mathematicization, which includes abstracting general rules and structures from the concrete background of things, but lacks another more basic mathematical model, which includes abstracting quantities, figures and concepts from the real world. Secondly, mathematical abstraction includes the abstraction of mathematical concepts and concepts from the relationship between quantity and quantity, graphics and graphics. This is the process of mathematical internalization. This process belongs to vertical Mathematicalization and is not possessed by horizontal mathematicization, but it will be reflected in mathematical problem solving. Thirdly, the focus of mathematical problem solving is problem solving, and the process of problem solving includes various mathematical thinking methods. Let's review the definition of core literacy: the essential character and key ability to adapt to life-long development and social development. The six core qualities of mathematics lay stress on the key ability, the essential character and the key ability of solving mathematical problems as well as the mathematical thought contained in them. By contrast, mathematical problem solving and its ideas are more suitable for solving problems, dealing with the intersection of mathematics and other disciplines, and adapting learners to life-long development and social development.

### V. TWO QUESTIONS ON MATHEMATICS CORE LITERACY

Social development has put forward new requirements for talent cultivation. Cultivating talents with innovative consciousness and critical thinking, interdisciplinary perspective to solve problems, team work and other core qualities has become the core focus of education reform in various countries. Therefore, the reform of world basic education curriculum with core literacy as its orientation has become the general trend. The research on the core qualities of students' development in basic education and higher education in China holds that learners should have the key ability and essential character to solve problems comprehensively using knowledge, skills and attitudes in specific situations. It is worth noting that most educational changes are not result-oriented events, but need to undergo a difficult process of transformation. Under the situation of unbalanced development of education, how to face the practical problems existing in classroom teaching practice, explore the development direction and practical path of teaching reform, and actively promote the cultivation of core literacy have

become the key issues of classroom teaching reform in primary and secondary schools [10].

#### Two Questions about Mathematics Core Literacy

Question 1: Since a complete mathematical activity necessarily includes two processes of horizontal and vertical mathematicalization, why not define the core mathematic literacy directly as horizontal and vertical mathematic literacy?

Senior high school students have a certain foundation for mathematics knowledge. They need to realize that mathematics comes from reality and solves practical problems. Therefore, it is reasonable to regard level mathematic literacy as the core literacy of mathematics. At the same time, we should note that vertical mathematicization contains too many abilities, not every student should become a mathematician. Mathematicalization is the whole organizational activity of mathematicians, but not every learner has to become a mathematician, so it is inappropriate to regard vertical Mathematicization as one of the core qualities of mathematics.

Question 2: What are the advantages of the new formulation of mathematical core literacy?

There are four advantages of the new mathematical core literacy. First, the new formulation is "core". Second, the new formulation emphasizes the integration of capabilities. Thirdly, the new formulation strengthens the connection with the real world. Fourthly, the new formulation is more clear, the level of mathematics and mathematical problem solving is a sequential relationship, more in line with people's habit of solving problems.

#### VI. CONCLUSION

This paper expounds the connotation of the new formulation of core literacy, analyses the core literacy of mathematics, and explains two problems about the new formulation of core literacy of mathematics.

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